CONCURRENT CONSTANT MODULUS ALGORITHM AND DECISION DIRECTED SCHEME FOR SYNCHRONOUS DS-CDMA EQUALISATION

Mahmoud Hadef and Stephan Weiss

School of Electronics & Computer Science
University of Southampton, Southampton SO17 1BJ, UK
{`m.hadef,s.weiss`}`@ecs.soton.ac.uk`

ABSTRACT

In this paper, we derive a concurrent constant modulus algorithm (CMA) and decision directed (DD) scheme for blind multiuser equalisation, suitable for downlink DS-CDMA systems. Adaptation is performed by concurrently minimising two cost functions based on either a CM criterion or a DD scheme for all active users. Computer simulations are used to assess the performance of the algorithm.

Keywords: constant modulus algorithm, decision directed adaptation, downlink, DS-CDMA system, FIRMER-CMA.

1. INTRODUCTION

In a DS-CDMA downlink scenario, transmission over a dispersive channel destroys the mutual orthogonality of the codes which are used to multiplex the various users in the system. As a result, the received code-demultiplexed user signals are subject not only to inter-symbol interference (ISI) due to channel dispersion but also to multiple access interference (MAI) due to the loss of code orthogonality. Consequently the conventional DS-CDMA code-matched filter receiver suffers sever performance degradation, thus motivating the need for better detection strategies.

There have been widespread investigations and extensive research efforts to introduce a reliable multiuser detector [1], apt to alleviate the effect of both MAI and ISI. While the maximum likelihood (ML) estimation based detector offers the best possible performance [1], its unrealistic complexity renders it unsuitable for downlink applications due to handset constraints. Alternatively, sub-optimal schemes such as multiuser equaliser offer moderate complexity and generally good performance. Commonly proposed multiuser equalisers [2, 3] are often based on training adaptation. These algorithms necessitate the implementation of training sequences or pilot signals which can be considered as a waste of the available bandwidth. A novel class of multiuser equalisers, equally referred to as blind detectors, do not require explicit knowledge of training sequence and channel parameters.

Various blind equalisation techniques, which can simultaneously suppress both MAI and ISI and improve bandwidth efficiency, have been proposed [4, 5, 6, 10]. The constant modulus algorithm (CMA) [7, 8] based multiuser equaliser is by far the most popular scheme. It has a very simple computational requirement and readily meets the real-time computational constraint. In [5, 6] self-recovering techniques have been performed using the CM criterion, whereby additional orthogonality constraints or mutual deccorelation of the recovered user sequence are required. Alternatively, in [10] a blind scheme, the so called filtered-R multiple error CM algorithm (FIRMER-CMA) has been developed, which is similar to [5, 6] but requires neither constraint nor mutual decorrelation are needed. However, since FIRMER-CMA is based on the CM criterion, it is prone to achieve only moderate levels of mean square error (MSE) after convergence, which may not be sufficiently low for the system to attain adequate bit error rate (BER) performance. A possible solution to the latter problem is to switch to a decision-directed (DD) mode in order to minimise the residual CMA steady state MSE. In order to avoid error propagation due to incorrect decisions, the CMA residual MSE should be sufficiently low. In practice such a low level of MSE may not always be achievable by the CMA [12, 9]. Consequently, a promising solution, suitable for single user transmission, has been proposed in [12]. Whereby, a DD equaliser is concurrently operating with CMA rather than switching to a DD adaptation after the CMA has converged. This concurrent CMA+DD equaliser is reported to achieve a significant enhancement in equalisation performance over the CMA [12].

In this paper, a concurrent FIRMER-CMA+DD algorithm is derived, which is similar to [12] but suitable for synchronous DS-CDMA systems. Based on the definition of a signal model in Sec. 2, two suitable multiuser CM and DD cost functions are discussed in Sec. 3. In Sec. 4 we derive the multiuser concurrent FIRMER-CMA+DD algorithm. Simulations of the proposed algorithm are presented in Sec. 5, and conclusions drawn in Sec. 6.

