

# Gain-Scheduling Techniques for Missile Autopilot Synthesis

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**Abstract—** This paper presents a comparison between the Controller Blending (CB) and the Gain Blending (GB) gain scheduling techniques, applied on a pitch-axis missile autopilot design problem.  $\mathcal{H}_\infty$  output feedback dynamic controllers with additional LMI pole placement constraints are designed for distinct operating points throughout the missile's operating domain for use with the (CB) technique. The same controllers are converted to state feedback/observer based controllers for use with the (GB) technique via Youla parameterization. In both cases, a global nonlinear control law giving satisfactory results is designed by interpolation according to Mach number and only output reference information. The results are motivated by the need of maintaining strict performance and disturbance rejection constraints for the missile model using linear controller synthesis tools.

## I. INTRODUCTION

GAIN scheduling is one of the most important design methodologies in the academic and industrial communities with many successful applications in both domains. The main idea behind this technique is to avoid using pure nonlinear control with its many drawbacks (complexity, lack of generality) and try, in a systematic manner, to adapt powerful linear design techniques directly to nonlinear systems. In general, this is performed by choosing certain 'important' operating points throughout the nonlinear system's operating domain and afterwards by computing a linearized model (and hence a controller) valid around each point. The global nonlinear controller is then obtained for any operating point by interpolating between the controllers being the closest to this point.

A systematic study of gain scheduling dates back to the work done by [20], [21] for a certain class of gain-scheduled systems (namely LPV systems) and has been considerably extended by other researchers [9], [17]. The important issue of stability is also considered in [19], [23] where theoretically justified algorithms for gain-scheduled systems are devised. A pair of surveys on the subject was recently published trying to summarize the results in the domain [13], [18].

On the other hand, there is a great number of articles that present different successful applications of gain scheduling

in many domains. The application of gain-scheduled  $\mathcal{H}_\infty$  controllers on missile autopilot design is considered in [15], [16]. In addition, an observer-based interpolation is used in [6], [11] whereas in [8] reduced order  $\mathcal{H}_\infty$  controllers using loop-shaping techniques are scheduled. Youla parameterization and LMI's are combined for multi-objective synthesis [5], and in [2] a gain-scheduled autopilot for a tail-controlled missile using  $\mu$ -synthesis techniques is presented. In [24] controller transfer function coefficients are interpolated for the autopilot problem of [15], whereas in [22] a comparison between an interesting technique (based on velocity-based linearization) and nonlinear dynamic inversion is presented. In [7] a comparison between  $\mathcal{H}_\infty$  and nonlinear dynamic inversion controllers is also performed. Finally, in [14] eigenvalue assignment is applied on the same missile model of [15].

In this paper, given that the controller interpolation issue is still open, a comparison between two interpolation methods is presented. They offer an alternative to LPV synthesis techniques, which remains very demanding in terms of computation resources and conservative in terms of finding a solution to the controller design problem. The first is the Controller Blending (CB) technique belonging to the family of ad-hoc techniques (in terms of stability guaranties) [12]. The second one is the Gain Blending (GB) technique that uses a state feedback/observer type control scheme for the interpolation, and belongs to the family of techniques for which stability guaranties for the gain-scheduled system do exist. Generic  $\mathcal{H}_\infty$  dynamic controllers are used for the first one and they are converted via Youla parameterization to an equivalent state feedback/observer form for use with the second one. The CB technique is chosen due to the advantages that it presents over other ad-hoc techniques such as transfer function coefficient and zero-pole-gain interpolation techniques [12]. The GB technique is chosen due to its wide use and the fact that it offers some important stability guaranties [5], [11], [23].

The techniques are applied to the missile-autopilot synthesis model introduced in [15]. This particular model is simple in form but it also stays 'sufficiently globally nonlinear' for gain-scheduling techniques to be tested. Several points are highlighted throughout the paper (such as open-loop control and loop shifting techniques), being motivated by the need of a systematic and repetitive procedure for gain scheduling and the results are presented in a comparative manner.

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The paper is organized as follows: In Section II the missile autopilot problem is reviewed, giving all the appropriate equations and performance-control constraints. In Section III the important problem of the open loop control unit is discussed. In Section IV the closed loop control unit is presented. The LTI synthesis model is detailed along with the LMI constraints imposed on the closed loop poles. In addition, the two scheduling techniques that are used for the global controller are detailed. Finally, in Section V the simulation results are presented comparing at first the two techniques and afterwards the GB technique is further investigated.

