

# A New Lyapunov Based Algorithm for Tuning BELBIC Controllers for a Group of Linear Systems

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**Abstract-** This paper proposes an algorithm for controlling first and second order linear systems using brain emotional learning based intelligent controller (BELBIC). BELBIC is an intelligent controller based on the model of the emotional part of brain. The algorithm proved to be able of globally asymptotically stabilizing the linear systems based on Lyapunov theorem. A simple example of first order linear system is simulated to show the capability of BELBIC controller with tuned parameters.

## I. INTRODUCTION

Lyapunov theorem is a useful theorem for proving systems' stability. But finding a good Lyapunov function is not that much simple, and without this function nothing could be said about the system [1],[2]. In this paper the parameters of derivative of Lyapunov function are determined in order to make it negative definite. Doing this, make parameters of controller determined. In fact, an algorithm for finding the parameters of controller to stabilize the system proposed.

The brain emotional learning based intelligent controller (BELBIC) is an intelligent controller which could be used in many fields [3-6]. The powerful aspect of this controller is laid in a function named reward. This reward function is the basic logic of this controller.

Finding a good stability theorem for BELBIC controller is hard to do because of relatively complicated relations that govern the BELBIC controller [7],[8].

A specific BELBIC controller defined and proved to be capable of regulating first order and part of second order linear systems. Using the proposed algorithm guarantee the stability of closed loop system because of using Lyapunov theory.

At first the Lyapunov theory is described, and then the BELBIC controller's configuration and function is discussed. In next section a whole view of the problem illustrated and after that control algorithms for first and second order linear systems are derived. Simulation result for a simple example for a first order case is shown. Finally the key points of paper mentioned.

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## II. LYAPUNOV THEORY

There are several stability criteria for analysis of systems. The most important and useful method in these criteria is Lyapunov method. Here we introduce this method briefly in a theorem. For further study refer to [1].

**Theorem 1:** Assume that there exists a scalar function  $L$  of the state  $\mathbf{x}$ , with continuous first order derivatives such that:

1.  $L(\mathbf{x})$  is positive definite
2.  $\dot{L}(\mathbf{x})$  is negative definite
3.  $L(\mathbf{x}) \rightarrow \infty$  as  $\|\mathbf{x}\| \rightarrow \infty$

Then the equilibrium at the origin is globally asymptotically stable.

**Proof:** Proof can be found in [1].

## III. BELBIC CONTROLLER

BELBIC is the abbreviation for brain emotional learning based intelligent controller. Motivated by the success in functional modeling of emotions in control engineering applications, a structural model based on the limbic system of mammalian brain, for decision making and control engineering applications has been developed [9]. The computational model of emotional learning in the amygdala, based on Moren and Balkenius model [10], is depicted in Figure 1. The main parts that are responsible for performing the learning algorithms are orbitofrontal cortex and amygdala.

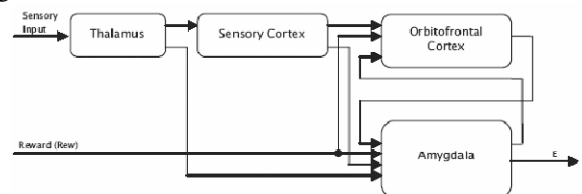


Figure 1: Computational model of emotional learning in the Amygdala

BELBIC controller has some sensory inputs. One of the designer's tasks is to determine the sensory inputs. BELBIC controller has two states for each sensory input. One of these two is amygdala's output and another is the output of orbitofrontal cortex. Therefore the number of sensory inputs has a key role in BELBIC controller. Usually the sensory inputs are rich signals [11].