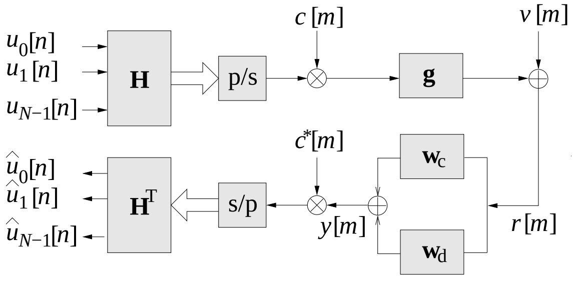

Fig. 1. DS-CDMA downlink signal model.

2. SIGNAL MODEL

We consider the DS-CDMA downlink system in Fig. 1 with multiple symbol-synchronous users, which for simplicity are assumed to have the same rate. The system is fully loaded with N user signals $u_l[n]$, $l = 0(1)N-1$, which are code multiplexed using Walsh sequences of length N extracted from a Hadamard matrix $\mathbf{H}$. The resulting chip rate signal, running at N times the symbol rate, is further scrambled by $c[m]$ prior to transmission over a channel with dispersive impulse response $g[m]$ and corrupted by additive white Gaussian noise $v[m]$, which is assumed to be independent of the transmitted signal.

The dispersive channel $g[m]$ destroys the orthogonality of the Walsh codes, such that direct decoding of the received signal $r[m]$ with descrambling by $c^*[m]$ and code-matched filtering by $\mathbf{H}^{\mathrm{T}}$ will lead to MAI and ISI corruption of the decoded user signals $\hat{u}_l[n]$, $l = 0(1)N-1$. In order to re-establish orthogonality of the codes, a chip rate equaliser $\mathbf{w}$ can be utilised [14, 13]. The equaliser $\mathbf{w}$ consists of a CMA equaliser $\mathbf{w}_c$ and a DD equaliser $\mathbf{w}_d$ operated in parallel, such that $\mathbf{w} = \mathbf{w}_c + \mathbf{w}_d$. Note that $\mathbf{w}$ contains the coefficients of the equalised, which is assumed to be a finite impulse response filter. In the following, we are concerned with the blind updating of $\mathbf{w}$.

3. MULTIUSER EQUALISATION CRITERION

We first derive the detected users' signals $\hat{u}_l[n]$ as a function of the chip-rate equaliser $\mathbf{w}$. Based on this, we state a suitable cost function on which the equaliser adaptation relies.

3.1. Demultiplexed User Signals

For the decoding, Walsh sequences are used as matched filters. The sequence for decoding the lth user, contained in a vector $\mathbf{h}_l$, can be taken from an $N \times N$ Hadamard matrix,

$$\mathbf{H}^{\mathrm{T}} = \left[\mathbf{h}_0 \;\; \mathbf{h}_1 \;\; \cdots \;\; \mathbf{h}_{N-1}\right]^{\mathrm{T}} . \tag{1}$$

The lth user is thus decoded as

$$\begin{aligned}
\hat{u}_l[n] &= \mathbf{h}_l^{\mathrm{T}} \cdot \begin{bmatrix} c^*[nN] & & & \mathbf{0} \\ & c^*[nN-1] & & \\ & & \ddots & \\ \mathbf{0} & & & c^*[nN-N+1] \end{bmatrix} \cdot \begin{bmatrix} y[nN] \\ y[nN-1] \\ \vdots \\ y[nN-N+1] \end{bmatrix} \\
&= \tilde{\mathbf{h}}_l^{\mathrm{T}}[nN] \cdot \begin{bmatrix} \mathbf{w}^{\mathrm{H}} & & & \mathbf{0} \\ & \mathbf{w}^{\mathrm{H}} & & \\ & & \ddots & \\ \mathbf{0} & & & \mathbf{w}^{\mathrm{H}} \end{bmatrix} \cdot \begin{bmatrix} r[nN] \\ r[nN-1] \\ \vdots \\ r[nN-L-N+2] \end{bmatrix}
\end{aligned}$$

whereby the descrambling code $c^*[m]$ has been absorbed into a modified and now time-varying code vector $\tilde{\mathbf{h}}_l[nN]$, and $\mathbf{w} \in \mathbb{C}^L$ contains the equaliser's L chip-spaced complex conjugate weights. Rearranging $\mathbf{w}$ and $\tilde{\mathbf{h}}_l[nN]$ yields