## II. PROBLEM DESCRIPTION

### A. Missile Modeling

The missile state dynamics (angle of attack  $\alpha$  in *rad* and pitch rate  $q$  in *rad/s*) are described by the following equations [15];  $\delta$  being the control variable (pitch fin deflection angle in *rad*)

$$\begin{aligned}\dot{\alpha} &= K_\alpha M C_n [\alpha, M, \delta] \cos \alpha + q \\ \dot{q} &= K_q M^2 C_m [\alpha, M, \delta]\end{aligned}\quad (1)$$

The measured outputs are the pitch rate  $q$  and the missile's normal acceleration  $\eta$  (in *g*)

$$\eta = K_\eta M^2 C_n [\alpha, M, \delta]\quad (2)$$

The functions  $C_n, C_m$  are the aerodynamic coefficients given by

$$\begin{aligned}C_n [\alpha, M, \delta] &= C_n^* [\alpha, M] + d_n \left( \frac{180}{\pi} \delta \right) \\ C_m [\alpha, M, \delta] &= C_m^* [\alpha, M] + d_m \left( \frac{180}{\pi} \delta \right)\end{aligned}\quad (3)$$

The functions  $C_n^*, C_m^*$  are polynomial functions of the following form (depending only on  $\alpha$  and  $M$ ):

$$\begin{aligned}C_n^* [\alpha, M] &= a_n \left( \frac{180}{\pi} \alpha \right)^3 + b_n \left( \frac{180}{\pi} \alpha \right)^2 + \\ &\quad + c_n \left( 2 - \frac{M}{3} \right) \left( \frac{180}{\pi} \alpha \right) \\ C_m^* [\alpha, M] &= a_m \left( \frac{180}{\pi} \alpha \right)^3 + b_m \left( \frac{180}{\pi} \alpha \right)^2 + \\ &\quad + c_m \left( -7 + \frac{8M}{3} \right) \left( \frac{180}{\pi} \alpha \right)\end{aligned}\quad (4)$$

The model constants and parameters can be found in [15]. In addition, the Mach number  $M$  is considered as an exogenous parameter produced by the following equation

$$\dot{M} = \frac{1}{U_s} [-|\eta| \sin |\alpha| + A_x M^2 \cos \alpha]\quad (5)$$

### B. Control and Performance Constraints

The goal for the autopilot is to track step reference signals  $\eta_{d,eq}(t)$  of different magnitudes with certain strict performance upper bounds, namely  $t_s = 0.35s$  for the settling time, 10% of maximum overshoot and 1% of static error. This must be done for  $M \in [1.5, 3]$  and  $\alpha \in [0^\circ, 20^\circ]$ . In addition, the controller must be able to absorb external disturbances acting on  $\alpha$  with reasonable control signal rates. A second-order low-pass filter is added to describe the actuator dynamics, having a damping ratio  $\zeta$  and an undamped natural frequency  $\omega_\alpha$ .

## III. OPEN LOOP CONTROL

The open loop control unit should be able to provide control inputs to keep the missile on its desired equilibrium trajectory  $\eta_{d,eq}$ . The approach here is different from [14] and the open loop control input  $\delta_{d,eq}$  is calculated from desired equilibrium point data parameterized directly from the desired output  $\eta_{d,eq}$  and the Mach number  $M$ . The first step is to solve for  $\delta_{d,eq}$  from (2), supposing that we are on the equilibrium trajectory

$$\delta_{d,eq} = \frac{\frac{\eta_{d,eq}}{K_\eta M_{eq}^2} - C_n^*|_{eq}}{d_n \left( \frac{180}{\pi} \right)}\quad (6)$$

Supposing that the Mach number does not vary fast, an equilibrium state  $x_{d,eq} = [\alpha_{d,eq} \quad q_{d,eq}]^T$  can be imposed and the left hand sides of (1) will go to zero on equilibrium. By replacing (6) to (1b),  $\alpha_{d,eq}$  is computed as a function of  $[\eta_{d,eq}, M_{eq}]$  by solving the following polynomial equation (which is of third order due to each function  $C_i^*$  being a third degree polynomial)

$$C_m^*|_{eq} - \frac{d_m}{d_n} C_n^*|_{eq} + \frac{d_m}{d_n} \frac{\eta_{d,eq}}{K_\eta M_{eq}} = 0\quad (7)$$

The above equation is solved explicitly; this is done by using the classic method of Cardano for the solution of third degree polynomial equations.

It must be also noted that only one of the solutions of this equation has a physical meaning. Once  $\alpha_{d,eq}$  is computed  $q_{d,eq}$  can also be computed from (1a). Finally, the equilibrium input  $\delta_{d,eq}$  may be calculated from (6) as a function of the scheduling vector  $[\eta_{d,eq}, M_{eq}]$  only (Fig. 1).

