RANK REDUCTION FOR RAPID TIMING ACQUISITION IN MULTIPLE ACCESS COMMUNICATIONS

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ABSTRACT

This work presents our new results on data-driven rank reduction for rapid timing acquisition in multiple access (MA) communications systems. The results generally apply to multiple access systems, such as code-division multiple access (CDMA) and the ultra-wide band (UWB) systems, operating over environments with severe radio interference. Our results show that when only a *limited amount* of data is available, *rather than* the true statistics required for timing acquisition, reduced rank solutions provide more reliable timing information than high-rank solutions. Our results demonstrate that a low-rank version of the objective function for timing acquisition, using the estimated statistic is less sensitive to spurious peaks than that of the high-rank solutions.

1. INTRODUCTION

Synchronization or timing acquisition is an important aspect of every communication system. From a signal processing perspective, timing acquisition is related to the traditional time delay estimation problem in the presence of multiple access interference and ambient noise. The time delay parameters are typically non-linearly entered in the data model, resulting in a non-linear parameter estimation problem. The maximum likelihood estimation of the non-linear parameters boils down to a peak search of an objective function, commonly named the compressed likelihood function(CLF), over the parameter space. For our problem, the CLF, is the ratio of quadratic forms involving data covariance matrix inversion. When only a finite amount of data rather than the true covariance matrix is available for timing acquisition, a reliable low-complexity data-driven solution is in need. In this work, we develop a reduced-rank data-driven solution that avoids the matrix inversion and only uses very few data snapshots, for rapid timing acquisition in a multiple access communications system.

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2. PROBLEM FORMULATION

In an asynchronous multiple access system over the additive Gaussian noise channel, the baseband data $r(t)$ can be modeled as,

$$r(t) = \sum_i \sum_{k=1}^{K} A_k \, b_k(i) \, s_k(t - \tau_k - i\,T) + n(t)\,, \quad (1)$$

where K is the number of active MA users; i is the symbol index; A_k, $b_k(i)$, τ_k, and $s_k(t)$ are the amplitude, BPSK information bit, propagation delay, and signature waveform (normalized within symbol interval T) of the kth user, respectively; $n(t)$ is a white Gaussian process with an average power of σ^2. The signature waveform is generated based on a binary signature sequence $s_k[l] \in \{-1, +1\}$ and the rectangular pulse of chip duration. That is,

$$s_k(t) = \frac{1}{L\,T_c} \sum_{l=1}^{L} s_k[l]\, p(t - l\,T_c), \qquad 0 \le t \le T.$$

with $T = L\,T_c$. Denote the k-th user's propagation delay as $\tau_k = \nu_k\,T_c + \gamma_k$, where ν_k and γ_k are the integer and the fractional parts of τ_k with respect to the chip duration T_c. In our further analysis and simulations, we choose a normalized chip interval, $T_c = 1$. Within the i-th processing interval of length T, which is commonly *not* aligned with the unknown delay, the chip-rate matched filtered and sampled data in (1) can be written in matrix form as,

$$\begin{aligned} \mathbf{r}(i) \;&= \underbrace{A_1 \left(b_1(i-1)\,\mathbf{u}_1^{(r)} + b_1(i)\,\mathbf{u}_1^{(l)} \right)}_{signal\ of\ interest} + \\ &\underbrace{\sum_{k=2}^{K} A_k \left(b_k(i-1)\,\mathbf{u}_k^{(r)} + b_k(i)\,\mathbf{u}_k^{(l)} \right)}_{MAI} + \mathbf{n}(i)\,, \end{aligned} \quad (2)$$

where the i.i.d. white noise vectors $\mathbf{n}(i) \sim \mathcal{N}(\mathbf{0}, \sigma^2\,\mathbf{I}_L)$. The $(L \times 1)$ vectors $\mathbf{u}_k^{(r)}$ and $\mathbf{u}_k^{(l)}$ are the effective signature vectors of the kth user, parameterized by the delay τ_k, i.e.,

