AN ALGORITHM FOR TRACKING MULTIPLE WIDEBAND CYCLOSTATIONARY SOURCES

*Huiqin Yan and H. Howard Fan**

Department of ECECS, University of Cincinnati
Cincinnati, OH 45221-0030, USA

ABSTRACT

In this paper, a new algorithm is presented for signal selective Direction of Arrival (DOA) tracking of multiple moving sources emitting narrowband or wideband cyclostationary signals. The most current array outputs are used to update the DOAs recursively and no data association is needed in this algorithm. Furthermore, by exploiting cyclostationarity, interference and noise are suppressed. Thus only sources of interest are tracked. The effectiveness of this new algorithm is demonstrated by simulations.

1. INTRODUCTION

Direction of Arrival (DOA) tracking of multiple moving sources has been of interest for decades due to its wide applications such as surveillance in military applications and air traffic control in civilian applications. An obvious method of DOA tracking is to find DOAs by an existing DOA estimation algorithm for each time frame, during which the directions are assumed to be fixed, then to associate each of the newly estimated DOAs to those of the old ones in order to keep tracking the DOA changes and the source movement. A key problem of this method is the computational complexity of correctly assigning the estimated DOAs to their corresponding sources, or data association. Recently, some DOA tracking algorithms which do not require data association have been proposed, such as [1–4]. [1] tracks the source movement by using the most current array output. DOA changes for each time frame, rather than new DOAs, are estimated by solving a Least Squares (LS) problem. [2] improves the performance of [1] by employing a source movement model. It updates the predicted DOAs, and refines the DOA estimates through a Kalman filter. [3] updates the DOA estimates for each time frame by solving a Maximum Likelihood (ML) problem of the most current array output. This approach also employs a source movement model and refines the DOA estimates through a Kalman filter as in [2]. [4] introduces Multiple Target States (MTS) to describe the target motion, and the DOA tracking is implemented through updating the MTS by maximizing the likelihood function of the array output. Whether by LS or ML method, whether using Kalman filter or not, all these algorithms implement the DOA tracking in a way that the order of the estimated DOAs for different time or time frame is maintained, thus data association is avoided. Note that all these methods assume the signals being narrowband stationary and the noise being white Gaussian.

Recently, a statistical property, cyclostationarity, which many types of man-made signals in communications such as BPSK, FSK, AM exhibit, has been exploited in DOA estimation [5–7]. By exploiting cyclostationarity, interference and noise that do not share the same cycle frequency as the desired signals or do not exhibit cyclostationarity can be suppressed, thus the performance of DOA estimation is improved, especially when the DOA of interference is close to the DOA of a desired signal. Similarly, cyclostationarity could also be exploited to suppress the interference and noise, thus to improve performance of DOA tracking. However, little work can be found on signal selective DOA tracking, or, exploiting cyclostationarity of the signals in DOA tracking except [8, 9], where the signals are assumed to be narrowband cyclostationary. Although there exists some algorithms for tracking multiple wideband targets such as [10, 11], these methods do not exploit cyclostationarity of the signals.

In this paper, we propose a new signal selective DOA tracking algorithm for multiple moving sources by exploiting cyclostationarity of the signals. Unlike [8, 9], in our algorithm, the signals emitted by the moving sources can be narrowband or wideband cyclostationary. Our algorithm assumes that the DOAs in each time frame are fixed and

*Corresponding author. E-mail: h.fan@uc.edu. This work is supported by the Office of Naval Research under Grant N00014-04-1-0059.

tracks the DOA changes from the last time frame to the current time frame by exploiting the difference of the averaged cyclic cross correlation of the array output. First, DOA tracking is initiated by applying once the DOA estimation method: averaged cyclic MUSIC (ACM) [7]. Then DOA changes for each time frame are estimated by finding the minimum solution to a cost function. To avoid inconsistent solutions when the DOAs are crossing, we assume that the sources are moving at constant speeds thus the DOA changes for two adjacent time frames are almost the same. Similar to [1–4], our signal selective DOA tracking algorithm does not require data association.

