

# SIMPLE AND HIGH-PERFORMING DOA ESTIMATION BY A FIELD OF BLIND BEEPERS

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## ABSTRACT

The estimation of the DOA of an acoustic wavefront is considered in the context of the Wireless Sensor Networks (WSNs). An ensemble of simple acoustic antennas, individually with no directional capability, is employed to the estimation aim. The network formed by these antennas (nodes) follows the SENMA paradigm: a mobile rover polls the nodes of the network and implement the estimation task after a certain number of queries of the nodes lying inside its mobile and eccentric field of view. Notwithstanding the DOA-blind character of individual nodes, their aggregation and the directivity of the rover's lobe makes possible inference of the acoustic DOA. The estimation error of the proposed method is close to that of the maximum likelihood (ML) approach, with the remarkable difference that the designed strategy is by far simpler to implement.

## I. INTRODUCTION

We are interested in estimating the Direction Of Arrival (DOA) of an acoustic wave by means of a wireless network of acoustic sensors. The elected network architecture follows the SENMA (SEnsor Network with Mobile Agents) paradigm [1]: a mobile agent (e.g., a rover) travels across the area where the sensors are located and collects information stored at the remote nodes, implementing the final task of DOA-finding. There are many advantages in using the SENMA architecture over other wireless network structures, and one that is particularly important is energy efficiency. It is well-known that a bottleneck of the Wireless Sensor Network (WSN) design is the amount of energy that any remote node uses for data transmission. This concern may be particularly acute in one-hop structures where all the remote nodes communicate with a single fusion center that can be spatially distant from many sensors. Alternatively, in the multi-hop design, the messages arrive to the fusion center after multiple intermediate steps over

neighboring nodes, and a possible drawback is that the nodes of the network must transmit *and receive* data: for any transmitting sensor there are many (those within a certain range) that must listen. This may be critical in terms of sensors' energy requirements. The SENMA model can be shown to be more energy-efficient with respect to ad-hoc flat multi-hop wireless networks, and the energy gain grows with the network size [1].

As to the nature of the employed devices, the general trend in WSN design is toward a large number of *dumb* (i.e., small, inexpensive, light, possibly unreliable, ...) nodes that, as a whole, guarantee system reliability, node masking, robustness etc., and are capable of far more complicated tasks with respect to a single, extremely powerful, *supernode*. In many practical scenarios the sensors cannot be located over a certain surveyed area in a pattern as one likes: instead they are randomly disseminated (e.g., broadcast by an aircraft). Note finally that, in complying with the required dumbness/cheapness of the devices, the sensors cannot carry out elaborate signal processing tasks, nor they may incorporate sophisticated acoustic antennas for DOA finding.

In designing the network we keep in mind the above constraints. As consequences, we rely upon the following assumptions:

- the remote nodes are acoustic sensors with no directional capabilities, i.e., they are completely *blind* to the DOA of a remotely generated (i.e., plane) wavefront;
- the sensors do not communicate with each other;
- they are randomly located over a certain area of interest (precisely: sensors are displaced according to a homogeneous Poisson field model);
- the density of the nodes (number of nodes per unit of area) may be unknown;
- each node's position is unknown to the rover, as well as to other nodes in the network;

- the capacity of the power supplies of individual nodes is extremely limited; consequently, most of the time the sensors must remain silent (this is also compatible with covertness);
- the nodes are designed for transmitting simple short electromagnetic pulses (called *beeps*); no digital communication schemes can be settled between nodes and rover.

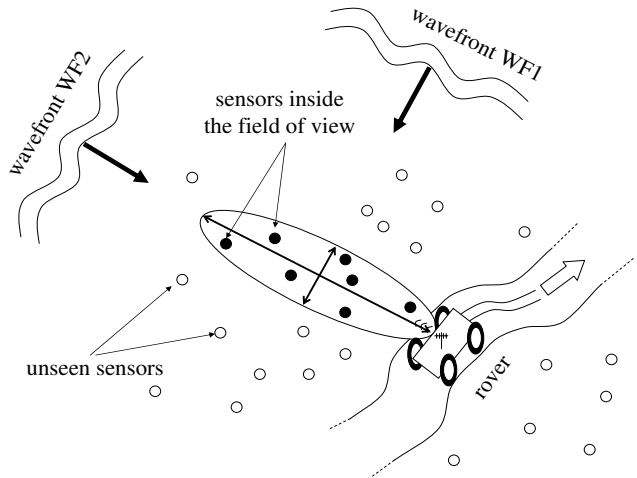
The rover, as should be clear, may be designed with much less stringent constraints. It polls the sensors lying inside its field of view, and waits for their “beeps”; then moves to a new position, polls again, records the beeps, and so on. The rover’s field of view (also referred to as the *lobe*), here assumed elliptical for simplicity, is the region around the rover where the sensors can be *heard*: all the pulses emitted by the sensors inside are collected at the rover, while no signal from outside can be recorded. It is assumed also that the field of view can be arbitrarily oriented and, in fact, successive rover dwells are taken with different (let us say *random*) orientations. To this aim one can assume that the mobile agent is equipped with sophisticated antenna arrays or, more simply, that the rover’s antenna might be rotated mechanically. Consider now the following questions.