$$\begin{aligned}\mathbf{u}_k^{(r)} &= (1 - \tfrac{\gamma_k}{T_c})\,\mathbf{s}_k^{(r)}(\nu_k) + \tfrac{\gamma_k}{T_c}\,\mathbf{s}_k^{(r)}(\nu_k + 1),\\ \mathbf{u}_k^{(l)} &= (1 - \tfrac{\gamma_k}{T_c})\,\mathbf{s}_k^{(l)}(\nu_k) + \tfrac{\gamma_k}{T_c}\,\mathbf{s}_k^{(l)}(\nu_k + 1),\end{aligned} \tag{3}$$

with $\mathbf{s}_k^{(r)}(\nu_k)$ and $\mathbf{s}_k^{(l)}(\nu_k)$ being the right and left portions of signature vector $\mathbf{s}_k$ partitioned by the integer part of delay ν_k. That is,

$$\mathbf{s}_k^{(l)}(\nu_k) = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ s_k[1] \\ s_k[2] \\ \vdots \\ s_k[L-\nu_k] \end{bmatrix}, \mathbf{s}_k^{(r)}(\nu_k) = \begin{bmatrix} s_k[L-\nu_k+1] \\ s_l[L-\nu_k+2] \\ \vdots \\ s_k[L] \\ 0 \\ \vdots \\ 0 \end{bmatrix}.$$

In this work, we combine the MAI and WGN of (2) into one colored noise vector. Estimating delay of the desired user τ_1 then becomes jointly estimating signal parameters ν_1 and γ_1 in colored noise of unknown covariance structure. The colored noise has zero mean and covariance matrix

$$\mathbf{Q} = \mathbf{U}\underline{\mathbf{A}}_1^2\,\mathbf{U}^T + \sigma^2\,\mathbf{I}_L,$$

where $\mathbf{U} = \begin{bmatrix} \mathbf{u}_2^{(r)} & \mathbf{u}_2^{(l)} & \cdots & \mathbf{u}_K^{(r)} & \mathbf{u}_K^{(l)} \end{bmatrix}$, and $\underline{\mathbf{A}}_1 = \mathbf{I}_2 \otimes \text{diag}\{A_2, A_3, \cdots, A_K\}$. Note that the actual structure of the noise covariance matrix depends on the nuisance parameters of all interfering users.

3. RANK REDUCTION FOR RAPID TIMING ACQUISITION

In most communications systems, a fixed preamble (M bits) associated with each user is used for timing acquisition and system synchronization. Then the signal of interest as well as the data model in (2) associated with the preamble bits can be simplified to

$$\mathbf{r}(i) = \beta_1\,\mathbf{u}_1(\tau_1) + \mathbf{e}(i), \qquad i = 1, 2, \ldots, M, \tag{4}$$

where β_1 is an unknown scalar; the signal vector $\mathbf{u}_1(\tau_1) = \mathbf{u}_1^{(r)} + \mathbf{u}_1^{(l)}$ is parameterized by delay $\boldsymbol{\theta} = [\nu_1\ \gamma_1]^T$; and the colored noise $\mathbf{e}(i)$ is of zero mean and unknown covariance, $E\{\mathbf{e}(i)\} = \mathbf{0},\ \text{cov}(\mathbf{e}(i)) = \mathbf{Q}$. Note that under the Gaussian assumption for the data vectors in (4), the sufficient statistics for estimating all the unknowns, τ_1, β_1 and $\mathbf{Q}$, are functions of the sample mean vector and the sample correlation matrix, i.e.

$$\frac{1}{M}\sum_{i=1}^{M}\mathbf{r}(i), \qquad \frac{1}{M}\sum_{i=1}^{M}\mathbf{r}(i)\,\mathbf{r}^T(i)$$

Hence, in this work, we use the Gaussian approximation to model the sample mean statistic. That is,

$$\hat{\mathbf{m}} = \frac{1}{M}\sum_{i=1}^{M}\mathbf{r}(i) \sim \mathcal{N}\left(\beta_1\,\mathbf{u}_1(\tau_1),\ \frac{1}{M}\mathbf{Q}\right).$$