Consider the  $i$ -th sensory input as  $s_i$ . Amygdala and orbitofrontal cortex outputs are as follows:

$$\begin{aligned} A_i &= s_i v_i \\ O_i &= s_i w_i \end{aligned}$$

$v, w$  are two states for the related sensory input. These two will be updated by following equations:

$$\begin{aligned} \dot{v}_i &= \alpha \cdot s_i \cdot \text{rew} \\ \dot{w}_i &= \beta \cdot s_i \cdot (\text{rew} + s_i + O_i - A_i) \end{aligned}$$

In these equations,  $\alpha$  and  $\beta$  are training coefficients. In BELBIC controller there is a function named *Reward*. This function has a great role in BELBIC controllers. Reward is like its name. The controller strives to increase this reward. Therefore the designer must define a reward function that has its maximum values in the most desired regions. This reward function could be either a frequency domain function or a normal mathematical function.

Amygdala acts as an actuator and orbitofrontal cortex acts as a preventer. Therefore the control effort of BELBIC controller is:

$$u = \sum A_i - \sum O_i$$

BELBIC is a controller that has only one output. Therefore for systems with more than one control inputs designer must use one BELBIC controller for each control input. As it can be seen there are several tuning parameters for each sensory input. The general algorithm for tuning these parameters is trial and error.

#### IV. PROBLEM FORMULATION

Consider a linear system that has one input and one output. Let's call its input  $u$  and its output  $y$ . Assume that we attach a BELBIC controller to this linear system and return the linear system's output as a feedback signal. It could be a tracking or regulation problem according to reference value. The whole control diagram is shown in figure 2.

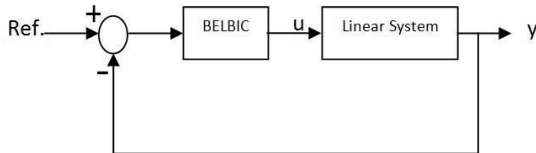


Figure 2: Problem Configuration

The input signal to BELBIC controller is error-type signal. Considering this fact and reward function properties, reward function could be chosen as follows:

$$\text{rew} = k_2 - k_1 s^2$$

The reward function's coefficients are real positive numbers to make the reward action in the way of reducing its sensory input which is error-type signal.

The  $s$  in this relation is the sensory input that is an error signal. Regarding the fact that reward function has its maximum value in origin, BELBIC controller makes this error as low as possible.

The regulation problem is considered here. Therefore we could write sensory input as follows:

$$s = -y$$

#### V. FIRST ORDER CASE

**Theorem 2:** A BELBIC controller with parameters as  $\alpha = \beta = -\frac{b}{2} < 0$  for a first order linear system in figure 2 with  $b > 2a$  can control the linear system.

**Proof:** Consider the linear system of first order with following state space description:

$$\begin{aligned} \dot{x} &= ax + bu \\ y &= x \end{aligned}$$

Therefore the sensory input for BELBIC controller become as follows:

$$s = -y = -x$$

Consider the positive definite Lyapunov function with following structure:

$$L = \frac{1}{2}(1 + w - v)^2 + \frac{1}{2}x^2$$

It can be seen that the Lyapunov function is zero at  $x = 0, w - v = -1$  that is equilibrium point of the closed loop system. Also the function tends to infinity when the closed loop's states goes to infinity. Thus if  $\dot{L}$  become negative definite under some conditions, the closed loop system would be globally asymptotically stable under those conditions.

It can be assumed that  $\alpha = \beta$  for BELBIC controller. With this assumption the  $\dot{L}$  would be as follows:

$$\dot{L} = \beta x^2 + 2\beta x^2(w - v) + \beta x^2(w - v)^2 + ax^2 + bx^2(w - v)$$

This mathematical function would be negative definite under these three assumptions:

$$\begin{aligned} \alpha &= \beta < 0 \\ b &> 2a \\ \beta &= -\frac{b}{2} \end{aligned}$$

If  $b < 0$  the first and third conditions couldn't be held simultaneously. Also for some first order systems the second condition may be not correct. These facts make the proof not to cover a group of first order linear systems. Putting a gain in the BELBIC's output could solve this problem. With this modification we could change the DC gain of the first order system to make conditions held. Now by choosing  $\alpha = \beta = -\frac{b}{2}$  for BELBIC controller any first order linear system can be regulated with the considered BELBIC controller.

It can be seen that there is no determined value for reward function's parameters.