- Is the DOA estimation possible with the design constraints detailed above? Can such a network of dumb and DOA-blind sensors perform the required task?

That the answer is affirmative can be found from [2]. But here we go further: we want the estimation be analytically simple and easy to implement. In the present work we not only provide a means for achieving these goals, but also show that the gain of our method may reach an order of magnitude in terms of MSE, with respect to the results of [2]. In fact, the estimation procedure that we propose is close in performance to the (asymptotically optimal) Maximum Likelihood (ML) approach, but is much simpler to implement. An more in depth treatment of the subject can be found in [3] to which the reader is referred for further details.

## II. PROPOSED APPROACH

Assume that the wavefront of an acoustic plane wave meets a generic sensor of the network at time  $\tau$ . Then, such a sensor *forms* the signal  $x(t) = \sum_k p(t - \tau - kT)$ , where  $p(t)$  is some kind of short pulse (beep) and  $T > 2r/v$ . Here  $v$  is the velocity of the sound in the considered medium, and  $r$  is the major axis of the elliptical field of view (i.e., the maximum distance at which the rover may hear a beep). When we say that the sensor forms the signal  $x(t)$ , we are not assuming that a real electromagnetic transmission is activated and in fact the sensor persists in its silent status until a specific polling signal is received. Only when polled does the virtual transmission become an actual emission



**Fig. 1.** Schematic representation of the addressed scenario showing the rationale behind the proposed estimation strategy, based on estimating the impinging-time support. The two extreme cases are the wavefronts WF1 and WF2, with the DOA respectively broadside and endfire with respect to the rover’s elliptical field of view. The lengths of the major and minor axis of the ellipse are  $r$  and  $h$ , respectively.

of the train of beeps. Such a *wake-up* message is emitted by the mobile agent and is received by all the sensors inside its field of view. After having alerted the nodes, the rover records the train of beeps for an amount of time not smaller than  $2T$ . A little thought reveals that such an interval allows the rover to *order* the received beeps in such a way that it can measure the time differences of the impinging times over different sensors, see [3] for details. In particular the rover may measure the *spread* of the impinging times, that is the difference between the maximum and the minimum. As we promptly show, the DOA estimation is built from this observable. Then, the rover moves to another position for a new polling/recording (snapshot) job. Successive rover positions are assumed far enough from each other, so that successive rover snapshots never involve a sensor already polled – that is, we assume independence.

The estimation procedure we propose relies upon the range of the observed impinging times, *integrated* over successive snapshots. The rationale is simple. With reference to figure 1, if the wavefront is that denoted by WF1, then the maximum spread of the impinging times cannot exceed  $h/v$ , with  $h$  the ellipse’s minor axis. If, conversely, the impinging wavefront is WF2, then the maximum time spread is bounded from above by  $r/v$ . More generally, the half-support of the impinging times for an arbitrary DOA

$\theta$  can be shown to be

$$\Omega_s = \frac{1}{2v} \sqrt{r^2 \cos^2(\theta - \phi_s) + h^2 \sin^2(\theta - \phi_s)},$$

where  $\phi_s$  is the rover's lobe orientation at snapshot  $s$ . (Assume for concreteness that the angles  $\theta$  and  $\phi_s$  are measured in figure 1 counterclockwise with respect to the vertical direction and "North".) Clearly, how much the measured time spread is close to the support  $2\Omega_s$  is a matter that depends upon the actual (unknown) sensor locations inside the lobe. However, on the average, the larger is the density of sensors (that is, more sensors lie in the field of view), the closer the measured values are to their bounds, in light of the uniform distribution of the nodes imposed by the Poisson law<sup>1</sup>. It is then clear that the DOA may be inferred by the spread of the impinging times. In fact, at successive snapshots, the range of allowable DOAs may be progressively restricted on the basis of the current estimate of  $\Omega_s$ , and the process eventually ends up with a very small interval inside which the actual DOA must be found.

