IMPROVED SYNCHRONISATION FOR SUPERIMPOSED TRAINING BASED CHANNEL ESTIMATION

E. Alameda-Hernandez, D. C. McLernon, M. Ghogho,*

The University of Leeds
School of Electronic and Electrical Engineering
LS2 9JT, Leeds, UK
{eenea,d.c.mclernon,m.ghogho}@leeds.ac.uk

A. G. Orozco-Lugo and M. Lara

CINVESTAV-IPN
Sección de Comunicaciones
C.P.07360, México City, México
{aorozco,mlara}@cinvestav.mx

ABSTRACT

This paper introduces a synchronisation method for superimposed training (ST) based channel estimation, using periodic ST sequences. The method exploits the particular structure, occurring when the ST sequence period is larger than the channel length, of the vector containing the received signal's first-order cyclostationary statistics. After synchronisation, any DC-offset can be removed and an unbiased channel estimate can be obtained. Necessary and sufficient conditions for synchronisation are provided. The problem of training sequence design for an improved synchronisation is also addressed. An expression for the variance of the channel estimate is obtained as well, assuming perfect synchronisation and using the designed training sequences. The proposed synchronisation method is computationally more efficient than existing methods, and yet its performance, in term of channel estimation MSE and BER, is not diminished as shown by simulations.

1. INTRODUCTION

Time-division multiplexed training (TDMT)-based methods are known to provide good channel estimates, but a loss in bandwidth is unavoidable. An alternative to TDMT is superimposed training (ST), where the training sequence is actually added to the data sequence. Note that the ST method may receive another names [1, 2]. Unlike TDMT, ST saves bandwidth at the expense of a small data power loss.

In order to simplify channel estimation, the ST sequence is chosen to be periodic, as in [3–8]. This periodicity induces cyclostationarity in the received sequence's first-order statistics. From these cyclic statistics, and knowledge of the training sequence, the channel can be extracted. In order for the cyclic mean to be unambiguously related to the channel, training sequence synchronisation (TSS) is required at the receiver.

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TSS was first addressed, together with RF receiver DC-offset removal, in [3]. The method in [3] is based on polynomial rooting and higher-order statistics, and assumed that the period of the ST sequence, P, equals the number of channel taps, M. A computationally simpler method based on first-order statistics and the DFT was presented in [5], where P was chosen to satisfy $P \geq 2M + 1$.

In this paper, we present a new method for TSS that shows a reduced computational burden when compared with the methods in [3, 5]. This new method for TSS is based on the particular structure of the vector containing the received signal's cyclic mean. It is not difficult to show that the cyclic mean vector contains all the information needed to solve the problem: information for synchronisation, DC-offset removal and channel determination. Assuming random channels with independent taps, we prove that to improve the performance (synchronisation capabilities and hence channel estimation) of the proposed method, the ST sequence should be chosen such that its DFT entries have a constant magnitude. Constant magnitude DFT entries ST sequences have been used as well, although for different reasons, in [3, 6]. An explicit expression for the channel estimation variance is provided when using these particular ST sequences. This expression for the variance shows for the first time in the ST scheme the effect of the DC-offset removal in the estimation of the channel estimate. Despite the computational burden reduction with respect to [3,5], the proposed method shows no performance worsening compared to them as simulations indicate.

This paper is organised as follows. The TSS problem in ST based channel estimation is presented in section 2. A geometric interpretation of the TTS problem allows for the new TSS method to be proposed in section 3. Then, the implementation of the new method is given in section 4. The performance of the new method is improved in section 5 through the appropiate design of the training sequence. Section 6 illustrates the performance of the proposed method. Finally, section 7 contains the conclusions.

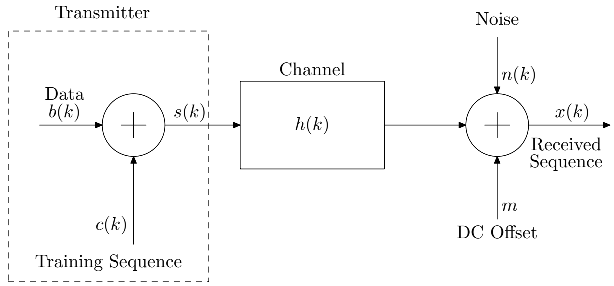

Fig. 1. Diagram of the communications system designed so that the ST method could be applied.