$$\begin{aligned}
\hat{u}_l[n] &= \mathbf{w}^{\mathrm{H}} \cdot \begin{bmatrix} \tilde{\mathbf{h}}_l^{\mathrm{T}}[nN] & & & \mathbf{0} \\ & \tilde{\mathbf{h}}_l^{\mathrm{T}}[nN] & & \\ & & \ddots & \\ \mathbf{0} & & & \tilde{\mathbf{h}}_l^{\mathrm{T}}[nN] \end{bmatrix} \cdot \begin{bmatrix} r[nN] \\ r[nN-1] \\ \vdots \\ r[nN-L-N+2] \end{bmatrix} \\
&= \mathbf{w}^{\mathrm{H}} \, \mathbf{H}_l[nN] \, \mathbf{r}_{nN} ,
\end{aligned} \tag{2}$$

with $\mathbf{H}_l[nN] \in \mathbb{Z}^{L \times (N+L-1)}$ being a convolutional matrix comprising the lth user's modified code vector $\tilde{\mathbf{h}}^{\mathrm{T}}[n]$ and $\mathbf{r}_{nN} \in \mathbb{C}^{N+L-1}$.

3.2. Cost Functions

The equaliser $\mathbf{w}$ consists of a CMA equaliser $\mathbf{w}_c$ and a DD equaliser $\mathbf{w}_d$ operating concurrently to minimise two cost functions ξ_c and ξ_d. We assume that the user signals $u_l[n]$, $l = 0(1)K-1$, consist of symbols with a constant modulus γ, such as BPSK, QPSK, or 8-PAM. Therefore, by forcing all decoded users $\hat{u}_l[n]$ onto a constant modulus γ the cost function ξ_c could be written as

$$\xi_c = \mathcal{E}\left\{ \sum_{l=0}^{N-1} (\gamma^2 - |\hat{u}_l[n]|^2)^2 \right\} , \tag{3}$$

where $\mathcal{E}\{\cdot\}$ denotes the expectation operator. The optimum equaliser coefficient vector w_c is therefore given by

$$\mathbf{w}_{c,\mathrm{opt}} = \arg\min_{\mathbf{w}_c} \xi_c \quad . \tag{4}$$

By employing a non-linearity $q(\cdot)$ that maps its input onto the the closest constellation alphabet, the multiuser decision directed cost function ξ_d could be formulated as

$$\xi_d = \mathcal{E}\left\{ \sum_{l=0}^{N-1} |q(\hat{u}_l[n]) - \hat{u}_l[n])|^2 \right\} , \tag{5}$$

whereby the optimum equaliser coefficient vector w_d is obtained from

$$\mathbf{w}_{d,\mathrm{opt}} = \arg\min_{\mathbf{w}_d} \xi_d \quad . \tag{6}$$

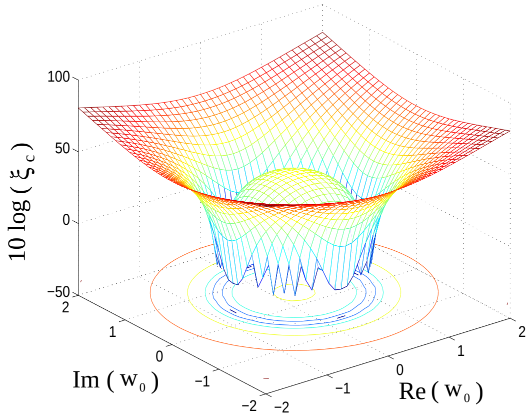

Fig. 2. Cost function ξ_c in dependency of a single complex valued coefficient w_0.

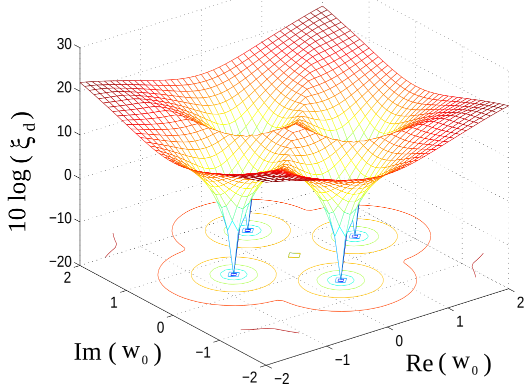

Fig. 3. Cost function ξ_d in dependency of a single complex valued coefficient w_0.