We now formalize the described procedure, making reference to figure 2. We want to estimate the half-support  $\Omega_s$ , and a possible estimator is the half-range (one half of the maximum spread) of the measured times. Thus, at snapshot  $s = 1$ , we have an estimate, say  $\hat{\Omega}_1$  and accordingly we can select a *range* of values of  $\theta$  that are *admissible*. Let us denote such a range with  $\Theta_1^*$ . At the next snapshot we have another estimate  $\hat{\Omega}_2$ , and a new range  $\Theta_2^*$  of admissible DOAs is found. However, only  $\Theta_2 = \Theta_2^* \cap \Theta_1^*$  is admissible after the second snapshot. Proceeding in this way, after  $M$  dwells we get  $\Theta_M = \bigcap_{s=1}^M \Theta_s^*$ . This is an interval estimator of the DOA, and the sought DOA estimate  $\hat{\theta}$  may be defined as an arbitrary (say, randomly chosen) value belonging to the set  $\Theta_M$ .

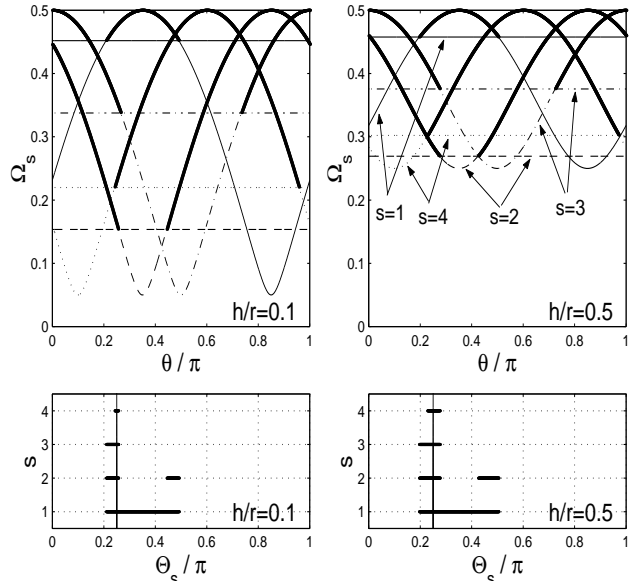
### III. RESULTS

The performances of the described DOA finding procedure, in terms of Mean Square Error (MSE) have been investigated by computer experiments. For the sake of comparison, we also consider the alternative estimator proposed in [2], as well as the ML estimator. This latter assumes as observables the impinging times over different sensors and exploits the whole probability density function of these observations<sup>2</sup>.

The MSE pertaining to the estimators are summarized in figure 3. It is seen that the MSE of the proposed method approaches that of the ML after some tens of snapshots,

<sup>1</sup>In this respect, it is worth noting that a homogeneous Poisson field assumption for the sensor location is not necessary, all that we need is a local property of uniformity.

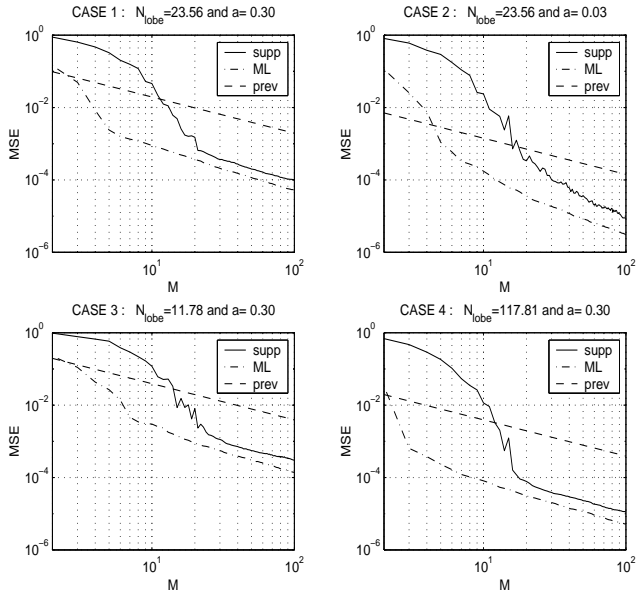
<sup>2</sup>Actually, the rover lacks of an absolute time reference, so that all the information it can retrieve consists of the differences between impinging times: the ML estimator uses these differences as observables.



**Fig. 2.** The rationale of support-based DOA estimation. Top plots illustrate the behavior of  $\Omega_s(\theta)$  versus  $\theta$ . Different linestyles refer to different snapshots  $s = 1, 2, 3, 4$  (i.e., to different  $\phi_s$ ), as indicated in the right plot. The horizontal lines represent the half-support estimation, say  $\hat{\Omega}_s$ , with the snapshot index  $s$  identified again by the linestyle. The bold portion of the curves highlights the regions such that  $\Omega_s \geq \hat{\Omega}_s$  and defines the range of values of  $\theta$  that are admissible. Successive intersections  $\Theta_s$  of such admissible values are correspondingly depicted in the bottom plots (horizontal axis), with respect to the snapshot index  $s$  (vertical axis). Left plots refer to  $r/v = 1$  (arbitrary units) and  $h/v = 0.1$ , while right plots refer to  $r/v = 1$  and  $h/v = 0.5$ . Note how the true value of  $\theta$  (vertical lines in bottom plots) is faster approached in the former case, in which  $a = h/r$  is smaller.

and then stays sufficiently close to that. The estimator proposed in [2] is considerably less efficient in terms of mean square error. The support-based estimator is by far the simplest to implement among the three.