Notation: Boldface uppercase and lowercase, denote matrices and vectors respectively. Vectors are assumed to be column vectors. The superscripts 'T' and 'H' denote respectively the transpose and conjugate transpose of a matrix or vector. For any matrix $\mathbf{A}$, then $\mathbf{A}^{[L]_c}$ and $\mathbf{A}_{[L]_c}$ correspond respectively to its first and last L columns. Furthermore, $\mathbf{A}^{[L]_r}$ and $\mathbf{A}_{[L]_r}$ respectively denote its first and last L *rows*. For any vector $\mathbf{v}$, $\mathbf{v}^{[L]}$ and $\mathbf{v}_{[L]}$ correspond respectively to its first and last L elements. $\mathbf{1}_{L\times Q}$ and $\mathbf{0}_{L\times Q}$ correspond respectively to $L \times Q$ matrices of ones and zeros. Finally, $\mathbf{I}_P$ is the $P \times P$ identity matrix.

2. PROBLEM DESCRIPTION

Consider the complex, lowpass equivalent digital communications system prepared for ST depicted in Fig. 1. Accordingly, the received data block has the following form [3,4,6]:

$$x(k) = \sum_{l=0}^{M-1} h(l)\left[b(k-\tau-l) + c(k-\tau-l)\right] + n(k) + m \tag{1}$$

where $k = 0, 1, \ldots, N-1$, $b(k)$ is the information-bearing sequence of power σ_b^2, $h(k)$ is the channel impulse response, $n(k)$ is the noise of power σ_n^2, m represents an unknown DC-offset term due to using first-order statistics with non-ideal RF receivers (see [3]), $c(k)$ is a known sequence with mean $\bar{c} = \frac{1}{P}\sum_{k=0}^{P-1} c(k)$ and power $\sigma_c^2 = \frac{1}{P}\sum_{k=0}^{P-1} |c(k)|^2$, which is taken to be periodic with period $P \geq M$, and τ is the time-offset (in number of samples) between the transmitter and the receiver. Since channel estimation is based on the first-order statistics of the receive signal, which is periodic in P, it is sufficient to limit the values of τ to the interval $(-\frac{P}{2}, \frac{P}{2}]$. All terms can be complex-valued.

In the analysis to follow, we will make the following assumptions:

A1) $b(k)$ is a zero mean independent and identically distributed (i.i.d.) random process. Likewise, $n(k)$ is an i.i.d. random process with zero mean. Furthermore, $b(k)$ and $n(k)$ are statistically independent of each other.

A2) The channel is of order $M-1$; i.e. $h(0) \neq 0$ and $h(M-1) \neq 0$.

A3) The channel order is known.

A4) The training sequence $c(k)$ is chosen so that the matrix $\mathbf{C} = \mathrm{circ}(c(0),\ c(P-1),\ c(P-2),\ \ldots,\ c(1))$, where the operation 'circ' produces a circulant matrix [9], is full rank.

A5) $P \geq 2M+1$. This will be known as the *strong* constraint.

A6) The channel taps are independent random variables.

Under A1 and the periodicity of $c(k)$, the mean of the output sequence $x(k)$ is periodic as well with the same period P. This cyclic mean is completely determined by

$$y(j) := \mathrm{E}[x(iP+j)] = \sum_{l=0}^{M-1} h(l)c(j-\tau-l) + m \tag{2}$$

with $j = 0, \ldots, P-1$. Knowing the training sequence $c(k)$ and $\mathbf{y} = [y(0),\ y(1),\ \ldots,\ y(P-1)]^T$, we show in the next section that the channel $h(k)$ can be computed even if m and τ are unknown.