2. DATA MODEL

Consider the tracking problem by a Uniform Linear Array (ULA) of size N with intersensor spacing d. I moving sources are assumed to generate I signals with cycle frequency α. These signals are considered as Signal of Interest (SOI). Other signals from other moving sources which do not exhibit cyclostationarity or have different cycle frequencies are considered as interference. Noise is assumed to be stationary. The signal received by the nth antenna in the array is

$$x_n(t) = \sum_{i=1}^{I} s_i(t + (n-1)\Delta_i(t)) e^{j2\pi f_0 (n-1)\Delta_i(t)} + \eta_n(t) \tag{1}$$

where $s_i(t)$ is the complex baseband signal of the ith SOI induced at the first antenna, $\eta_n(t)$ is the interference and noise induced at the nth antenna, f_0 is the carrier frequency, and $\Delta_i(t) = d \sin\theta_i(t)/c$ is the time delay between two adjacent antennas. Here $\theta_i(t)$ is the impinging direction of the ith SOI at time t, and c is the propagation speed.

Now assume that the DOAs of the sources change little during a time frame T, i.e., $\theta_i(t)$ or $\Delta_i(t)$ are constant during the kth time frame $[(k-1)T, kT]$, where $k = 1, \cdots, K$. The total tracking time is assumed to be KT seconds. Then we have $\Delta_i(k) = d \sin\theta_i(k)/c$ for the kth time frame. Note that (1) imposes no restriction on the signal bandwidth.

We define the following vectors and matrices which shall be used later in this paper.

$$\mathbf{s}(t) = [s_1(t) \ \cdots \ s_I(t)]^T \tag{2}$$

$$\mathbf{x}(t) = [x_1(t) \ \cdots \ x_N(t)]^T \tag{3}$$

$$\mathbf{A}(f,k) = [\mathbf{a}_1(f,k), \ \cdots \ \mathbf{a}_I(f,k)] \tag{4}$$

$$\mathbf{a}_i(f,k) = [1, \ e^{j2\pi f \Delta_i(k)}, \ \cdots, \ e^{j2\pi f (N-1)\Delta_i(k)}]^T \tag{5}$$

where $[\cdot]^T$ denotes matrix transpose, and $\mathbf{s}(t)$ is the source signal vector, $\mathbf{x}(t)$ is the received signal vector, $\mathbf{A}(f,k)$ is the steering matrix evaluated at the frequency f for the kth time frame, and $\mathbf{a}_i(f,k)$ is the steering vector for the ith SOI evaluated at f for the kth time frame.

3. TRACKING ALGORITHM

For the kth time frame, calculate the cross cyclic correlation of $x_1(t)$ and $x_n(t)$, where $x_n(t)$ is the signal received at the nth antenna. By varying n from 2 to N, we can obtain $N-1$ such cross cyclic correlations as

$$\begin{aligned}
& r^{\alpha}_{x_1 x_n}(\tau, k) \\
&= \int_k x_1(t + \frac{\tau}{2}) x_n^*(t - \frac{\tau}{2}) e^{-j2\pi\alpha t} dt \\
&= \int_k \sum_{p=1}^{I} s_p(t + \frac{\tau}{2}) \sum_{i=1}^{I} s_i^*(t - \frac{\tau}{2} + (n-1)\Delta_i(k)) \\
&\quad \cdot e^{-j2\pi f_0 (n-1)\Delta_i(k)} e^{-j2\pi\alpha t} dt \\
&= \sum_{i=1}^{I}\sum_{p=1}^{I} \int_k s_p(t + \frac{\tau}{2}) s_i^*(t - \frac{\tau}{2} + (n-1)\Delta_i(k)) \\
&\quad \cdot e^{-j2\pi\alpha t} e^{-j2\pi f_0 (n-1)\Delta_i(k)} dt \\
&= \sum_{i=1}^{I} \left[\sum_{p=1}^{I} r^{\alpha}_{s_p s_i}(\tau - (n-1)\Delta_i(k), k) \right] \\
&\quad \cdot e^{-j2\pi(f_0 - \frac{\alpha}{2})(n-1)\Delta_i(k)}
\end{aligned} \tag{6}$$