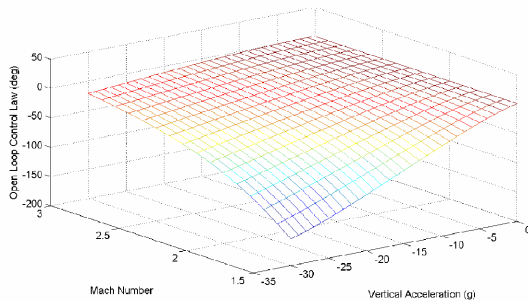


Fig. 1. Open loop control  $\delta_{d,eq}$  as a function of the desired output (vertical acceleration  $\eta_{d,eq}$ ) and Mach number  $M$ .

#### IV. CLOSED LOOP CONTROL

##### A. LTI Synthesis

The goal for the closed loop control  $\delta\delta$  is to restore the system's output trajectory  $\eta$  onto the desired equilibrium trajectory  $\eta_{d,eq}$  in case of a sudden disturbance, leaving to the open loop control  $\delta_{d,eq}$  the task of trajectory following.

The first step in gain scheduling is obtaining a linearized representation of the original system (1),(2). However, before linearization, the direct feedthrough term must be deducted from (2), in order to obtain a new system whose linearization will give strictly proper plants. This 'loop shifting' technique [3] will yield a new output  $\eta^*$  being

$$\eta^* = \eta - K_{\eta} d_n M^2 \left( \frac{180}{\pi} \delta \right) \quad (8)$$

The original system (1), (2) is linearized around a specific set of values  $P = [\rho_{i,j}]_{i=1,\dots,g_M, j=1,\dots,g_{\eta}}$  for the scheduling

vector  $\rho = [\eta_{d,eq} \quad M]^T$  [13], in order to give linear models  $S_{ij}$  of the form

$$\begin{aligned} \delta\dot{x} &= \mathbf{A}(\eta_{d,eq}, M) \delta x + \mathbf{B}(\eta_{d,eq}, M) \delta\delta \\ \delta\eta &= \mathbf{C}(\eta_{d,eq}, M) \delta x \end{aligned} \quad (9)$$

The quantities  $\delta x, \delta\delta, \delta\eta$  are deviations from the equilibrium values and are computed by subtracting the current value from the corresponding equilibrium value (either for the state, the control or the output)

$$\begin{aligned} \delta x &= x - x(\eta_{d,eq}, M) \\ \delta\delta &= \delta - \delta(\eta_{d,eq}, M) \\ \delta\eta &= \eta - \eta(\eta_{d,eq}, M) \end{aligned} \quad (10)$$

The equations (9) are complemented by appropriate matrices in order to create an augmented plant  $P_{ij}$  (standard synthesis scheme) (Fig. 2).

The  $\mathcal{H}_{\infty}$  machinery is applied to compute the corresponding output feedback controllers satisfying  $\|T_{w\zeta_{\infty}}\|_{\infty} < \gamma$ .

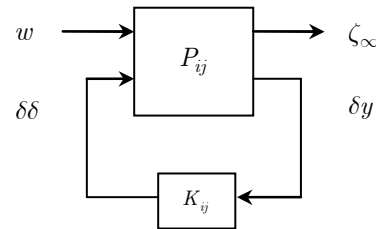


Fig. 2. The  $\mathcal{H}_{\infty}$  synthesis scheme

The synthesis scheme has as a goal to minimize the effect of external perturbations  $w$ , acting directly on the state, on the output tracking error  $\delta e = \delta\eta - \delta\eta_{d,eq}$  while imposing constraints on the control input  $\delta\delta$ .

The corresponding synthesis model for each synthesis point is

$$\begin{aligned} \delta\dot{x} &= \mathbf{A}_{ij} \delta x + \mathbf{B}_1 w + \mathbf{B}_{2,ij} \delta\delta \\ \zeta_{\infty} &= \mathbf{C}_1 \delta x + \mathbf{D}_{12} \delta\delta \\ \delta y &= \mathbf{C}_{2,ij} \delta x \end{aligned} \quad (11)$$

With

$$\begin{aligned} \mathbf{B}_1 &= \begin{bmatrix} w_1 \\ 0 \end{bmatrix} & \mathbf{C}_{1,ij} &= \begin{bmatrix} l_1 \mathbf{C}_{ij} \\ 0 \quad 0 \end{bmatrix} \\ \mathbf{D}_{12} &= \begin{bmatrix} 0 \\ l_2 \end{bmatrix} & \mathbf{C}_{2,ij} &= \begin{bmatrix} \mathbf{C}_{ij} \\ 0 \quad 1 \end{bmatrix} \end{aligned} \quad (12)$$

Here  $\mathbf{A}_{ij}, \mathbf{B}_{2,ij}, \mathbf{C}_{ij}$  are the matrices of the linearized model (9) at each operating point  $\rho_{ij}$ . In addition, matrices  $\mathbf{B}_1, \mathbf{C}_1, \mathbf{D}_{12}$  introduce a disturbance on the angle of attack and select the signals to be treated, namely  $\delta\eta$  and  $\delta\delta$ , with weighting parameters  $w_1, l_1, l_2$ . Finally the matrix  $\mathbf{C}_{2,ij}$  defines  $\delta\eta^*$  and  $\delta q$  to be available for feedback. The values for the weighting parameters are chosen to be  $w_1 = 0.0005, l_1 = l_2 = 1$ .