3. A CIRCULAR GEOMETRY APPROACH

For convenience in later algebraic derivations, let us write (2) in matrix form:

$$\mathbf{y} = \mathbf{C}_\tau^{[M]_c}\mathbf{h} + m\mathbf{1}_{P\times 1} \tag{3}$$

where $\mathbf{h} = [h(0),\ h(1),\ \ldots,\ h(M-1)]^T$ and $\mathbf{C}_\tau = \mathrm{circ}(c(-\tau),\ c(-\tau-1),\ c(-\tau-2)\ldots,\ c(-\tau+1))$. Note that $\mathbf{C}_\tau$ is obtained by cyclically permuting the columns or rows of $\mathbf{C}$ by τ; hence, $\mathbf{C}_\tau$ is also full-rank under assumption A4.

Now, we would like to isolate the terms in (3) related to the channel $\mathbf{h}$ and the DC-offset m, respect to the synchronisation offset τ. So first let us rewrite (3) as

$$\mathbf{y} = \mathbf{C}_\tau\left([\mathbf{h}^{\mathrm{T}}\ \mathbf{0}_{1\times(P-M)}]^{\mathrm{T}} + \tilde{m}\mathbf{1}_{P\times 1}\right) \tag{4}$$

where $\tilde{m} = \frac{m}{P\bar{c}}$. Then, from (4) it follows that

$$\mathbf{C}_\tau^{-1}\mathbf{y} = [\mathbf{h}^{\mathrm{T}}\ \mathbf{0}_{1\times(P-M)}]^{\mathrm{T}} + \tilde{m}\mathbf{1}_{P\times 1}. \tag{5}$$

The special structure of the RHS of (5) is used to compute $\mathbf{h}$ exactly even if $m \neq 0$. To start with, in the next subsection τ is assumed known.

3.1. DC-offset removal and channel computation

To extract the channel coefficients from (5), $\tilde{m}$ is computed first. Note that $\tilde{m}$ can be obtained from any of the last $P-M$ elements of $\mathbf{C}_\tau^{-1}\mathbf{y}$ in (5), but since the cyclic mean will be replaced by its finite-sample estimate later, we suggest to use the following average

$$\tilde{m} = \frac{1}{P-M}\mathbf{1}_{1\times(P-M)}\left(\mathbf{C}_\tau^{-1}\mathbf{y}\right)_{[P-M]}. \qquad (6)$$

Then, the channel coefficients can be computed from the first M elements of $\mathbf{C}_\tau^{-1}\mathbf{y}$ once the effect of m has been removed using (6):

$$\mathbf{h} = \left((\mathbf{C}_\tau^{-1})^{[M]_r} - \frac{\mathbf{1}_{M\times(P-M)}}{P-M}(\mathbf{C}_\tau^{-1})_{[P-M]_r}\right)\mathbf{y}. \qquad (7)$$

Note that if the DC-offset m is known (or non-existent), the channel can be computed just from

$$\mathbf{h} = (\mathbf{C}_\tau^{-1})^{[M]_r}(\mathbf{y} - m\mathbf{1}_{P\times 1}). \qquad (8)$$

In the next subsection we explain how $\mathbf{h}$ can be obtained if TSS is required in the presence of a DC-offset. For the particular case of known DC-offset, the method presented in [7] must be considered the precursor of the method to be presented next.

3.2. Training sequence synchronisation

In using (6) and (7), we have assumed that the offset τ was known. Next, we develop a method for estimating τ. The basic idea of the TSS technique is based on the observation that the last $P-M$ elements of $\mathbf{C}_\tau^{-1}\mathbf{y}$ in (5) are all equal to $\tilde{m}$. This special structure would be lost if the assumed τ, which we denote by τ', was different from the actual τ. Indeed, from (4) we can readily obtain the following equation

$$\mathbf{C}_{\tau'}^{-1}\mathbf{y} = \mathbf{P}_{\tau'}\left([\mathbf{h}^{\mathrm{T}}\,\mathbf{0}_{P-M}^{\mathrm{T}}]^{\mathrm{T}} + \tilde{\mathbf{m}}\right) \qquad (9)$$