where $[\cdot]^*$ denotes complex conjugate and $\int_k$ denotes integral from $(k-1)T$ to kT. Since evaluation of cyclic correlation will only retain those SOI, interference and noise are suppressed in the above equation. Also in the above equation, the shift property of cyclic correlation is applied, i.e., if $y_1(t) = x_1(t+T)$ and $y_2(t) = x_2(t+T)$, then $r^{\alpha}_{y_1 y_2}(\tau) = r^{\alpha}_{x_1 x_2}(\tau) e^{j2\pi\alpha T}$. Note that the cross correlation $r^{\alpha}_{x_1 x_n}(\tau, k)$ is a function of $\Delta_i(k)$. To simplify this function, we can average $r^{\alpha}_{x_1 x_n}(\tau, k)$ over the time delay τ, and obtain the averaged cyclic correlation between $x_1(t)$ and $x_n(t)$ at the kth time frame as

$$\langle r^{\alpha}_{x_1 x_n}(k) \rangle_\tau = \sum_{i=1}^{I} \left[\sum_{p=1}^{I} \langle r^{\alpha}_{s_p s_i}(k) \rangle_\tau \right] e^{-j2\pi(f_0 - \frac{\alpha}{2})(n-1)\Delta_i(k)} \tag{7}$$

where $\langle r^{\alpha}_{s_p s_i}(k) \rangle_\tau$ is the averaged cyclic correlation of $s_p(t)$ and $s_i(t)$ at the kth time frame. $\langle \cdot \rangle_\tau$ is used to denote averaging over τ. If the duration of a time frame is long enough, $\langle r^{\alpha}_{s_p s_i}(k) \rangle_\tau$ can be assumed to be independent of k. Therefore, we drop k, and define

$$E_i = \sum_{p=1}^{I} \langle r^{\alpha}_{s_p s_i} \rangle_\tau \tag{8}$$

Also define

$$g_n(\theta) = e^{-j2\pi(f_0-\frac{\alpha}{2})(n-1)d\sin\theta/c} \tag{9}$$

Then, (7) can be written as

$$\langle r^{\alpha}_{x_1 x_n}(k)\rangle_\tau = \sum_{i=1}^{I} E_i g_n(\theta_i(k)) \tag{10}$$

To derive our algorithm, we need to know E_i. Therefore, let us illustrate how to estimate E_i which can be done during initialization.

First, we apply the signal selective DOA estimation algorithm ACM [7] to estimate the initial DOAs. The number of sources emitting SOI are assumed to be known or estimated by the MDL criteria [12]. Note that ACM works for both narrowband and wideband signals. Below is a summary of this algorithm.

1. Compute the cyclic correlation matrix $\mathbf{R}^{\alpha}_{\mathbf{xx}}(\tau,1)$ during the first time frame by

$$\mathbf{R}^{\alpha}_{\mathbf{xx}}(\tau,1) = \int_1 \mathbf{x}(t+\tau/2)\mathbf{x}^H(t-\tau/2)e^{-j2\pi\alpha t}dt$$

   where $[\cdot]^H$ denotes Hermitian transpose and $\int_1$ denotes integral from 0 to T.
2. Average $\mathbf{R}^{\alpha}_{\mathbf{xx}}(\tau,1)$ over τ to obtain the averaged cyclic correlation matrix of $\mathbf{x}(t)$ for the first time frame, i.e., $\langle\mathbf{R}^{\alpha}_{\mathbf{xx}}(1)\rangle_\tau$.
3. Apply the Singular Value Decomposition (SVD) to $\langle\mathbf{R}^{\alpha}_{\mathbf{xx}}(1)\rangle_\tau$ to estimate all the DOAs of SOI for the first time frame, i.e., $\theta_i(1)$, where $i = 1,\cdots,I$.