This turns out to be a reasonable assumption when the combined effect from M preamble bits and the number of MA users K is large. The delay estimate is then given by [7],

$$\hat{\tau}_1 = \arg\max_{\tau} J(\tau) = \arg\max_{\tau} \frac{\left|\hat{\mathbf{m}}^T\,\hat{\mathbf{Q}}^{-1}\,\mathbf{u}_1(\tau)\right|^2}{\mathbf{u}_1^T(\tau)\,\hat{\mathbf{Q}}^{-1}\,\mathbf{u}_1(\tau)}, \tag{5}$$

where $\hat{\mathbf{Q}} = \frac{1}{M}\sum_{i=1}^{M}\mathbf{r}(i)\,\mathbf{r}^T(i) - \hat{\mathbf{m}}\,\hat{\mathbf{m}}^T$ is the sample covariance estimate of the unknown $\mathbf{Q}$ matrix. Note that without the Gaussian assumption, the above delay estimate is the non-linear weighted least squares solution for τ_1 in the model of equation (4). Also note that normalizing the objective function $J(\tau)$ by a term $\hat{\mathbf{m}}^T\,\hat{\mathbf{Q}}^{-1}\,\hat{\mathbf{m}}$ (not affecting the delay estimate) leads to the equivalent adaptive coherence estimator (ACE) [9, 10].

In communication applications, the preamble bits are scarce, therefore we propose to use a rank reduction technique to facilitate the rapid timing acquisition. The rank reduction technique from our recent work [5, 6] can alleviate the problem encountered in sample covariance matrix inversion (especially when $M < L$), yet at the same time to deliver data-driven solutions to low-complexity timing acquisition. In doing so, the rank-r solution to timing acquisition is obtained from,

$$\hat{\tau}_1 = \arg\max_{\tau} J^{(r)}(\tau)\,,$$

where the rank-r version of the objective function becomes,

$$J^{(r)}(\tau) = \frac{\left|\hat{\mathbf{m}}^T\,\mathbf{w}_{\mathbf{u}_1}^{(r)}(\tau)\right|^2}{\left|\mathbf{u}_1^T(\tau)\,\mathbf{w}_{\mathbf{u}_1}^{(r)}(\tau)\right|}. \tag{6}$$

In (6), the vector $\mathbf{w}_{\mathbf{u}_1}^{(r)}(\tau)$ is a rank-r approximation to the Wiener filter vector defined as $\mathbf{w}_{\mathbf{u}_1}(\tau) = \hat{\mathbf{Q}}^{-1}\,\mathbf{u}_1(\tau)$. It can be iteratively calculated using the conjugate direction vectors [2, 3, 5],

$$\mathbf{w}_{\mathbf{u}_1}^{(r)}(\tau) = \sum_{i=1}^{r}\alpha_i\,\mathbf{d}_i \in \mathcal{K}_r(\hat{\mathbf{Q}},\,\mathbf{u}_1(\tau))\,,$$

where $\mathcal{K}_r(\hat{\mathbf{Q}},\,\mathbf{u}_1(\tau))$ denotes the rank-r Krylov subspace; the vectors $\mathbf{d}_i \in \mathcal{K}_r(\hat{\mathbf{Q}},\,\mathbf{u}_1(\tau))$ are the $\hat{\mathbf{Q}}$-conjugate directions; and the scalars $\alpha_i = \mathbf{u}_1^T(\tau)\,\mathbf{d}_i/\mathbf{d}_i^T\,\hat{\mathbf{Q}}\,\mathbf{d}_i$ are the best linear combination coefficients (the scalar Wiener filters working on the decorrelated internal variables $z_i = \mathbf{d}_i^T\,\hat{\mathbf{m}}$) for the set of r given conjugate direction vectors. Specifically, at the r-th step of iteration, out of the rank-r Krylov subspace $\mathcal{K}_r(\hat{\mathbf{Q}},\,\mathbf{u}_1(\tau))$, a rank-$r$ approximation to the Wiener filter is constructed as an optimal linear combination of the r conjugate direction vectors generated by the vector conjugate gradient (V-CG) method. The initial direction vector is chosen as $\mathbf{d}_1 = \mathbf{u}_1(\tau)$, and the subsequent direction vectors are chosen as the $\hat{\mathbf{Q}}$-conjugate directions. That is $[2, 5]$, the new direction vectors are updated using the innovation contained in the residue (gradient) vector,

$$\mathbf{d}_{k+1} = \mathbf{g}_{k+1} + \frac{\|\mathbf{g}_{k+1}\|^2}{\|\mathbf{g}_k\|^2}\,\mathbf{d}_k\,, \tag{7}$$

where $\mathbf{d}_k^T\,\mathbf{R}_{\mathbf{yy}}\,\mathbf{d}_m = 0$ for $k \neq m$. The residue vector is updated accordingly as

$$\mathbf{g}_{k+1} = \mathbf{g}_k - \alpha_k \mathbf{R}_{\mathbf{yy}}\,\mathbf{d}_k\,, \tag{8}$$