#### VI. FIRST ORDER EXAMPLE

In this section a first order system used to show how BELBIC controller could stabilize a first order linear plant.

The normal physical systems in nature are nonlinear and have not a determined order, but linearizing these systems is a useful way to simplify controlling these systems.

State space description for considered system is as follows:

$$\begin{aligned} \dot{x} &= x + 3u \\ x_0 &= 2 \end{aligned}$$

The considered system is linear without using controller. The BELBIC controller's parameters based on proposed algorithm are:

$$\alpha = \beta = -1.5$$

The state trajectory for this system using the BELBIC controller tuned by proposed algorithm is shown in figure 3.

As it can be seen the first order linear system is regulated using BELBIC controller.

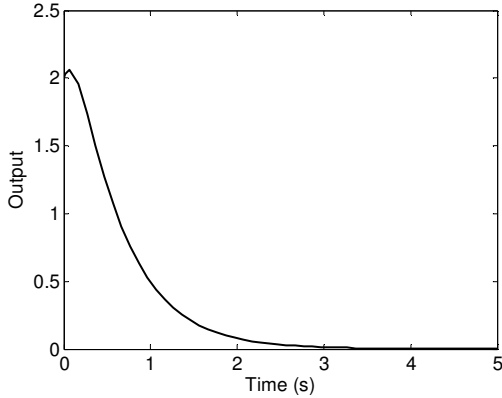


Figure 3: Simulation Result

## VII. SECOND ORDER CASE

**Theorem 3:** A BELBIC controller with parameters as  $\alpha = \beta = -\frac{b_1}{2} < 0, \beta < -a_1$  for a second order linear system in figure 2 with  $b_2 = 0, a_2 = -1$  can control the linear system.

**Proof:** Once again consider a linear system but a system of second order with following state space relations:

$$\begin{aligned}\dot{x}_1 &= a_1x_1 + x_2 + b_1u \\ \dot{x}_2 &= a_2x_1 + b_2u \\ y &= x_1\end{aligned}$$

Without losing generality the controllable state space description has been considered here to simplify the calculations. The sensory input for BELBIC controller in this case would be:

$$s = -y = -x_1$$

Consider the following positive definite Lyapunov function:

$$L = \frac{1}{2}(1 + w - v)^2 + \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2$$

If  $\dot{L}$  become negative definite under some conditions, the closed loop system would be globally asymptotically stable under those conditions.

It can be assumed that  $\alpha = \beta$  for BELBIC controller. With this assumption the  $\dot{L}$  would be as follows:

$$\begin{aligned}\dot{L} &= b_1x_1^2(w - v) + b_2x_1x_2(w - v) + x_1x_2(a_2 + 1) + a_1x_1^2 \\ &\quad + 2\beta x_1^2(w - v) + \beta x_1^2(w - v)^2 + \beta x_1^2\end{aligned}$$

The following conditions make the Lyapunov function's derivative negative definite:

$$\begin{aligned}\alpha &= \beta < 0 \\ \beta &< -a_1 \\ \beta &= -\frac{b_1}{2} \\ b_2 &= 0 \\ a_2 &= -1\end{aligned}$$

In first order case all first order linear systems comprised in the proof using a simple gain, but in this case a specific

group of second order linear systems could be proved to be controlled by the mentioned BELBIC controller.  $b_1$  could be changed by a gain like the one in first order case, but  $a_2$  and  $b_2$  should take their specified values. Therefore this method proposes a BELBIC controller for the regulation problem of any second order linear system with mentioned values.

## VIII. CONCLUSION

Although normal systems in nature are nonlinear, linearizing nonlinear systems before designing controller is usual in several control techniques. Therefore finding a controller for linear systems could be useful.

In this paper a Lyapunov based algorithm introduced to tune BELBIC controllers for first and second order linear systems. The proposed algorithm used for controlling an unstable first order linear system and simulation results shown.

The proposed algorithm does not cover all linear second order systems. Therefore in future this algorithm could be extended using other Lyapunov functions.

Another future work in this field is to determine the group of nonlinear systems which could be stabilized using the proposed algorithm.

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