where $\mathbf{P}_{\tau'}$ is the cyclic permutation matrix such that $\mathbf{C}_\tau = \mathbf{P}_{\tau'}\mathbf{C}_{\tau'}$, and hence $\mathbf{P}_{\tau'} = \mathbf{I}_P$ if $\tau' = \tau$. In the above equation, we have used the commutative property of circulant matrices [9]. Given that the channel and DC-offset information contained in the RHS of (9) is distorted by a cyclic permutation, we find it useful to sketch this effect in Fig. 2 where the entries of $\mathbf{C}_{\tau'}^{-1}\mathbf{y}$ are placed on a circle. The proposed TSS is achieved by simply rotating the circle until the $(P-M)$-element long "constant" section coincides with the last $(P-M)$ entries of $\mathbf{C}_{\tau'}^{-1}\mathbf{y}$.

We summarise our TSS method in the following proposition. First, we define $\tilde{m}_{\tau'}$ as in eq. (6) where $\mathbf{C}_\tau$ is substituted for $\mathbf{C}_{\tau'}$. Recall that τ is the true synchronisation offset.

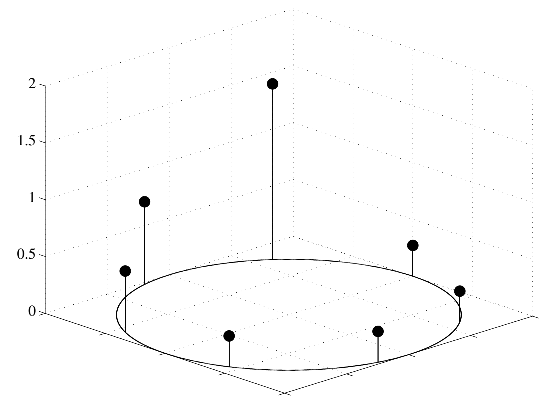

Fig. 2. Graphical representation of $\mathbf{y}$ after a transformation by $\mathbf{C}_{\tau'}^{-1}$ —see (9). Here: $P = 7$, $M = 3$, $\sigma_c^2 = 0.2$, $m = \sqrt{0.1}$ and $\mathbf{h} = [1.2735,\ 0.4562,\ 0.2661]^{\mathrm{T}}$.

Proposition 1 *Let $-\frac{P}{2} < \tau' \leq \frac{P}{2}$. Then, under A2, A3, A4 and A5, $\mathcal{J}(\tau') := \left\|(\boldsymbol{C}_{\tau'}^{-1}\boldsymbol{y})_{[P-M]} - \boldsymbol{1}_{P-M\times 1}\tilde{m}_{\tau'}\right\| = 0$ iff $\tau' = \tau$.*

Proof: The necessary condition ($\mathcal{J}(\tau') = 0 \Leftarrow \tau' = \tau$) is proved by realising that $\tilde{m}_{\tau'=\tau} = \tilde{m}$ and then using (5). Proof of sufficiency ($\mathcal{J}(\tau') = 0 \Rightarrow \tau' = \tau$) begins by noting, from the definition of $\tilde{m}_{\tau'}$, that $\mathcal{J}(\tau') = \|\mathbf{V}(\mathbf{C}_{\tau'}^{-1}\mathbf{y})_{[P-M]}\|$, where

$$\mathbf{V} := \mathbf{I}_{(P-M)} - \frac{1}{P-M}\mathbf{1}_{(P-M)\times(P-M)} \qquad (10)$$

acting on a vector produces the same vector with its mean removed from each element. So it follows from (9) that

$$\mathcal{J}(\tau') = \|\mathbf{V}(\mathbf{P}_{\tau'}[\mathbf{h}^{\mathrm{T}}\,\mathbf{0}_{1\times(P-M)}]^{\mathrm{T}})_{[P-M]}\| \qquad (11)$$