Note that when the number of available preamble bits M is very small, making the sample covariance matrix $\hat{\mathbf{Q}}$ rank deficient ($M < L$), the proposed reduced-rank version of the objective function can still be calculated using the above procedures. In such cases, we observe from the simulations the advantages of the low-rank timing acquisition scheme over the high-rank solutions.

4. SIMULATION EXPERIMENTS

The application problem considered in this work is the propagation delay estimation of a desired user in a code-division multiple-access communication environment [7, 8]. In such applications, the presence of multiple-access interference (MAI) from other users renders the conventional correlator based delay estimator useless. As mentioned earlier, we treat the *desired user* as the *signal of interest* and other interfering users as *interference* of *unknown* covariance structure. Under the condition of the near-Gaussian interference-plus-noise, the maximum likelihood estimate of τ_1 simply corresponds to the maximum of an objective function $J(\tau_1)$ in (5) or its scaled version $J_{ACE}(\tau_1)$. This estimator is near-far resistant due to the fact that it makes use of the structure of the MAI. In our approximation of this objective function, we use a sample covariance matrix $\hat{\mathbf{Q}}$ and *an implicit* approximation to its inverse, by using a sequence of recursively approximated Krylov subspaces.

For the examples used, we choose a multiple access system with $K = 10$ users. The signature length is chosen as $L = 31$. The SNR for the desired user is fixed as $SNR(1) = 11dB$, and the SNRs for all the interfering users are chosen as $SNR(k) = SNR(1) + NFRdB, k = 2, \ldots, K$, with near-far ratio chosen as $NFR = 0, 10dB$. We vary the number of preamble bits $M = 10, 20, 31$. In all the figures, we show the objective functions at different ranks and different NFRs as a function of delay parameter τ_1. The true delay is marked by a dashed line.

This work demonstrates the applicability of the fast converging reduce-rank Wiener filter to low-complexity data-driven rapid timing acquisition in CDMA systems.

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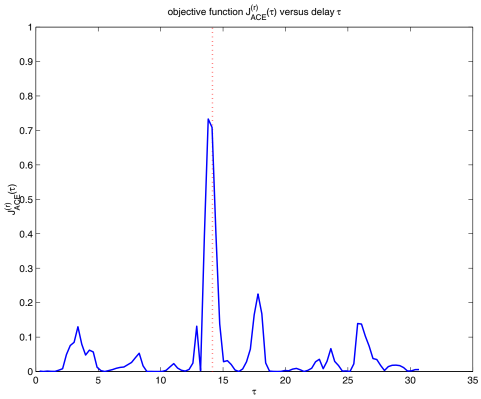

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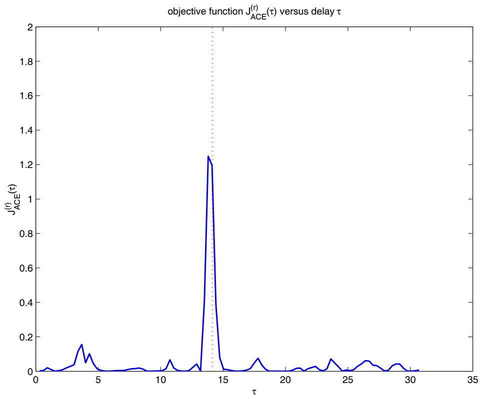

