BUILDING A BAYESIAN FRAMEWORK TO ANALYSE MEASUREMENTS FROM THE LIGNE D'INTÉGRATION LASER FACILITY

X. Vallières, R. André and P.Minvielle-Larrousse*

Commissariat à l'Energie Atomique, Centre d'Etudes Scientifiques et Techniques d'Aquitaine, France
* Laboratoire de Physique et Chimie de l'Environnement, CNRS, Orléans, France

ABSTRACT

The LIL facility is the small-scale prototype of the Laser MégaJoule (LMJ) that is one of the two high power laser facilities for nuclear fusion by inertial confinement. Several diagnostics have been set up to characterise both laser performances and laser-matter interactions. Therefore, we wish to describe the state of a physical system knowing noisy measurements, by using a probabilistic approach. Following Clapp et al. [1], we have set up a simulated annealed particle filter that determine the posterior probability density function given measurement, calibration data and a priori knowledge on the analysed system. This implementation has been applied to the characterization of focal spot intensity.

1. INTRODUCTION

The LIL facility is the small-scale prototype of the Laser Méga Joule (LMJ) that is one of the two high power laser facilities for nuclear fusion by inertial confinement. LIL has been designed to validate technological concepts that will be used by the LMJ. Actually, it includes 4 identical laser beams (40 x 40 cm wide) that deliver an energy of approximately 15 kJ each at a 350 nm wavelength when focused in the experiment chamber.

Several diagnostics have been set up to characterise both laser performances and laser-matter interactions. Therefore, we have to deal with the classical problem of describing the state of a physical system knowing noisy measurements. To get a valuable comparison of different experiments or diagnostics results, we have to determine precisely the measurement uncertainties. More, because several measurements are by nature highly interrelated, we have to use a framework that could naturally take this property into account, to increase the measurement reliability.

We have chosen to follow the probabilistic approach, and built a general Bayesian framework to analyse our measurements [2,4,7,8]. Therefore, we need to determine the posterior probability density function (pdf) of the system state, given measurements, diagnostics calibration data and our knowledge of the system under our interest (a priori).

In order to get a computational scheme that could be applied (with variations) to the major part of our measurements, we have explored the simulated annealing particle filter [1]. This paper describes the computational scheme used and shows the results obtained in a feasibility study devoted to the characterisation of the laser focal spot.

2. IMPLEMENTATION OF THE BAYESIAN FRAMEWORK

Given a model that is assumed to generate the observed measurement *m* from a system parameterised by θ, Bayes' theorem relates the posterior probability distribution $p(\theta/m)$ of the model parameter given *m* to the parameter prior $p(\theta)$ and to the likelihood $p(m/\theta)$, through

$$p(\theta|m) = \frac{p(m|\theta)\,p(\theta)}{\int_{\Sigma} p(m|\theta)\,p(\theta)\,d\theta}$$

Taking the general case, we consider that we do not have any specific a priori on the parameter values and take a uniform distribution ($p(\theta) = cste$). Likelihood then plays a major role and will be evaluated depending on the considered problem.

2.1. Measurement model

This first analysis is dedicated to the analysis of the laser focal spot, using a specific diagnostic called Système d'Analyse de la Tache focale (SAT). One of the task of this device is to make an image of the laser beams at the focal plane. The image analyse should give pertinent parameter such as its maximum intensity, its size, location and shape.

The intensity model consists in a two-dimensional SuperGaussian shape parameterised by the centre location (x_c, y_c), the radius (r), the maximum amplitude (I_0) and the SuperGaussian order (n) and is given by

$$I(x,y) = I_0 e^{-\left(\frac{(x-x_c)+(y-y_c)}{(2r)^2}\right)^n}$$

The likelihood is then given by the joint likelihood of each pixel. Since the intensities of pixels can be seen as independent, the likelihood is given by

$$L(\theta|m) = p(\theta|m) = \prod_i p_i(\theta|m)$$

The individual likelihood $p_i(\theta/m)$ of each pixel is supposed to follow a Gaussian distribution whose mean value is the model intensity for θ at the considered pixel and whose variance is supposed to be representative of the image noise.

2.2. Simulated annealing

Particle filters are commonly used to explore high-dimensional state space in a Bayesian framework. They provide a statistical representation of the posterior density. Several authors propose to draw the posterior density with a particle filter in which the likelihood is introduced gradually, starting from the prior [1,3,11]. The successive steps are inspired by Simulated Annealing. This algorithm then introduces continuously the influence of narrow peaks in the target density and has proven its efficiency in the case of non Gaussian and multimodal posterior densities.