As to the parameters that influence the estimation performances of our method, we expect that the larger the average number of sensors captured by the rover field of view at each snapshot, the better the range of the impinging time estimates the support  $\Omega_s$ . Thus, the performances are expected to increase by increasing  $N_{lobe} = \lambda \pi r h / 4$ , with  $\lambda$  being the average number of sensors per unit of area. On the other hand, it is intuitive that the sensitivity of the estimation procedure to the DOA is a function of the aspect ratio  $a = h/r$ , with better performances achieved with smaller  $a$ . Finally, it is obvious that the estimate error should reduce as the number  $M$  of total (independent)



**Fig. 3.** The MSE of the proposed estimator  $\hat{\theta}_M$  (labelled as *supp*). For comparison purposes, the MSE attained with the estimation procedure proposed in [2] (see *prev*) is also drawn. Noticeably, almost an order of magnitude is gained with the present estimator, which is also considerably simpler to implement. The MSE pertaining to the asymptotically optimum ML estimate shows that the proposed support-based procedure is nearly optimal. The four cases considered reproduce the scenarios addressed in [2]. Case 1 assumes  $\lambda = 100$ ,  $r = 1$ ,  $h = 0.3$  (in arbitrary measure units); in case 2 we have  $\lambda = 10$ ,  $r = 10$ ,  $h = 0.3$ ; case 3 refers to  $\lambda = 50$ ,  $r = 1$ ,  $h = 0.3$ ; finally, in case 4 we set  $\lambda = 500$ ,  $r = 1$ ,  $h = 0.3$ .

snapshots taken increases. All these intuitions are confirmed by our numerical investigations, as also witnessed by the curves in figure 3.

#### IV. CONCLUSIONS

A sensor network of DOA-blind sensors is employed for an estimation task aimed to discover the direction of arrival of an acoustic signal. The elected network architecture follows the SENMA paradigm, where sensors (nodes) are queried by a mobile agent that travels across the surveyed area. When polled for, any sensor lying in the rover's field of view transmits a conventional periodic signal (say a train of *beeps*), from which the rover may extract the relative times of the wavefront impinging over individual sensors. Using the maximum spread of such impinging times the DOA may be inferred, provided that the rover's lobe exhibits an eccentricity. The lobe's shape is here taken as an ellipse for simplicity, but this is no loss of generality. The rover takes successive independent snapshots and

ends up with the estimate  $\hat{\theta}$  sought by reducing, dwell after dwell, the range of candidate DOAs. This is made more efficiently if the rover's lobe orientation is arbitrarily rotated at each snapshot, as we have assumed.

The proposed method is simple to implement and achieves performance reasonably close to those of the (much more computationally demanding) maximum likelihood approach. Comparison with estimators previously proposed in the literature shows that the present method substantially outperforms the one offered in [2], gaining near one order of magnitude in terms of MSE.

While clearly appealing in the present form, the proposed DOA estimation might be generalized to take into account some further aspect of practical relevance. In the following we emphasize the issues that, in the author's opinion, may merit further attention. First of all, we assume that the rover's field of view is elliptical, and this assumption seems not critical. However, such a lobe is *deterministic* in the sense that all is visible inside and nothing outside. A more reasonable model should account for some randomness, with any sensor being awakened and heard by the rover according to a certain probability. Second, we limit the analysis to a single wavefront: at the present our method is able to estimate the DOA of a single emission. Third, our wavefronts are here straight lines, meaning that the design assumes that the sensors lie in the far field with respect to the acoustic source: practically, one should admit near-field sources. Finally, note that sensors should maintain a certain synchronism, as the DOA estimation is performed by exploiting the difference of the impinging times of the wavefront over different nodes. It should also be emphasized that the sensors' beeps might remain virtual for a long time before being actually sent to the rover, so that moderate drifts of the sensors' internal clock may be problematic. In this respect we note that estimating the DOA of an electromagnetic source seems a far more complicated task, in that the required timing accuracy is orders of magnitude more stringent with respect to the case here treated of acoustic waves. Currently, many of the above issues are under investigation.

#### V. REFERENCES

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