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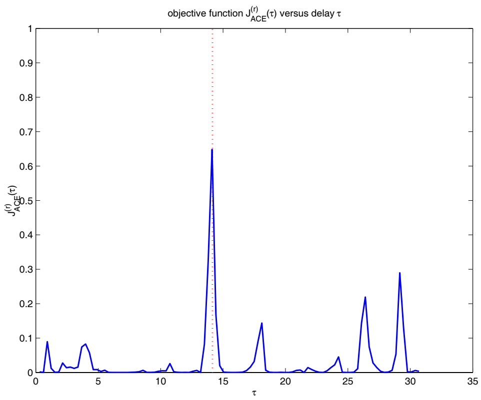

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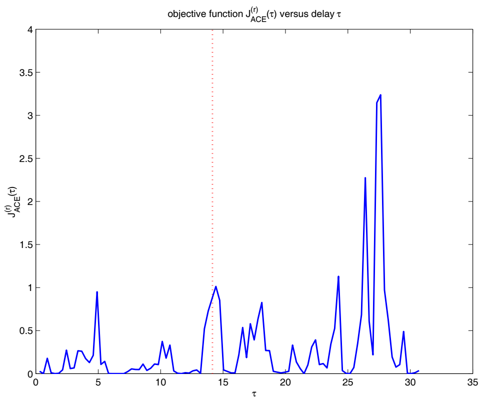
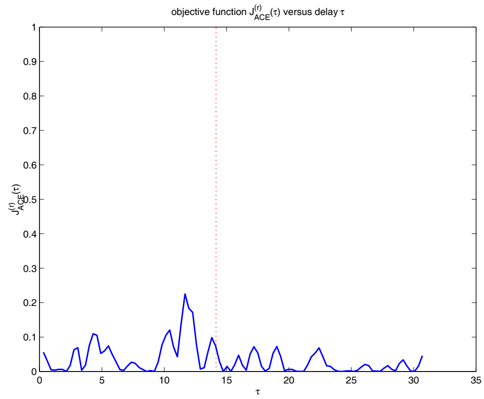


(d). data sample $M = 10$, rank $r = 10$.

Fig. 1. Rank-r approximation to the objective function using $M = 10 < L$ data samples for the case NFR=0 dB. The rank ranges from 1 to 10 in fig. (a) through (d). In all the figures, the true delay for the desired user is marked by a red dotted line.



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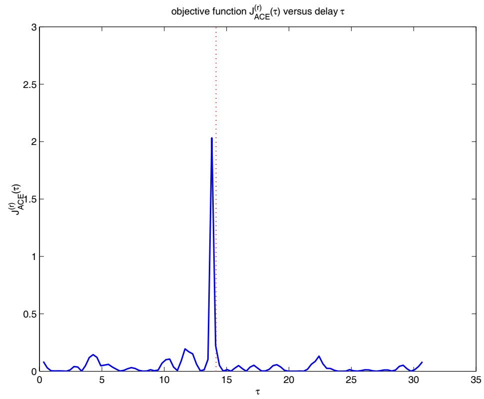

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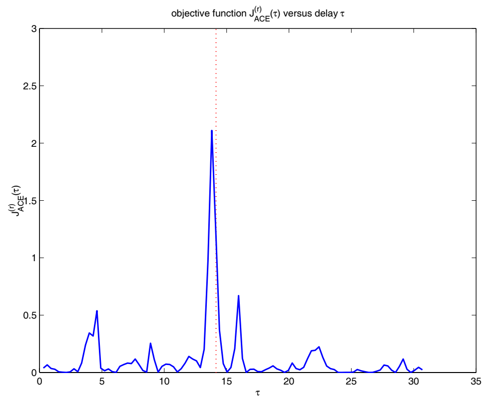

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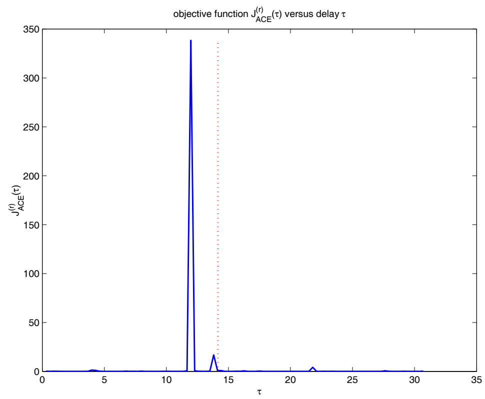
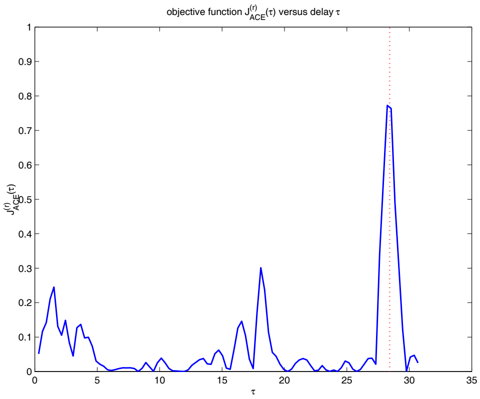