2.3. Importance Sampling

Markov Chain Monte Carlo (MCMC) sampling is often applied to generate samples (or particles) while exploring a state space using a Markov transition kernel that leaves invariant the target distribution (e.g. [5]). To limit the degeneracy of the algorithm, an importance sampling / resampling scheme is adopted, prior to the MCMC moves. The effect of importance sampling is to discard small weight trajectories and to concentrate upon large weight trajectories in the state space.

As shown in Figure 1, a set of samples from the prior density is drawn. Each sample is weighted according to a function depending on the current annealing rate and given by $q(\theta) = p(\theta).p(m/\theta)^{1-\alpha_m}$, where α_m is the rate of annealing at step *m*, ranging from 1 to 0. At step 0, the distribution reduces to the prior. When $\alpha = 0$, the annealing process stops and the distribution of particles gives the posterior density.

The effective sample size, or survival rate, is a good estimate of the algorithm degeneracy. When the effective sample size is too low, the number of samples used to describe the target density is not sufficient. To maintain a sufficient sample size, a multinomial resampling is applied [6].

The annealing schedule is self-calibrated by maintaining a minimum survival rate through the successive steps of the algorithm. A classical dichotomy search gives the current annealing rate.

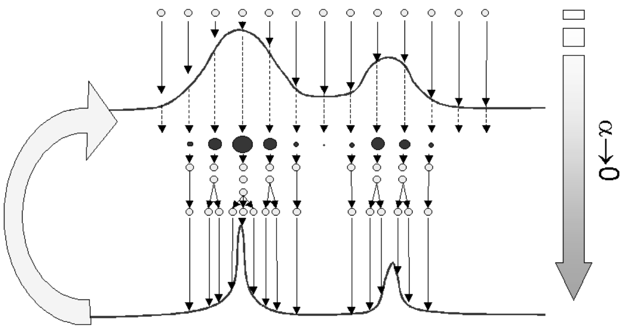

Figure 1: Details of the algorithm used: importance Sampling / Resampling + MCMC moves in a Simulated Annealing scheme.

2.4. Markov Chain Monte Carlo

MCMC moves that leave $q(\theta)$ invariant are then applied to the particles to generate the samples for the next annealing step. The proposal density for these moves is a Gaussian with a self-calibrated width. Several tries are performed with a decreasing width, depending on the annealing rate and on the number of tries, until a sufficient rate of acceptance for the moves is reached. The range for the proposal width is given as an input of the algorithm.

The annealing process stops when $\alpha = 0$. The set of particles is then representative for the posterior density and the posterior maximum or other statistical estimates can be computed.

3. RESULTS

The annealed particle filter algorithm is applied to the problem of the intensity model parameterization over a simulated SAT image.

3.1. Average Values

Figure 2 shows 4 steps of the annealed search: step 0 $(\alpha = 1)$, step 4 $(\alpha = 0.94)$, step 8 $(\alpha = 0.56)$ and step 12$(\alpha = 0)$. Upper plots represent the position in the (x_c, y_c, r) space of the 500 samples with the location of the most likely sample (dotted lines) and lower plots show their position in the (I_0, n) plane.

Results obtained on the centre, the radius and the maximum intensity are good and samples concentrate on the correct solution.

Since the pdf is known, one can easily extract all useful quantities, such as mode, mean and uncertainties. For example, Figure 3 shows the pdf associated to the focal spot position.