since $\mathbf{V}\mathbf{1}_{(P-M)\times 1} = \mathbf{0}_{(P-M)\times 1}$. With this new structure for $\mathcal{J}(\tau')$ it is clear that $\mathcal{J}(\tau') = 0$ iff the last $P-M$ elements of $\mathbf{P}_{\tau'}[\mathbf{h}^{\mathrm{T}}\,\mathbf{0}_{1\times(P-M)}]^{\mathrm{T}}$ are all equal. Now as letting $\tau' = \tau$ is equivalent to setting $\mathbf{P}_{\tau'} = \mathbf{I}_P$ then the last $P-M$ elements of $\mathbf{P}_{\tau'}[\mathbf{h}^{\mathrm{T}}\,\mathbf{0}_{1\times(P-M)}]^{\mathrm{T}}$ should come from the vector $\mathbf{0}_{1\times(P-M)}$ —i.e. $\mathbf{0}_{1\times(P-M)}$ should represent the only $P-M$ equal elements in $\mathbf{P}_{\tau'}[\mathbf{h}^{\mathrm{T}}\,\mathbf{0}_{1\times(P-M)}]^{\mathrm{T}}$. And to ensure that $P-M$ equal elements cannot occur in $\mathbf{h}$, we let $P-M > M$ —i.e. the previously stated assumption A5. Q.E.D.

Note that if we knew that the number of contiguous equal taps in $\mathbf{h}$ is Q, we could have required $P-M > Q$ instead. For a non-contiguous equal tap situation ($Q = 1$) we get $P \geq M + 2$, referred to as the *soft* constraint.

3.3. Relaxing assumption A3 —conditions for equalisation

Exact knowledge of the channel order, as required by Proposition 1, seems to be a very restrictive requirement for a practical implementation of the method. An unknown channel order will affect two aspects in the method proposed here: first, how to make A5 hold, and second, Proposition 1. Let us study these two aspects now.

If the channel order is replaced by an upper bound K, any value of P satisfying A5 with K, will satisfy A5 with M as well. So, even without knowing M, A5 can be fulfilled.

What happens now if we apply operator $\mathcal{J}$ in Proposition 1 with M replaced by K? From (11), what we obtain is a new form for $\mathcal{J}$:

$$\mathcal{J}^*(\tau') = \|\mathbf{V}^*(\mathbf{P}_{\tau'}[\mathbf{h}^{\mathrm{T}}\ \mathbf{0}_{1\times(P-M)}]^{\mathrm{T}})_{[P-K]}\| \tag{12}$$

where $\mathbf{V}^*$ is defined as in (10) but with M replaced by K. It is important to note that the length of $\mathbf{h}$ is not changed, even if it is unknown it is still M. So, as required in our formalism, in (12) we need to add $(P-M)$ zeros to $\mathbf{h}$ to make it of length P. This reflects that the structure of the vectors we are working with do not change, the only thing that changes is the way we look at it: we are looking for a shorter flat part —$(P-K)$ long instead of $(P-M)$— in $\mathbf{P}_{\tau'}[\mathbf{h}^{\mathrm{T}}\ \mathbf{0}_{1\times(P-M)}]^{\mathrm{T}}$ than we should. This means that, apart of the solution $\tau'=\tau$, there are $\tau'\neq\tau$ zeroing operator $\mathcal{J}^*$. Anyway, all the obtained solutions τ' provide delayed versions of the true $\mathbf{h}$. This delay is not important in equalisation.

Note that in a practical implementation, due to the noise, this identification delay can be present as well if the channel taps at the tails are very small. This can happen even if the channel order is known exactly.

4. IMPLEMENTATION OF THE METHOD

In practice, the cyclic mean vector $\mathbf{y}$ is estimated from the measured channel's output $\{x(k)\}_{k=0}^{N-1}$ using, as usual, time averages:

$$\hat{y}(j) = \frac{1}{N_P}\sum_{i=0}^{N_P-1} x(iP+j)$$

with $j=0,\ 1,\ \ldots,\ P-1$ and $N_P=\frac{N}{P}$ being the number of training sequence periods contained in the length-N record. Based on this estimate, channel estimation is carried out as shown in Table 1.