(d). data sample $M = 10$, rank $r = 10$.

Fig. 2. Rank-r approximation to the objective function using $M = 10 < L$ data samples for the case NFR=10 dB. The rank ranges from 1 to 10 in fig. (a) through (d). In all the figures, the true delay for the desired user is marked by a red dotted line.



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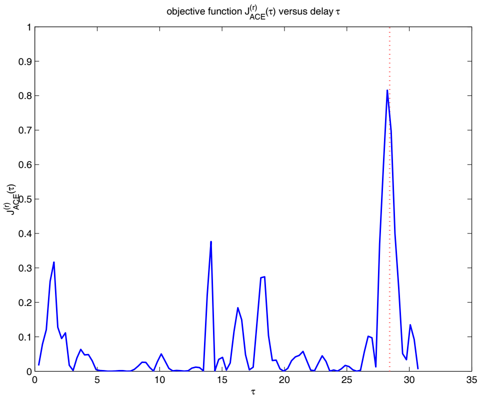

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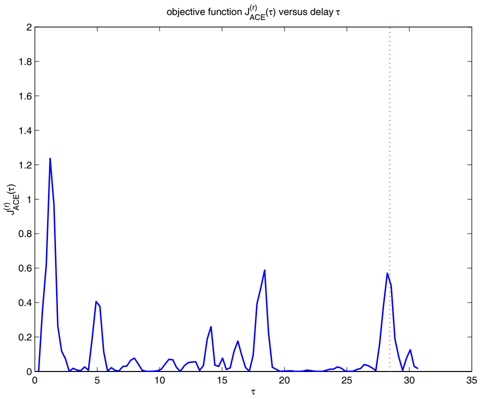

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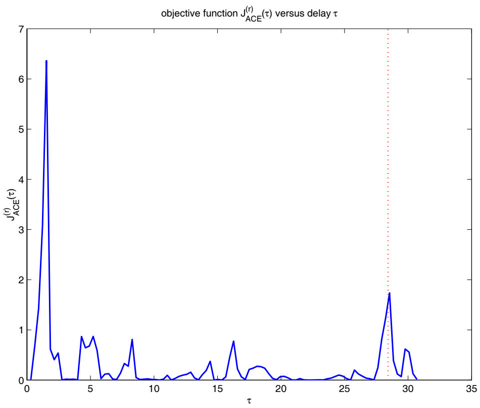
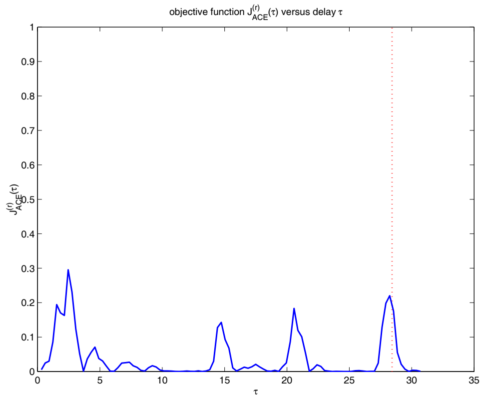


(d). data sample $M = 20$, rank $r = 10$.

Fig. 3. Rank-r approximation to the objective function using $M = 20 < L$ data samples for the case NFR=0 dB. The rank ranges from 1 to 10 in fig. (a) through (d). In all the figures, the true delay for the desired user is marked by a red dotted line.