Now we can estimate E_i. From [7], we have

$$\langle\mathbf{R}^{\alpha}_{\mathbf{xx}}(1)\rangle_\tau = \mathbf{A}(f_0+\alpha/2,1)\langle\mathbf{R}^{\alpha}_{\mathbf{ss}}(1)\rangle_\tau\mathbf{A}^H(f_0-\alpha/2,1) \tag{11}$$

where $\mathbf{A}$ is defined in (4), $\langle\mathbf{R}^{\alpha}_{\mathbf{ss}}(1)\rangle_\tau$ is the averaged cyclic correlation matrix of $\mathbf{s}(t)$ for the first time frame, which can be estimated by, based on (11),

$$\langle\mathbf{R}^{\alpha}_{\mathbf{ss}}(1)\rangle_\tau = \mathbf{A}(f_0+\alpha/2,1)^\dagger\langle\mathbf{R}^{\alpha}_{\mathbf{xx}}(1)\rangle_\tau\mathbf{A}^H(f_0-\alpha/2,1)^\dagger \tag{12}$$

where $[\cdot]^\dagger$ denotes pseudo inverse. If the duration of the time frame is long enough, $\langle\mathbf{R}^{\alpha}_{\mathbf{ss}}(1)\rangle_\tau$ can be viewed as an estimate of $\langle\mathbf{R}^{\alpha}_{\mathbf{ss}}\rangle_\tau$. Obviously, the (p,i)th element of $\langle\mathbf{R}^{\alpha}_{\mathbf{ss}}\rangle_\tau$ is $\langle r^{\alpha}_{s_p s_i}\rangle_\tau$. Therefore an estimate of E_i as in (8) is obtained.

The tracking algorithm can be developed as follows. For iteration k, or for the kth time frame, compute $\langle r^{\alpha}_{x_1 x_n}(k)\rangle_\tau$, for $n = 2,\cdots,N$. Now instead of estimating $\theta_i(k)$ anew, we update it by

$$\theta_i(k) = \theta_i(k-1) + \widetilde{\theta}_i(k) \tag{13}$$

where $\theta_i(k-1)$ is assumed to be estimated up to the $(k-1)$th iteration. Since $\theta_i(1)$ has been estimated in the initial step, $\theta_i(k)$ can be obtained if $\widetilde{\theta}_i(k)$ can be computed for all k. Now assume that $\widetilde{\theta}_i(k)$ is small, using Taylor expansion, dropping the terms of higher orders, we obtain

$$g_n(\theta_i(k)) = g_n(\theta_i(k-1)) + \frac{\partial g_n(\theta)}{\partial\theta}|_{\theta=\theta_i(k-1)}\widetilde{\theta}_i(k) \tag{14}$$

where

$$\begin{aligned}&\frac{\partial g_n(\theta)}{\partial\theta}|_{\theta=\theta_i(k-1)}\\ &\quad= \left[-j2\pi(f_0-\frac{\alpha}{2})(n-1)\frac{d\cos\theta_i(k-1)}{c}\right]g_n(\theta_i(k-1))\end{aligned} \tag{15}$$

Using (10) and (14), we obtain

$$\begin{aligned}&\langle r^{\alpha}_{x_1 x_n}(k)\rangle_\tau\\ &\quad= \sum_{i=1}^{I} E_i g_n(\theta_i(k-1)) + \sum_{i=1}^{I} E_i\frac{\partial g_n(\theta)}{\partial\theta}|_{\theta=\theta_i(k-1)}\widetilde{\theta}_i(k)\\ &\quad= \langle r^{\alpha}_{x_1 x_n}(k-1)\rangle_\tau + \sum_{i=1}^{I} c_{n,i}(k-1)\widetilde{\theta}_i(k)\end{aligned} \tag{16}$$

where

$$c_{n,i}(k-1) = E_i\frac{\partial g_n(\theta)}{\partial\theta}|_{\theta=\theta_i(k-1)} \tag{17}$$