There are no unique solutions to either (4) or (6), since minimising (3) or (5) is ambiguous due to an indeterminism in phase rotation. Also, note that erroneous decisions are possible in (5) and therefore affect (5).

Example. In this example the two cost functions ξ_c and ξ_d are plotted in Figs. 2 and 3 respectively, in dependency of an equaliser $\mathbf{w}$ with a single complex coefficient w_0. The system adopted here is a fully loaded DS-CDMA system with $N = 16$ users transmitting their signals over a distortionless and delayless channel with $SNR = 30dB$. The modulation scheme employed here is QPSK with $\gamma = 1$. Fig. 2 shows that ξ_c exhibits a manifold of optimum solutions satisfying $|w_0| = \gamma$. Yet, only four solutions can be seen in ξ_d due to the four possible QPSK decisions.

4. CONCURRENT ADAPTATION

In this section, we derive the concurrent FIRMER-CMA+DD algorithm which updates the multiuser equaliser vector $\mathbf{w}$, similarly to the concurrent CMA+DD described in [12] for single user systems. The main idea is to update the CMA part $\mathbf{w}_c$, which is followed by a DD adaptation step only if the previous CMA adaptation step is deemed successful. First, it is convenient to define

$$\mathbf{x}_l[n] = \mathbf{H}_l[nN]\mathbf{r}_{nN} . \tag{7}$$

The proposed algorithm, which is updated at the symbol rate with symbol time index N, can be described by the following steps:

1. The decoded signals $\hat{u}_l[n]$ are calculated for all users according to

   $$\hat{u}_l[n] = \mathbf{w}_c^{\mathrm{H}}[n]\mathbf{x}_l[n] + \mathbf{w}_d^{\mathrm{H}}[n]\mathbf{x}_l[n], \tag{8}$$

   for $l = 0(1)N-1$.

2. The CMA part $\mathbf{w}_c$ is adapted according to the rule

   $$\mathbf{w}_c[n+1] = \mathbf{w}_c[n] + \mu_c \sum_{l=0}^{N-1} e_{l,c}^{*}\mathbf{x}_l[n] \tag{9}$$

   where $e_{l,c} = \hat{u}_l[n](\gamma^2 - |\hat{u}_l[n]|^2)$ and μ_c is the CMA step size. This stochastic gradient algorithm, identical to FIRMER, is based on optimising an instantaneous cost function derived from (3) by dropping expectations [10].

3. Intermediate signals $\tilde{u}_l[n]$, $l = 0(1)N-1$, are evaluated by exploiting the previously calculated $\mathbf{w}_c[n+1]$, suc that

   $$\tilde{u}_l[n] = \mathbf{w}_c^{\mathrm{H}}[n+1]\mathbf{x}_l[n] + \mathbf{w}_d^{\mathrm{H}}[n]\mathbf{x}_l[n] . \tag{10}$$

4. Finally, the DD part of the algorithm adjusts $\mathbf{w}_d$ as

   $$\begin{aligned}\mathbf{w}_d[n+1] = \mathbf{w}_d[n] \quad & + \\ + \mu_d \sum_{l=0}^{N-1} \delta(q(\tilde{u}_l[n]) - q(\hat{u}_l[n]))\, & e_{l,d}^{*}\mathbf{x}_l[n] ,\end{aligned} \tag{11}$$

   where $e_{d,c} = (q(\hat{u}_l[n]) - \hat{u}_l[n])$, and μ_d is the DD step size. The indicator $\delta(\cdot)$ is defined as

   $$\delta(\alpha) = \begin{cases} 1 & \text{if} \quad \alpha = 0 \\ 0 & \text{if} \quad \alpha \neq 0 \end{cases} , \tag{12}$$

   and therefore disables the DD adaptation step for a specific user if the CMA adaptation step leads was to alter the decision.