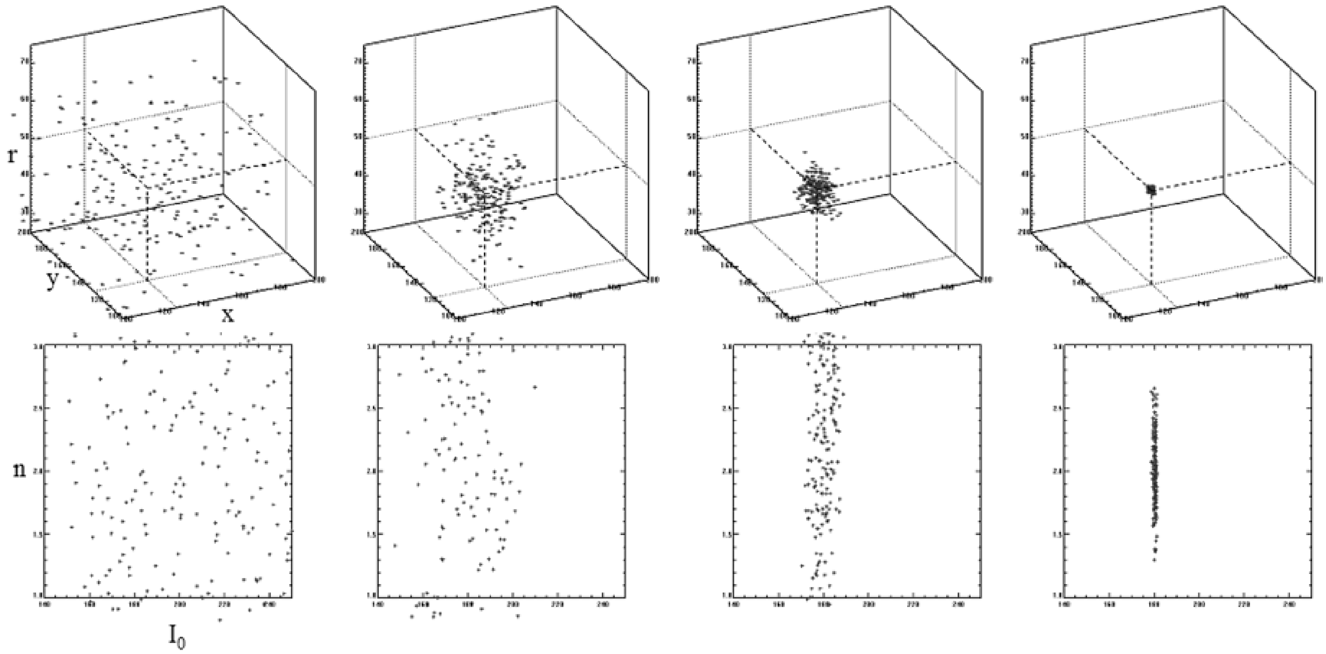

Figure 2: Annealed search of the SAT intensity model parameters from step 0 to step 12

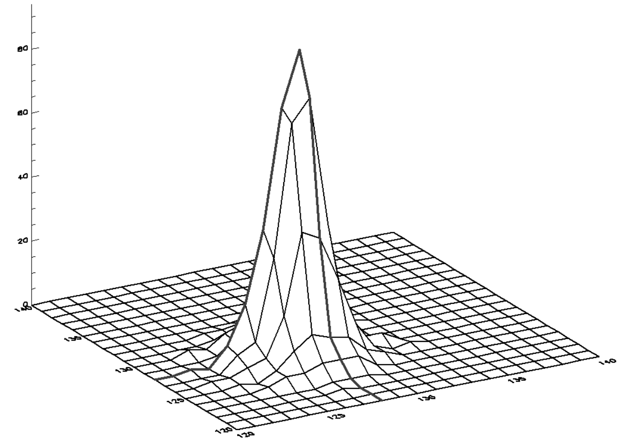

Figure 3: pdf associated to the focal spot position (x_0, y_0)

3.2. Model selection

On the SuperGaussian order, the dispersion is larger. The filter failed to concentrate samples because small variations of *n* implies very small variations of the intensity shape.

Since *n* has to be an integer, one can define different models (*n*=1, 2 or 3) and for each, compute its marginal probability of occurrence. Doing so, we have found that the model n=2 has a probability of 67% whereas models 1 and 3 have respectively a probability of 4% and 29%

4. CONCLUSION

Some topics regarding the processing of the LIL signals are addressed in a Bayesian-based approach. An annealed particle filter, based upon MCMC sampling and Simulated Annealing, is implemented for the characterization of focal spot intensity and for a classical pose estimation problem. Several parameters of the algorithm like the annealing rate, the width of proposal distributions are self-tuned. At this stage of the study, results are shown on simulated images where it shows good results on the intensity characterization.

Next step is the implementation of a model selection technique (reversible jump [9, 12]) that could distinguish if the whole image is better characterised by 1 or several spots.

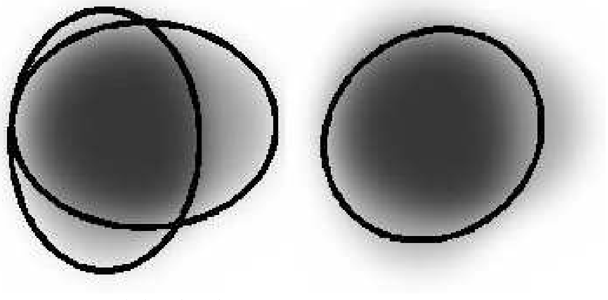

Figure 4: Model selection: 1 or 2 spots ?

5. REFERENCES

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