Let

$$\widetilde{r}_n(k) = \langle r^{\alpha}_{x_1 x_n}(k)\rangle_\tau - \langle r^{\alpha}_{x_1 x_n}(k-1)\rangle_\tau \tag{18}$$

Then, from (16), $\widetilde{r}_n(k)$ can be written as

$$\widetilde{r}_n(k) = [c_{n,1}(k-1),\cdots,c_{n,I}(k-1)]\,\widetilde{\Theta}(k) \tag{19}$$

where

$$\widetilde{\Theta}(k) = \left[\widetilde{\theta}_1(k),\cdots,\widetilde{\theta}_I(k)\right]^T \tag{20}$$

Stack $\widetilde{r}_n(k)$ for $n = 2,\cdots,N$, we obtain

$$\widetilde{\mathbf{r}}(k) = [\widetilde{r}_2(k),\cdots,\widetilde{r}_N(k)]^T = \mathbf{C}(k-1)\widetilde{\Theta}(k) \tag{21}$$

Here $\mathbf{C}(k-1)$ is an $(N-1)$ by I matrix whose $(n-1,i)$th element is $c_{n,i}(k-1)$ as in (17). $\mathbf{C}(k-1)$ is known since E_i is estimated during initialization, and DOAs at the $(k-1)$th time frame are assumed to be updated through our recursive algorithm. Therefore, $\widetilde{\Theta}(k)$, or the DOA changes can be estimated by solving the LS problem of (21), and the DOAs at the kth time frame are updated using (13).

One problem of the above method occurs when two or more DOAs are crossing at the $(k-1)$th time frame. In this case, $\mathbf{C}(k-1)$ in (21) will be rank deficient, and the solution of $\widetilde{\Theta}(k)$ may become unreliable. To overcome this problem, we further assume that the sources are moving at constant speeds, thus the DOA changes for two adjacent time frames are almost the same, i.e.,

$$\widetilde{\Theta}(k) \approx \widetilde{\Theta}(k-1) \tag{22}$$

Now define a revised LS cost function

$$\begin{aligned} & f(\widetilde{\Theta}(k)) \\ & = \left[\mathbf{C}(k-1)\widetilde{\Theta}(k) - \widetilde{\mathbf{r}}(k)\right]^H \left[\mathbf{C}(k-1)\widetilde{\Theta}(k) - \widetilde{\mathbf{r}}(k)\right] + \\ & \quad \left[\widetilde{\Theta}(k) - \widetilde{\Theta}(k-1)\right]^H \Lambda(k) \left[\widetilde{\Theta}(k) - \widetilde{\Theta}(k-1)\right] \end{aligned} \tag{23}$$

The first term of this cost function reflects the LS criterion of equation (21), and the second term reflects the assumption (22). Here $\Lambda(k)$ is an I by I diagonal matrix whose ith element is $\lambda_i(k)$. The value of $\lambda_i(k)$ can be chosen by the designer to further optimize performance.

Now the $\widetilde{\Theta}(k)$ which minimizes the cost function (23) is the estimated DOA changes for the kth time frame. The solution to this is

$$\begin{aligned} \widetilde{\Theta}(k) = & \left[\mathbf{C}^H(k-1)\mathbf{C}(k-1) + \Lambda(k)\right]^{-1} \\ & \left[\mathbf{C}^H(k-1)\widetilde{\mathbf{r}}(k) + \Lambda(k)\widetilde{\Theta}(k-1)\right] \end{aligned} \tag{24}$$

The role of $\Lambda(k)$ is clear from (24). When $\mathbf{C}(k-1)$ is rank deficient at DOA crossing, $\Lambda(k)$ in (24) provides regularization effect to the solution $\widetilde{\Theta}(k)$. Normally, entries of $\Lambda(k)$ should be small positive numbers. In our simulation, we choose $\lambda_i(k)$ equal to $1/5 \sim 1/2$ of the ith diagonal element of $\mathbf{C}^H(k-1)\mathbf{C}(k-1)$.

Below is a summary of our signal selective tracking algorithm.