Data acquisition and cyclic mean computation
Obtain $\{x(k)\}_{k=0}^{N-1}$ compute $\hat{y}(j)=\frac{1}{N_P}\sum_{i=0}^{N_P-1} x(iP+j),\ j=0,\ \ldots\ P-1$
Training sequence synchronisation
$\hat{\tau}=\arg\min_{-\frac{P}{2}<\tau'\le\frac{P}{2}} \left\Vert(\mathbf{C}_{\tau'}^{-1}\hat{\mathbf{y}})_{[P-M]}-\mathbf{1}_{(P-M)\times 1}\hat{\hat{m}}_{\tau'}\right\Vert$ with $\hat{\hat{m}}_{\tau'}=\frac{1}{P-M}\mathbf{1}_{1\times(P-M)}\left(\mathbf{C}_{\tau'}^{-1}\hat{\mathbf{y}}\right)_{[P-M]}$
Channel computation
$\hat{\mathbf{h}}=\left((\mathbf{C}_{\hat{\tau}}^{-1})^{[M]_r}-\frac{1}{P-M}\mathbf{1}_{M\times(P-M)}(\mathbf{C}_{\hat{\tau}}^{-1})_{[P-M]_r}\right)\hat{\mathbf{y}}$

Table 1. Proposed method for ST-based channel estimation in the presence of a DC-offset with no TSS.

4.1. Computational burden

Consider the overall computational burden of the practical approach, in terms of total products and divisions. For the proposed method this is $P^3+(1-M)P^2+2P+3$ —i.e. $\mathcal{O}(P^3)$. For the method in [5] the computational burden is $MNP+2P^3+(M+1)P^2-(M+2)P+1$ —i.e. $\mathcal{O}(MNP)$. The filtering steps required in [5], contributing toward the MNP term, can be a significant part of the computational burden of [5]; for example, let $N=\mathcal{O}(P^3)$ and $M=\mathcal{O}(P)$ as in [5] and in the following simulation, then $\mathcal{O}(MNP)$ becomes $\mathcal{O}(P^5)$.

Comparing $\mathcal{O}(P^3)$ of the proposed method with $\mathcal{O}(P^5)$ of the method in [5], it is evident that the computational burden reduction achieved with the new method is significant. And could be bigger if the number of samples N used in the estimation is increased.

5. TRAINING SEQUENCE DESIGN

Given that TSS is based on an estimate (of cyclic permutations) of $\mathbf{C}^{-1}\mathbf{y}$, it is desirable to design $c(k)$ such that the estimation error power $\sigma^2 = \mathrm{E}\left[\|\mathbf{C}^{-1}(\hat{\mathbf{y}}-\mathbf{y})\|^2\right]$ is minimised. With a little of algebra, it is not difficult to show that

$$\sigma^2 = \mathrm{Tr}\left\{\mathbf{C}^{-\mathrm{H}}\mathbf{C}^{-1}\mathbf{R}_y\right\} \tag{13}$$

where $\mathbf{R}_y = \mathrm{E}\left[(\hat{\mathbf{y}}-\mathbf{y})(\hat{\mathbf{y}}-\mathbf{y})^{\mathrm{H}}\right]$ is the covariance matrix of the (unbiased) estimator $\hat{\mathbf{y}}$. Under A6, the $(ij)^{\mathrm{th}}$ element of $\mathbf{R}_y$ is $(\mathbf{R}_y)_{ij} = \delta_{ij}\left(\sigma_b^2\epsilon_h+\sigma_n^2\right)/N_P$ where $\epsilon_h = \mathrm{E}\left[\sum_{l=0}^{M-1}|h(l)|^2\right]$ is the mean energy of the channel, so (13) reduces to

$$\sigma^2 = \frac{\sigma_b^2 \epsilon_h + \sigma_n^2}{N_P} \mathrm{Tr}\left\{\mathbf{C}^{-\mathrm{H}}\mathbf{C}^{-1}\right\}. \tag{14}$$