The convergence of this concurrent scheme is governed by the step sizes in the algorithm. In practise, the DD step size μ_d can be often be chosen much larger than the CMA step size μ_c. However, choosing too large values can cause

	Concurrent FIRMER-CMA+DD Algorithm
1:	update $\mathbf{x}_l[n] = \mathbf{H}_l[nN]\mathbf{r}_{nN}$, for $l = 0(1)N-1$
2:	$\hat{u}_l[n] = \mathbf{w}^{\mathrm{H}}[n]\mathbf{x}_l[n]$, for $l = 0(1)N-1$
3:	$e_{l,c} = \hat{u}_l[n](\gamma^2 - \|\hat{u}_l[n]\|^2)$, $l = 0(1)N-1$
4:	$\mathbf{w}_c[n+1] = \mathbf{w}_c[n] + \mu_c \sum_{l=0}^{N-1} e^*_{l,c}\mathbf{x}_l[n]$
5:	$\tilde{u}_l[n] = \mathbf{w}_c^{\mathrm{H}}[n+1]\mathbf{x}_l[n] + \mathbf{w}_d^{\mathrm{H}}[n]\mathbf{x}_l[n]$
6:	$e_{l,d} = (q(\hat{u}_l[n]) - \hat{u}_l[n])$, for $l = 0(1)N-1$,
7:	$\mathbf{w}_d[n+1] = \mathbf{w}_d[n] +$
	$+\mu_d \sum_{l=0}^{N-1} \delta(q(\tilde{u}_l[n]) - q(\hat{u}_l[n]))e^*_{l,d}\mathbf{x}_l[n]$
8:	$\mathbf{w}[n+1] = \mathbf{w}_c[n+1] + \mathbf{w}_d[n+1]$

Table 1. Concurrent FIRMER-CMA+DD multiuser equalisation algorithm.

serious error propagation due to incorrect decisions. Tab. 1 summarises main equations of the proposed concurrent FIRMER-CMA+DD.

The potential drawback of DD adaptation is that if the hard decision is incorrect, error propagation occurs which subsequently degrades the equaliser performance. It was shown that if the equaliser hard decisions before and after the CMA adaptation are the same then the decision is likely to be correct [9]. For this reason, the $\mathbf{w}_d$ is only updated in the latter case, similar to [12]. Hence, by employing the concurrent FIRMER-CMA+DD, a considerably enhanced convergence speed and a lower steady state MSE can be achieved compared to the standard FIRMER-CMA.

5. SIMULATION RESULTS

For the simulations below, we apply the proposed concurrent FIRMER-CMA+DD to two different stationary channel impulse responses, a short $g_1[m]$ and a more dispersive $g_2[m]$, as characterised by the zeros of their transfer functions in Fig. 4. As can be seen in Fig. 4, the non-minimum phase nature and zero positions close to the unit circle render $g_2[m]$ a difficult task for equalisation. In the following, we first demonstrate the convergence behaviour and the steady state MSE over noise-free channels, and thereafter characterise the evolution of the received costellation in a noisy environment.

Experiment 1: In order to demonstrate the convergence behaviour of the proposed algorithm, we transmit $N = 16$ QPSK user signals over $g_1[m]$ in the absence of channel noise. The length of the equaliser is $L = 10$, and the relaxation factors are chosen to be $\mu_c = 10^{-4}$ and $\mu_d = 10^{-2}$. The adaptation is initialised with the fifth coefficient in the weight vector set to unity. The MSE performances of the proposed concurrent FIRMER-CMA+DD and the standard FIRMER-CMA algorithms are shown in Fig. 5. Evidently a faster convergence and lower steady state can be achieved by the proposed concurrent algorithm compared to an adaptation based on FIRMER-CMA.

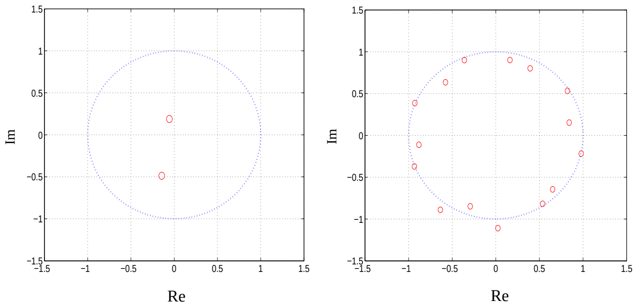

Fig. 4. Zeros locations of the channels (left) $g_1[m]$ and (right) $g_2[m]$.