However, due to strict performance constraints, the  $\mathcal{H}_{\infty}$  controller is not adequate by its own, so further constraints

on the location of the systems' closed loop poles are imposed. These constraints can be incorporated according to [4] to the initial dynamic output feedback synthesis problem. The system's poles are chosen to be confined to a convex LMI region  $\mathcal{S}$  being the union of three regions: a stable vertical strip (for minimum decay rate  $\lambda$ ), a conic section (for minimum damping  $\zeta$ ) and a semicircle (for maximum undamped natural frequency  $r$ ). The values chosen are  $\lambda = -18, r = 19, \zeta = 0.99$ . In brief, the problem is finding an LTI control law  $\delta\delta = K_{ij}(s)\delta y$ , such that

- The closed-loop poles lie in the convex region  $\mathcal{S}$
- $\|T_{w\zeta\infty}\|_{\infty} < \gamma$  where  $\|T_{w\zeta\infty}\|$  is the transfer function from the external disturbances to the treated signals

The solution of this problem can be found in [4] and it yields dynamic controllers  $K_{ij}$  of the form as in (13)

$$\begin{aligned} \dot{x}_K &= \mathbf{A}_{K,ij}x_K + \mathbf{B}_{K,ij}\delta y \\ \delta\delta &= \mathbf{C}_{K,ij}x_K \end{aligned} \quad (13)$$

The following figure (Fig. 3) shows the poles of each closed system  $F_l(P_{ij}, K_{ij})$  (the constraints being fulfilled), whereas in (Fig. 4) we can see the achieved values for  $\gamma$  for each operating point  $\rho_{ij}$  (for both cases we have used a gridding corresponding to  $g_M = g_{\eta} = 4$ ).

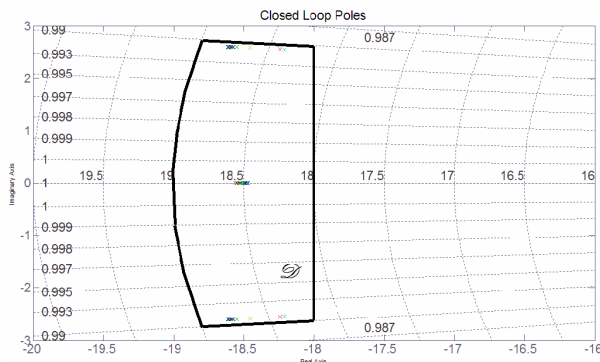


Fig. 3. Closed loop poles of all systems  $F_l(P_{ij}, K_{ij})$  lying in the convex LMI region  $\mathcal{S}$

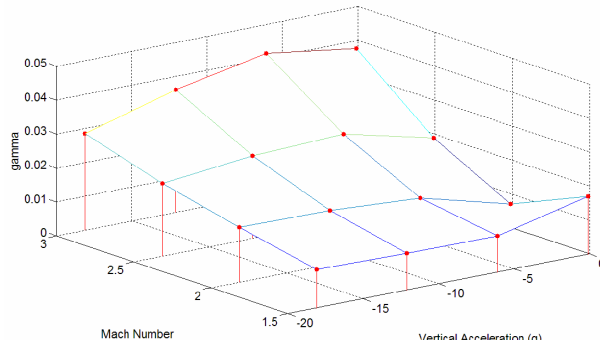


Fig. 4. The optimal  $\gamma$  achieved for every synthesis point

The dynamic controllers in (13) will be scheduled according to either the Controller Blending (CB), or the Gain Blending (GB) technique. For the latter case, they should be converted to state feedback/observer form, in order to perform the gain interpolation. This can be done using Youla parameterization [1], [3].