The trace in the previous expression can be expressed explicitly as a function of the DFT coefficients of the training sequence $c(k)$. This is achieved taking into account that $\mathbf{C}$ is a circulant matrix, so it is diagonalised by the FFT matrix $(\mathbf{F})_{ki} = \exp(-j2\pi i \frac{k}{P})$ after a similarity transformation. The eigenvalues are the DFT coefficients $\{C(k)\}_{k=0}^{P-1}$. So, (14) can be rewritten as:

$$\sigma^2 = \frac{\sigma_b^2 \epsilon_h + \sigma_n^2}{N_P} \sum_{j=0}^{P-1} \frac{1}{|C(j)|^2}. \tag{15}$$

Then, the minimum value for σ^2 in (15), constraint to a fix training sequence power —i.e. $\sigma_c^2 = \sum_{k=0}^{P-1} |C(k)|^2 =$ constant— is attained when the DFT entries of $\{c(n)\}_{n=0}^{P-1}$ have the same magnitude. This means that the training sequence power is equally distributed among its cycles. This kind of sequence is used in the rest of the paper. A TSS method based on the properties of this kind of sequence was presented in [8]. Note that the method presented here could be considered as a generalisation of the method in [8].

Assuming now perfect TSS, the channel estimation error power $\sigma_h^2 = \mathrm{E}[\|\mathbf{h} - \hat{\mathbf{h}}\|^2]$ can be computed following a similar procedure as the one in the previous paragraph. So, from expressions (7) and (8), depending if the DC-offset is known or not, we obtain

$$\sigma_h^2 = \frac{M}{N} \{1 + a\} \frac{\sigma_b^2 \epsilon_h + \sigma_n^2}{\sigma_c^2} \tag{16}$$

where $a = 0$ for known or non-existent DC-offset and $a = \frac{1}{P-M}$ otherwise. Note that the variance in (16) when $a = 0$ does not depend on P, as previously reported in [6].

6. SIMULATION

Three-tap Rayleigh fading channels were simulated. The channel coefficients were complex Gaussian, i.i.d. with unit variance. The average energy of the channel ϵ_h was set to unity. The data was a BPSK sequence of $\{+1, -1\}$. To the data sequence, following section 5, a training sequence with its power σ_c^2 equally distributed among its DFT bins was added before transmission. The training to information power ratio TIR $= \sigma_c^2/\sigma_b^2$ was set to -6.9798 dB, the training sequence period to $P = 7$ and the number of samples to $N = 399$ —the same values that were used in [5]. We generated $N_B = 300$ blocks at the transmitter, where (1) represents just one of these blocks. Note that only N samples were used for channel estimation, but all the blocks were used for BER computation. A deterministic DC-offset (m) was added at the channel output, together with a zero-mean white Gaussian noise. The value of the DC-offset was determined by the DC-offset to signal AC-component (DCAC) power ratio as defined in [4]

$$\mathrm{DCAC} = \frac{m^2}{\mathrm{E}\left[|x(k) - n(k) - m|^2\right]}.$$

In these simulations this was set to DCAC$= 0.1$. At each realisation, a random synchronisation offset between 0 and $P - 1$ was introduced between transmitter and receiver, so we could be at *any* sample index within the first *period*. After channel estimation, an MMSE equaliser, based on the channel estimates, of length 11 and optimum delay was used to compute the BER; 1000 realisations were averaged. As already mentioned at the end of subsection 3.3, we may have an unknown delay of the estimated channel with respect to the true one. In this particular case, it may happen because of the randomness of the channel taps, so the first and last channel tap could be close to, or even, zero. This *identification delay*, which in practice has no major consequences, can worsen the simulated BER, misleading the performance analysis of the method. To avoid this, the identification delay was computed by comparing the equalised symbols with the true ones, for different delays, and choosing the delay giving the smallest BER—this problem was reported in [5] as well. For a comparison between the TSS methods in [3] and [5], please refer to [5], where simulations show that the later clearly outperforms the former.