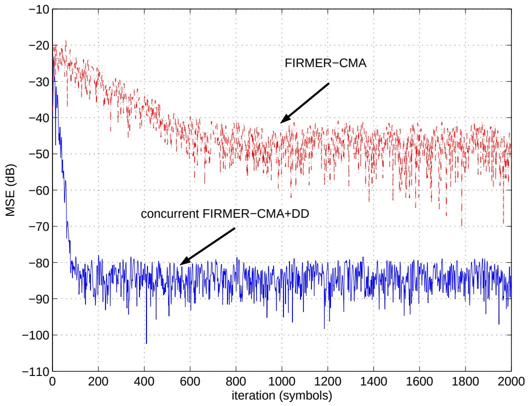

Fig. 5. Comparison of convergence speed and steady state MSE between the proposed concurrent FIRMER-CMA+DD algorithm and the standard FIRMER-CMA.

Experiment 2: For $N = 16$ QPSK users, we have adapted the concurrent FIRMER-CMA under SNR=10dB over the dipersive channel $g_2[m]$. The length of the equaliser is $L = 64$, and the relaxation factors are chosen to be $\mu_c = 10^{-6}$ and $\mu_d = 10^{-4}$. Fig. 6 depicts the decoded signal constellations of user $l = 0$ after adaptation of 10^4 symbols with (a) no equalisation performed (b) a standard FIRMER-CMA equaliser and (c) the concurrent FIRMER-CMA+DD. The results clearly show that the concurrent algorithm achieves to open the initially closed eye and to overcome the phase ambiguity encountered in the CMA scheme by locking onto the constellation pattern prescribed by $q(\cdot)$.

6. CONCLUSIONS

A concurrent FIRMER-CMA+DD algorithm for blind multiuser equalisation suitable for a DS-CDMA downlink scenario has been derived. The proposed algorithm combines advantages of both the CMA and DD adaptation and enables faster convergence and lower steady state mean square error compared to the standard FIRMER-CMA approach. Furthermore, the algorithm can mitigate phase found in FIRMER-

CMA case by locking the solution onto the prescribed constellation pattern.

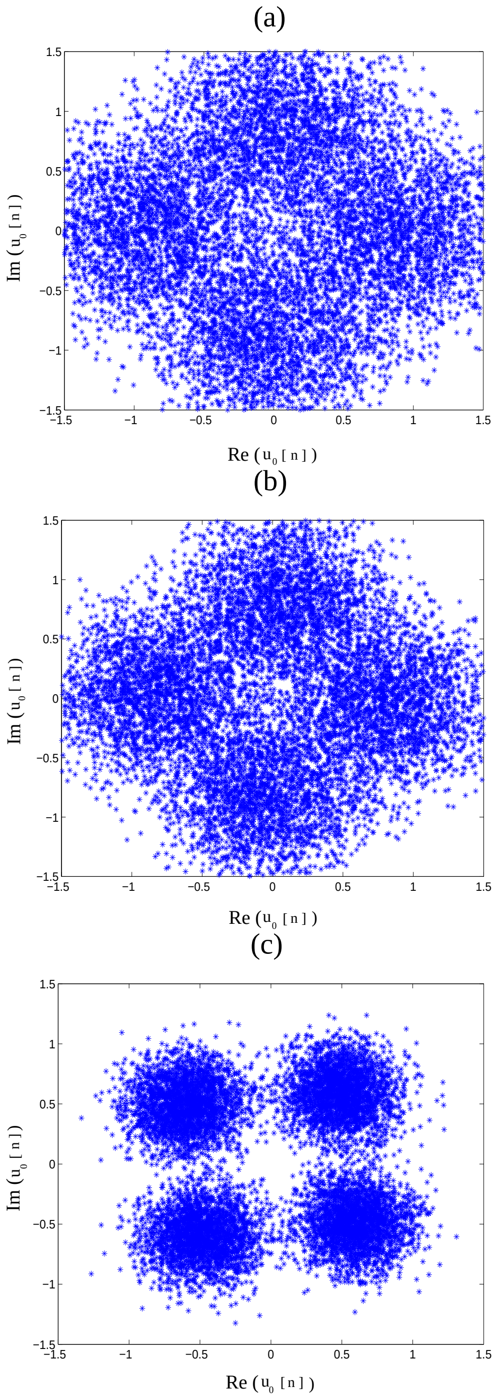

Fig. 6. The decoded signal constellations of user $l = 0$ after adaptation of 10^4 symbols with (a) no equalisation (b) standard FIRMER-CMA equaliser and (c) consurrent FIRMER-CMA+DD.

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