The controllers are of the same order as the plant so the Youla parameter is reduced to a static gain. Furthermore, the controllers being strictly proper force this static gain to zero. Therefore, the state feedback/observer gains are the only ones to be interpolated, thus greatly reducing computational burden. The procedure passes from the solution  $\mathbf{T}_{ij}$  of a generalized non-symmetric rectangular Ricatti equation for every synthesis point

$$\mathbf{A}_{K,ij}\mathbf{T}_{ij} - \mathbf{T}_{ij}\mathbf{A}_{ij} - \mathbf{T}_{ij}\mathbf{B}_{2,ij}\mathbf{C}_{K,ij}\mathbf{T}_{ij} + \mathbf{B}_{K,ij}\mathbf{C}_{2,ij} = \mathbf{0} \quad (14)$$

The solution of the above equation is found with standard invariant subspace techniques [1], [3] and the transformed controller's gains are given by

$$\begin{aligned} \mathbf{K}_{c,ij} &= -\mathbf{C}_{K,ij}\mathbf{T}_{ij} \\ \mathbf{K}_{o,ij} &= \mathbf{T}_{ij}^{-1}\mathbf{B}_{K,ij} \end{aligned} \quad (15)$$

### B. Gain Scheduling

#### 1) Controller Blending (CB)

The Controller Blending technique belongs to the family of ad-hoc interpolating techniques and thus offers no stability guaranties [12].

In brief, the technique blends the outputs of adjacent controllers to produce the corrective closed loop control for the system, in order to achieve output tracking. The controllers here are blended in rectangular blending domains; each such domain having four controllers (Fig. 5).

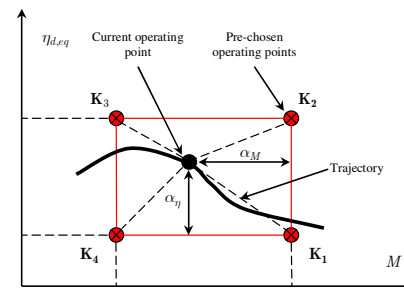


Fig. 5. A rectangular blending domain

The main advantage of this technique is that it requires no order-structure similarities for each controller designed thus permitting more flexibility in the overall design. As a result, different types of controllers can be calculated for any operating point, according to the system's characteristics.

Supposing that the distance of the current operating point from the nearest four design points can be measured, then the

scheduling variables can be used to update the closed loop control law. Suppose also that four controllers  $\mathbf{K}_1, \mathbf{K}_2, \mathbf{K}_3, \mathbf{K}_4$  of the form as in (12) are available and  $0 \leq \alpha_M, \alpha_\eta \leq 1$  are normalized quantities showing the closeness to each corner of the rectangle (referring to the Mach number and the output respectively).

The closed loop control law  $\delta\delta_{1234}$  (with the total control being  $\delta = \delta_{d,eq} + \delta\delta_{1234}$ ) will be of the following form ( $\delta\delta_1, \delta\delta_2, \delta\delta_3, \delta\delta_4$  are the outputs of each controller)

$$\left. \begin{aligned} \delta\delta_{12} &= \alpha_M \delta\delta_1 + (1 - \alpha_M) \delta\delta_2 \\ \delta\delta_{34} &= \alpha_M \delta\delta_3 + (1 - \alpha_M) \delta\delta_4 \end{aligned} \right\} \Rightarrow \quad (16)$$

$$\delta\delta_{1234} = \alpha_\eta \delta\delta_{12} + (1 - \alpha_\eta) \delta\delta_{34}$$

An important implementation issue here has to do with the initial conditions of the new controllers introduced when we pass from one region to the next. Instead of giving them zero initial conditions a different initialization technique is used in order to ameliorate the controller's performance (taking into account the output of the controller  $\delta\delta_{1234}$  just before region switching). The initial condition  $x_{K,0}(t_c)$  given to all controllers  $K_{ij}$  can be computed from the output equation (12b) ( $\delta\delta_{1234}(t_c^-)$  being the controller's output just before switching and  $\mathbf{C}_{K,ij}^+$  the Moore-Penrose pseudoinverse of each controller's output matrix)

$$x_{K,0}(t_c) = \mathbf{C}_{K,ij}^+ \delta\delta_{1234}(t_c^-) \quad (17)$$

With this technique, the phenomenon of unwanted initial controller transients [12] may be reduced and better performance may be achieved.

## 2) Gain Blending (GB)

The Gain Blending (GB) technique is a widely used interpolation technique. It can offer certain stability guaranties for the overall gain-scheduled design [19], [23], here however the simplest possible form will be implemented. Assume the following structure for the global controller, where  $\delta\hat{x}, \delta\delta, \delta y$  represent the estimated state, the total control input and the measurement as in (10)

$$\begin{aligned} \delta\dot{\hat{x}} &= \mathbf{A}\delta\hat{x} + \mathbf{B}_2\delta\delta + \mathbf{K}_o(\delta y - \mathbf{C}_2\delta\hat{x}) \\ \delta\delta &= -\mathbf{K}_c\delta\hat{x} \end{aligned} \quad (18)$$