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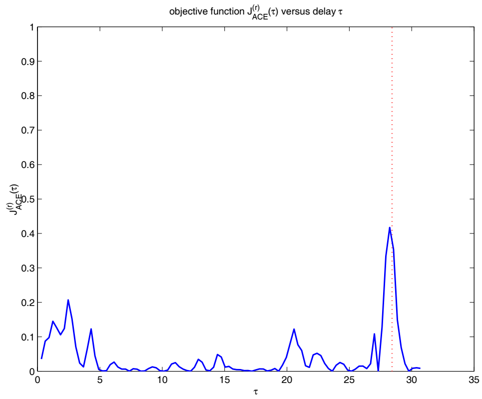

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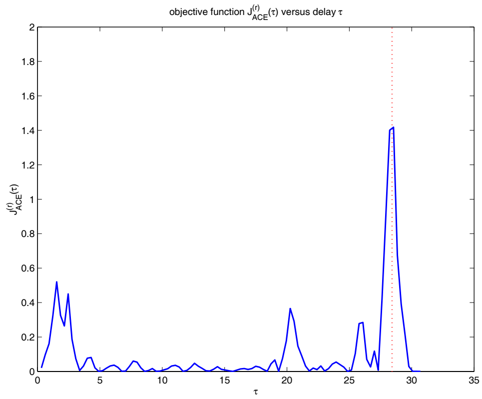

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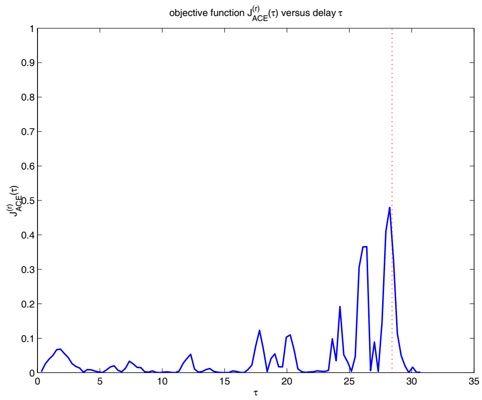
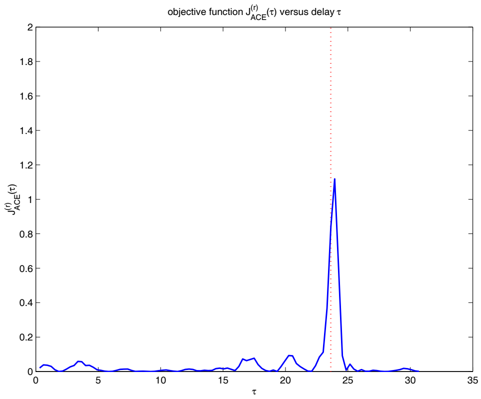


(d). data sample $M = 20$, rank $r = 10$.

Fig. 4. Rank-r approximation to the objective function using $M = 20 < L$ data samples for the case NFR=10 dB. The rank ranges from 1 to 10 in fig. (a) through (d). In all the figures, the true delay for the desired user is marked by a red dotted line.



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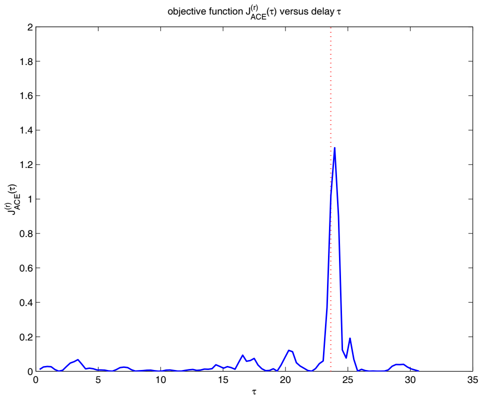

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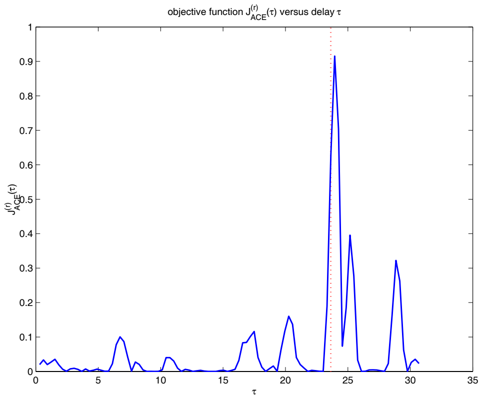