1. At time interval $[0, T]$, i.e., $k = 1$

- **a.** Apply ACM [7] to estimate $\Theta(1) = [\theta_1(1), \cdots, \theta_I(1)]^T$. $\langle r^{\alpha}_{x_1 x_n}(1)\rangle_\tau$ can also be obtained during this procedure.
- **b.** Estimate E_i from (12) and then (8).
- **c.** Set $\widetilde{\Theta}(1)$ to 0 or a small value.

2. At time interval $[(k-1)T, kT]$, for $k = 2, \cdots, K$

- **a.** Estimate $\mathbf{C}(k-1)$, whose $(n-1, i)$th element is $c_{n,i}(k-1)$ as in (17), for $n = 2, \cdots, N$, $i = 1, \cdots, I$
- **b.** Estimate $\langle r^{\alpha}_{x_1 x_n}(k)\rangle_\tau$ from the data samples collected in this time interval, and then obtain $\widetilde{\mathbf{r}}(k)$ whose $(n-1)$th element is as in (18), for $n = 2, \cdots, N$.
- **c.** Estimate $\widetilde{\Theta}(k)$ from (24).
- **d.** Obtain $\Theta(k) = \Theta(k-1) + \widetilde{\Theta}(k)$

Note that when calculating the cross cyclic correlation $r^{\alpha}_{x_1 x_n}(\tau, k)$ as in (6), no assumption is made that $s_p(t)$ and $s_i(t)$ are cyclically uncorrelated. Therefore, our recursive algorithm works even when the sources are cyclically correlated, or coherent. The only modification needed in such a situation is the initialization. We need some technique such as Spatial Smoothing (SS) [13, 14] to decorrelate the coherent sources in order to estimate the initial DOAs and $\langle r^{\alpha}_{s_p s_i}\rangle_\tau$.

4. SIMULATIONS

The following simulation is carried out to illustrate our signal selective tracking algorithm. Three sources are assumed to emit three wideband BPSK signals with raised cosine pulse shaping. Two of them are SOI with baud rate 20 MHz and carrier frequency 100 MHz. The other is interference with baud rate 6 MHz and carrier frequency 80 MHz. Furthermore the two SOI are coherent. A ULA with 7 antennas are used. The subarray size is 6 for SS during intialization. The duration of the time frame is 0.5 second. The Signal to Noise Ratio (SNR) of one SOI is 1 dB lower than the other. The SNR of the interference is 5 dB lower than the higher powered SOI. Now three tests are carried out with the SNR of the higher powered SOI as 5 dB, 10 dB and 15 dB. The results are shown in Fig. 1, Fig. 2 and Fig. 3, respectively. It can be seen that our algorithm can suppress the interference and track the DOA changes of coherent wideband cyclostationary signals successfully even when the DOAs are crossing.

5. REFERENCES

[1] C. K. Sword, M. Simaan, and E. W. Kamen, "Multiple target angle tracking using sensor array outputs," *IEEE Trans. Aerospace and Electronic Systems*, vol. 26, pp. 367–373, Mar. 1990.

[2] S. B. Park, C. S. Ryu, and K. K. Lee, "Multiple target angle tracking algorithm using predicted angles ,“ *IEEE Trans. Aerospace and Electronic Systems*, vol. 30, pp. 643-648, Apr. 1994.

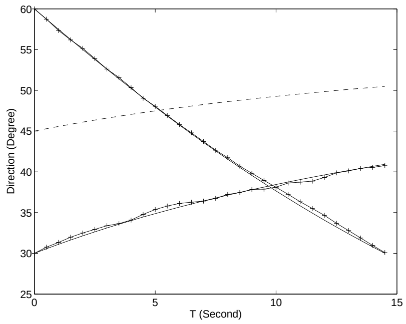

Fig. 1. DOA tracking for two SOI with one interefence. − actual tracks of the SOI, − ∗ − estimated tracks, −− track of the interefence, SNR of the higher powered SOI: 5 dB.