In contrast with the CB technique, here a unique controller is implemented, its gains  $\mathbf{K}_c, \mathbf{K}_o$  now being found by interpolation similar to(16). For instance, the observer gain is calculated (using also the measurements on the scheduling vector) as the following linear interpolation

$$\left. \begin{aligned} \mathbf{K}_{o,12} &= \alpha_M \mathbf{K}_{o,1} + (1 - \alpha_M) \mathbf{K}_{o,2} \\ \mathbf{K}_{o,34} &= \alpha_M \mathbf{K}_{o,4} + (1 - \alpha_M) \mathbf{K}_{o,3} \end{aligned} \right\} \Rightarrow \quad (19)$$

$$\mathbf{K}_{o,1234} = \alpha_\eta \mathbf{K}_{o,12} + (1 - \alpha_\eta) \mathbf{K}_{o,34}$$

An important issue here is which system matrices  $\mathbf{A}, \mathbf{B}_2, \mathbf{C}_2$  will be put in the controller in (17). As the parameter vector trajectory moves around the missile's operating domain, the system matrices used correspond to the desired output reference value and not to the current since only the former corresponds to the desired equilibrium trajectory. A final issue concerning both interpolation methods has to do with the scheduling variable  $\eta$ . The scheduling operation can be done by using either the desired output trajectory  $\eta_{d,eq}$  or the actual one  $\eta$  (Fig. 6). The differences in the performance will be shown in Section V.

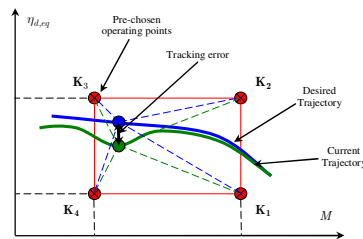


Fig. 6. Scheduling on the current or the desired trajectory

## V. SIMULATION RESULTS

### A. Controller Blending vs. Gain Blending

In this section the simulation results will be presented. The two techniques are compared following a typical scenario for the desired equilibrium trajectory. The specifications for the LTI controllers are those given in Section IV (gridding on  $\eta, M$  is each 6g and 0.25 units). In (Fig. 7) the output  $\eta$  is shown when using the CB or GB techniques (blue/red).

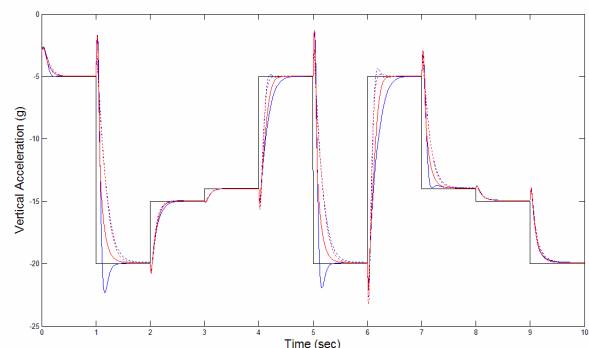


Fig. 7. Vertical acceleration  $\eta$  (plant output) as a function of time. CB (blue) and GB (red). Scheduling on the current output (solid) or on the desired (dotted).

Furthermore, for each technique, the use of either the current output  $\eta$  (solid line) or the desired output  $\eta_{d,eq}$  (dotted line) for scheduling is presented. The Mach varies almost linearly from 3 to 2.5 according to (5).

It is evident (Fig. 7) that the GB technique is clearly superior from the CB technique across the missile's flight envelope. Even though the controllers in both techniques are the same, the interpolation scheme makes the difference in terms of performance (settling time, overshoot, etc.). The following figures show the open loop control  $\delta_{d,eq}$  (Fig. 8) and the total control signal (Fig. 9) after the actuator.

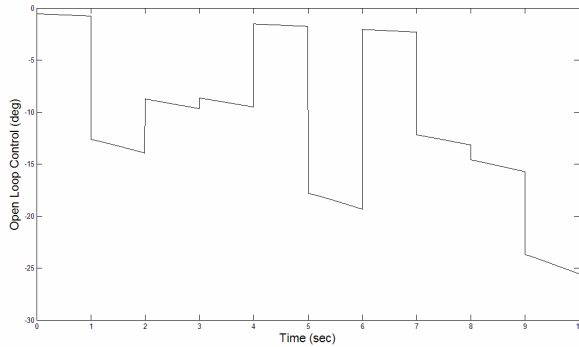


Fig. 8 Open loop control  $\delta_{eq,d}$

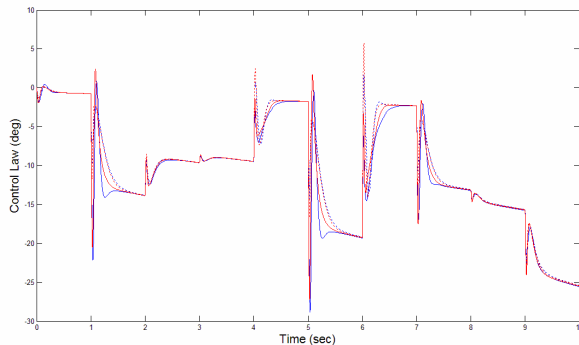


Fig. 9 Closed loop control law  $\delta$

In addition, scheduling on the current output  $\eta$  instead of the reference  $\eta_{d,eq}$ , gives better performance since the controller takes into account the current state of the plant and tries to make it equal with the reference one.