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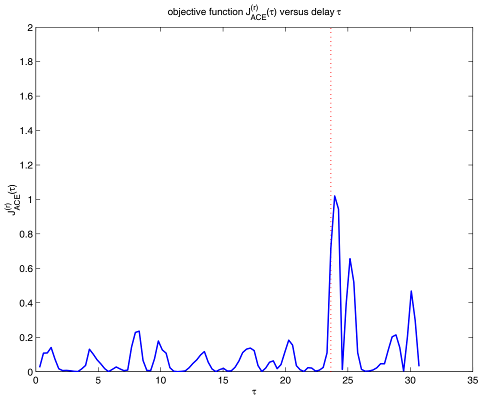
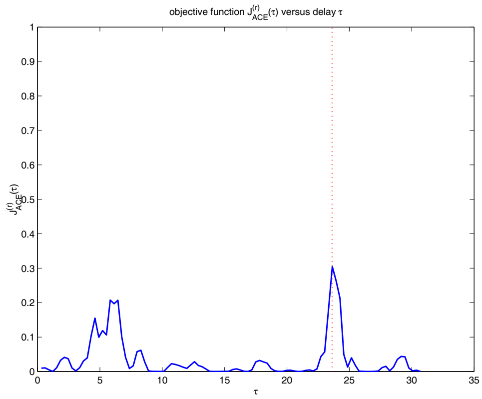


(d). data sample $M = 31$, rank $r = 10$.

Fig. 5. Rank-r approximation to the objective function using $M = 31 = L$ data samples for the case NFR=0 dB. The rank ranges from 1 to 10 in fig. (a) through (d). In all the figures, the true delay for the desired user is marked by a red dotted line.



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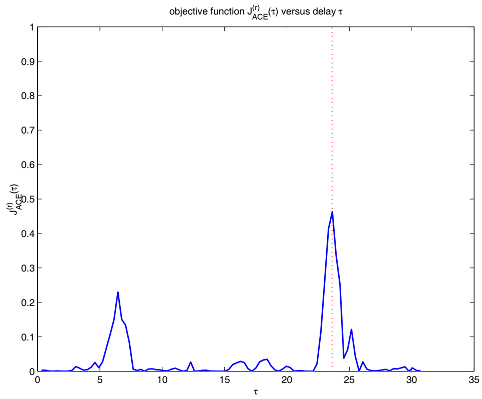

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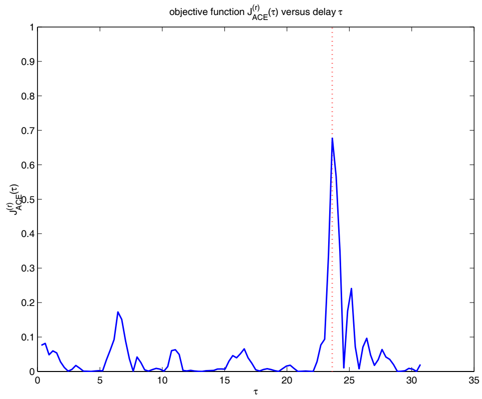

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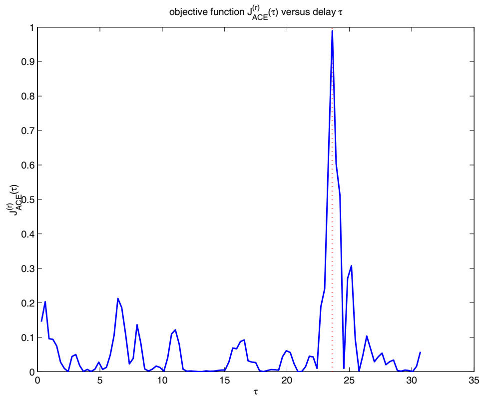

(d). data sample $M = 31$, rank $r = 10$.

Fig. 6. Rank-r approximation to the objective function using $M = 31 = L$ data samples for the case NFR=10 dB. The rank ranges from 1 to 10 in fig. (a) through (d). In all the figures, the true delay for the desired user is marked by a red dotted line.