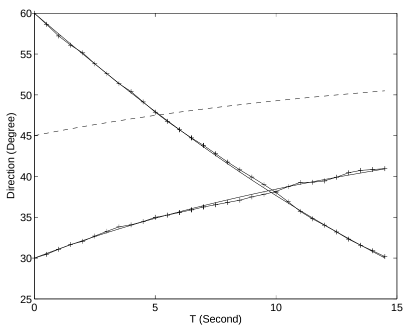

Fig. 2. DOA tracking for two SOI with one interefence. − actual tracks of the SOI, − ∗ − estimated tracks, −− track of the interefence, SNR of the higher powered SOI: 10 dB.

[3] C. R. Rao, C. R. Sastry, and B. Zhou, “Tracking the direction of arrival of multiple moving targets,” *IEEE Trans. Signal Processing*, vol. 42, pp. 1133–1144, May 1994.

[4] Y. Zhou, P. C. Yip, and H. Leung, “Tracking the direction-of-arrival of multiple moving targets by passive arrays: algorithm,” *IEEE Trans. Signal Processing*, vol. 47, pp. 2655–2666, Oct. 1999.

[5] W. A. Gardner, “Simplification of MUSIC and ESPRIT by exploitation of cyclostationarity,” *Proc. IEEE*, vol. 76, pp. 845-847, July 1988.

[6] P. Charge, Yide Wang and J. Saillard, “An extended cyclic MUSIC algorithm,” *IEEE Trans. Signal Processing*, vol. 51, July 2003.

[7] H. Yan and H. Fan, “Wideband cyclic MUSIC algorithms,” *Signal Processing (Elsevier)*, vol. 85, pp. 643-649, Mar. 2005.

[8] J. Riba, J. Goldberg, G. Vazquez, and M. A. Lagunas, “Signal selective DOA tracking for multiple moving targets,” in *Proc. of IEEE International Conference on Acoust., Speech, and Signal Processing*, vol. 5, May 1996, pp. 2559–2562.

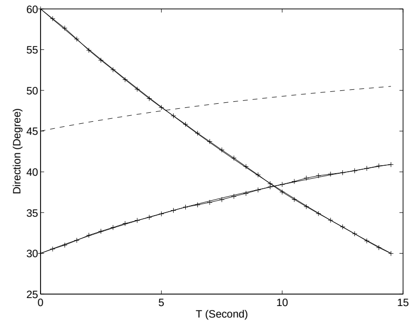

Fig. 3. DOA tracking for two SOI with one interefence. − actual tracks of the SOI, − ∗ − estimated tracks, −− track of the interefence, SNR of the higher powered SOI: 15 dB.

[9] J. Xin, H. Tsuji, H. Ohmori, and A. Sano, “Directions-of-arrival tracking of coherent cyclostationary signals without eigendecomposition,” in *Proc. of IEEE Third Workshop on Signal Processing Advances in Wireless Communications*, Mar. 2001, pp. 318–321.

[10] W. Ng, J. P. Reilly, and T. Kirubarajan, “A Bayesian approach to tracking wideband targets using sensor arrays and particle filters,” *Proc. of IEEE Workshop on Statistical Signal Processing*, pp. 510-513, Sept. 2003.

[11] A. Satish, and R. L. Kashyap, “Wideband multiple target tracking,” *Proc. of IEEE International Conference on Acoust., Speech, and Signal Processing*, vol.4, pp.517-520, April 2004.

[12] M. Wax and T. Kailath, “Detection of signals by information theoretic criteria,“ *IEEE Trans. Acoustics, Speech, and Signal Processing*, vol. 33 , pp. 387-392, Apr. 1985.

[13] T.-J. Shan, M. Wax, and T. Kailath, “On spatial smoothing for direction-of-arrival estimation of coherent signals,” *IEEE Trans. Acoust., Speech, and Signal Processing*, vol. 33, pp. 806-811, Aug. 1985.

[14] S. U. Pillai and B. H. Kwon, “Forward/backward spatial smoothing techniques for coherent signal identification,” *IEEE Trans. Acoust., Speech, and Signal Processing*, vol. 37, pp. 8-15, Jan. 1989.