**B. Investigation of the GB technique**

The GB technique will be chosen henceforth (selecting only scheduling on the current output) in order to investigate the dependence of the plant's performance on the gridding interval on the desired output equilibrium  $\eta_{eq,d}$  (the gridding on  $M$  is chosen to be 0.1 units). The influence of gridding on  $M$  is not considered since it does not affect system performance significantly. The following figure (Fig. 10) shows the plant's output for three different gridding intervals for  $\eta$ , namely for each 40,20,1g (blue, green, red respectively), whereas in (Fig. 11) we can see the control input  $\delta$  and the system's trajectory on the parameter space (Fig. 12). In this latter figure, the LTI synthesis points are presented with green dots (for gridding each 20g for  $\eta_{eq,d}$ ), whereas the arrows show the distance of the operating point from the synthesis points. This has an influence on how

much weight is posed on each adjacent controller of the corresponding interpolation region. The settling time for the three gridding scenarios can be seen on Fig. 13 (blue, green and red lines correspond to a gridding of 40,20,1g).

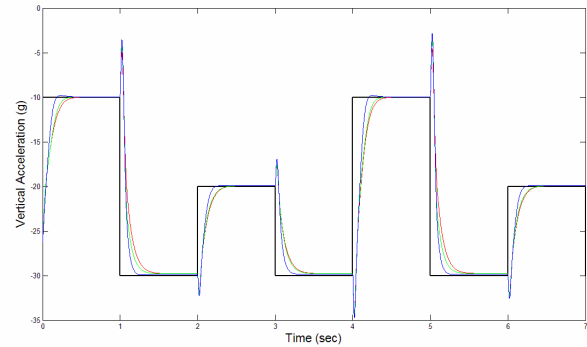


Fig. 10. Vertical acceleration  $\eta$  (plant output) as a function of the gridding performed on  $\eta_{d,eq}$

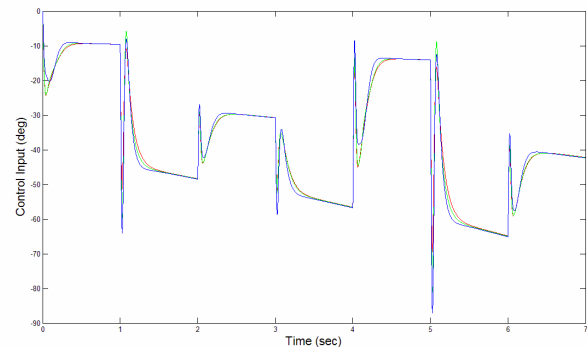


Fig. 11. The control input  $\delta$  as a function of the gridding performed on  $\eta_{d,eq}$

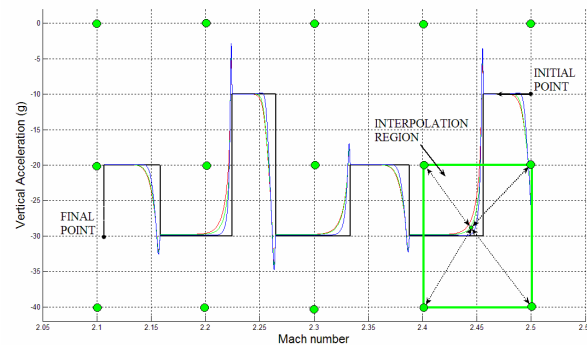


Fig. 12. The plant's trajectory on the parameter space as a function of the gridding performed on  $\eta_{d,eq}$ . An interpolation region is shown

From the above figures, it seems that the sparser the gridding gets, the faster the system becomes. However, the correct interpretation is that the denser the gridding becomes, the more the nonlinear system's response time approaches the one corresponding to the LTI systems (Fig. 13). Increasing the gridding step may or may not result in better performance depending on system's dynamics. A finer gridding may increase computational burden but it helps ensure that the LTI synthesis specifications are carried over to the nonlinear system.