The MSE of the channel estimates obtained using the

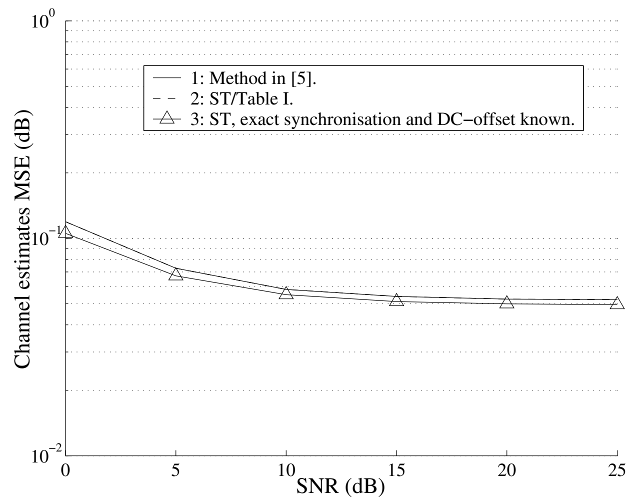

Fig. 3. MSE of channel estimates, as a function of the SNR, computed following Table 1. The identification delay has been considered. The estimates assuming known DC-offset and perfect synchronisation are included, together with the method in [5], for comparison purposes. Note that methods 1 and 2 are indistinguishable on the graph.

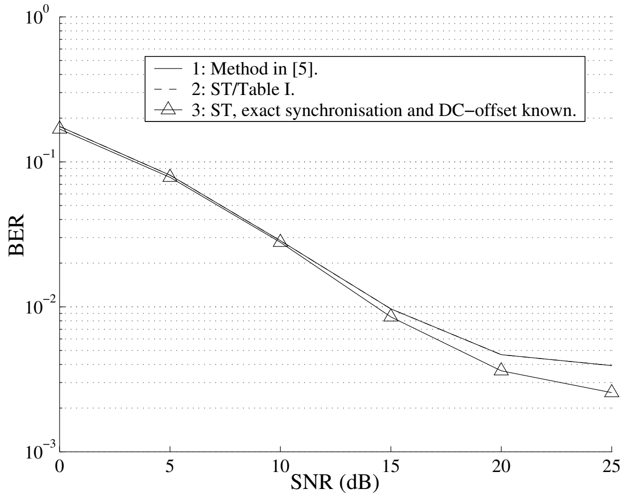

Fig. 4. BER of channel estimates, as a function of the SNR. A MMSE equaliser was computed from the channel estimates obtained following Table 1. The identification delay has been considered. Note that methods 1 and 2 are indistinguishable on the graph.

method described in Table 1 is plotted in Fig. 3. As a benchmark, the MSE obtained assuming perfect synchronisation and known DC-offset is included —i.e. as predicted by (16) with $a = 0$. We can see that the difference between them two are small indeed, so the proposed method works very well. The TSS method of [5] is also included for comparison. As can be checked, both the method in [5] and the method presented here give identical results in what respect the MSE. But, as previously mentioned, the computational burden of the proposed method is much smaller than that of the method in [5].

The BER after equalisation is shown in Fig. 4. The conclusions drawn in the previous paragraph can be translated here.

7. CONCLUSIONS

Training sequence synchronisation (TSS), required for a correct channel estimation when using ST, has been investigated. The proposed TSS method is motivated by a graphical interpretation using circular geometry, and its mayor advantage with respect to existing TSS methods is its computational low burden. Then, conditions for TSS have been derived. Although these require the channel order to be known, it is shown that, for equalisation purposes, only an upper bound of the channel order is actually needed. It has been shown that training sequences with their powers equally distributed among its DFT bins are desired to achieve a better TSS when using the proposed method. Using this kind of training sequence, expressions for the channel estimate error power assuming known/unknown DC-offset, and perfect TSS, have been given. Finally, simulations show that the performance of the proposed method —MSE and BER— equals that of existing algorithms, but with the already mentioned advantage of being computationally more appealing.

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