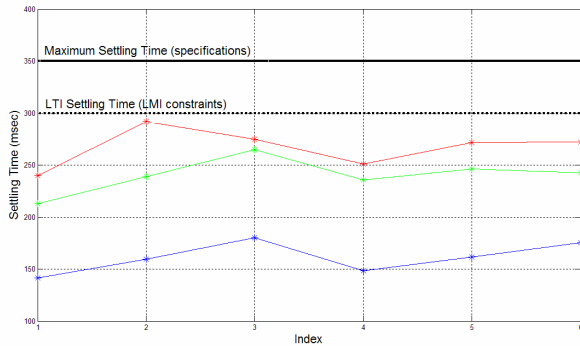


Fig. 13. The settling time corresponding to the responses in Fig. 10. Each star on the figure represents a change on the output reference trajectory  $\eta_{d,eq}$ . The solid black line represents the maximum admissible settling time whereas the dotted one represents the settling time of the linearized closed loop systems (due to the LMI

In the following figure (Fig. 14), the elements of the controller matrix  $K_c$  (17) over the parameter space  $[\eta_{d,eq} \ M]$  are shown. In addition, the same elements are presented for the same three gridding scenarios (Fig. 15).

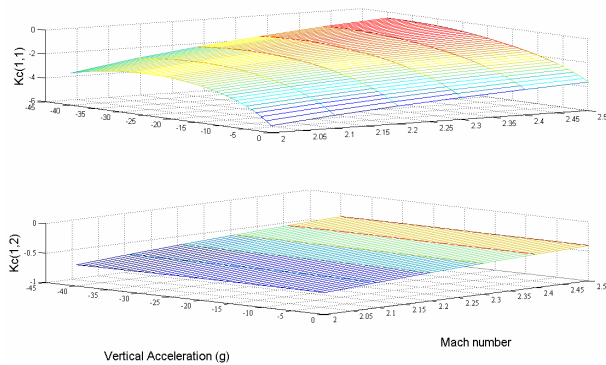


Fig. 14. The elements of the state feedback controller matrix  $K_c$  over the parameter space.

The synthesis technique yields controller gains that are smooth over the parameter space. Consequently, their time interpolation (during region transitions) is also smooth (Fig. 15). However, due to the different gridding step, the interpolation yields different results for the output  $\eta$ .

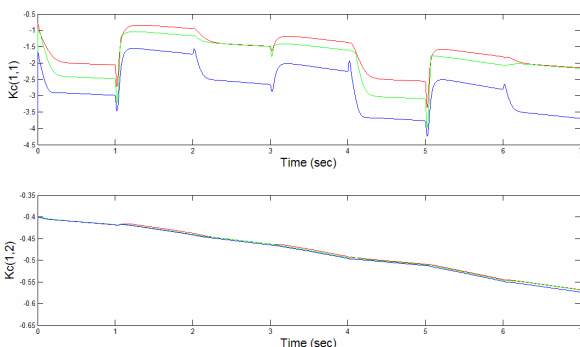


Fig. 15. The elements of the state feedback controller matrix  $K_c$  as a function of time.

Finally, the (GB) control scheme used in our case is shown (Fig. 16). On the one hand, there is the control loop

(fast), being comprised of the state feedback/observer control scheme and being complemented by an additional block supplying the correct inputs to the observer.

On the other hand, a scheduling loop (slow) is necessary in order to update the controller/observer matrices in accordance with scheduling vector.

In addition, an external block, generating the Mach number as a function of the AOA (5) and the output  $\eta$  is added.

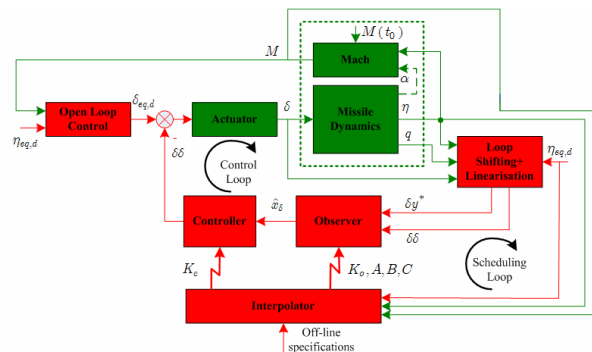


Fig. 16. Block diagram of the GB nonlinear controller

## VI. CONCLUSION

In this paper, two gain scheduling techniques, namely the Controller Blending (CB) and the Gain Blending (GB) techniques, were tested on a missile autopilot synthesis problem. Optimal output feedback controllers minimizing the  $\mathcal{H}_\infty$  norm with additional pole placement constraints were computed (for use with the CB technique) and scheduled according to the Mach number and plant output, in order to construct a global nonlinear controller. The same controllers were converted to state feedback/observer type ones (for use with the GB technique) via Youla parameterization.

Several facts concerning these two methods were highlighted such as open loop control, initial conditions on the controllers when using the CB technique and system matrix computation when using the GB technique.

A comparison was also made on whether the controllers should be scheduled according to the desired or the current output trajectory. Finally, the effect of the gridding step over the system's performance was investigated.

The future work will focus on different LTI synthesis (such as loop  $\mathcal{H}_\infty$  shaping control) and/or other scheduling methods (such as fixed structure controller interpolation). In addition, the important fact of stability of the gain-scheduled system [23] will be investigated.

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