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AN (r, Q) INVENTORY POLICY FOR PERISHABLE PRODUCTS HAVING A FIXED LIFETIME

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ABSTRACT: This paper deals with the inventory control of perishable products having a fixed lifetime. In the first part of the paper, we review the relevant literature on the problem of determining appropriate ordering policies for fixed life perishable inventory. Then, we propose a continuous review inventory policy; i.e. an *(r, Q)* policy, extending an earlier investigation (i.e. Chiu, 1995). Expressions of the different components of the expected total cost are derived and the problem of finding the optimal replenishment level *r* and the optimal incoming quantity *Q* is formulated. A comparative numerical study shows that the model we propose significantly outperforms the classical *(r, Q)* policy and Chiu's model.

KEYWORDS: *Continuous review inventory model; Perishable products*

1. INTRODUCTION

This paper is motivated by the potential applications of perishable inventory control for foodstuff, pharmaceutical industry and blood bank management. The inventory of such products, characterised by a limited lifetime, should be managed carefully. Figures from industry outline the cost associated with such perishable products: $30 billion of products are lost due to perishability in US grocery industry (Lystad et al., 2006). Furthermore, the perishability issue leads also to stockouts that are induced not only from a forecast error in estimating customer demand but also from the fact that products are perished in stock. Therefore, the economic importance of managing perishable products led researchers to conduct substantial work in perishable inventory control; however the fixed life perishability problem still remains difficult when the product lifetime is greater than 2 periods. It involves complex recursive computations and finding an analytical solution becomes extremely complex (Nahmias, 1982). Thus, it is required to develop an approximate inventory control model that deals with perishable products. In this paper we consider a continuous review perishable inventory model for items with fixed lifetime and operating under positive lead time. Our interests are in determining the optimal ordering quantity *Q* and the replenishment level *r* and finding the optimal long run average expected costs.

Several inventory models of perishable products have been proposed and studied extensively. An excellent summary of perishable inventory theory can be found in (Nahmias, 1982; Raafat, 1991; Goyal and Giri, 2001). The fixed life perishability problem can be classified into two classes: periodic review models and continuous review models. For periodic review systems, if the products can be retained in stock no more than one period, the problem is reduced to the newsboy problem. When products are kept in stock for more than one period, the problem turns out to be extremely complex. To deal with this complexity, (Nahmias and Pierskalla, 1973) studied a perishable inventory model where product lifetime is known to be two periods. (Fries, 1975) and (Nahmias, 1975) extended independently the model to the case of three or more periods, they showed that the computation of an optimal policy requires the solution of a dynamic program whose state variable has dimension *m-1* (*m* is the product lifetime). These works assume that the ordering cost is proportional to the number of units ordered. (Nahmias, 1978) relaxed this assumption by including the fixed ordering cost. The implementation of these models becomes difficult when the product lifetime is superior to three periods. In order to avoid this difficulty induced by the need to track the different-aged inventory, research has been focused on the development of heuristic approximations. Several papers have been appeared that trying to find an approximate solution. For example, (Nahmias, 1976) considered only the total quantity of inventory on hand

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in order to simplify the computations rather than the different elements of age categories. (Nansakumar and Morton, 1993) derived myopic upper and lower bounds on the order quantities and used the bounds to obtain a heuristic. A comprehensive survey of works under periodic review can be found in (Nahmias, 1982).

For the continuous review systems, (Weiss, 1980) showed that the *(s, S)* policy is optimal under zero lead times and Poisson demand. (Liu and Lian, 1999) extended Weiss's model by considering a general renewal demand process. (Lian and liu, 1999) analysed a discrete time *(s, S)* model with zero lead times, geometric inter-demand and batch demands. Allowing backlogs, they showed that the discrete time models are an accurate approximation of the continuous times models. When a positive order lead time is introduced, the problem becomes extremely complex, the literature reveal that there exist a limited number of works dealing with positive lead time. (Kalpakam and Spana, 1994) have studied an *(s, S)* model with Poisson demand, exponential lead time and exponential lifetime. (Liu and Yang, 1999) have considered a similar model with backorders and no restriction on the number of outstanding replenishment orders; they analysed the problem using matrix-analytical approach. (Kalpakam and Spana, 1996) studied an *(S-1, S)* system with renewal demand and exponential lifetime and (Kalpakam and Shanthi, 1998) propose an *(s, S)* policy with Poisson demand and exponential lifetime where orders are placed only at demand epochs. Recently, (Kalpakam and Shanthi, 2006) generalized there model of 1998 by considering a renewal demand. For the deterministic lead times, (Schmidt and Nahmias, 1985) proposed an *(S-1, S)* system with Poisson demand and fixed life and lead times. Later (Lian and Liu, 2001) considered a general *(s, S)* perishable inventory model with batch demands. Assuming that the lead time is zero, they constructed a Markov renewal model and proposed a heuristic approach to deal with the case of a constant lead time.

An important contribution is the one of (Tekin et al, 2001) and (Chiu, 1995). (Tekin et al., 2001) introduced a *(Q,r,T)* policy, in which a replenishment order of size *Q* is placed either when the inventory level drops to *r* , or when *T* units of time have elapsed since the last instance at which the inventory level hit *Q*, whichever occurs first. On the other hand, (Chiu, 1995) proposed an *(r, Q)* inventory model with positive and constant lead time and demand follows a nonnegative random variable. (Chiu, 1999) also proposed a similar model with modified and accurate expression of the expected inventory level per unit time. In order to take into account the effect of perishability on the total expected average cost, Chiu developed an approximation to the expected outdating units. This approximation might be established by analysing the net inventory after order arrives. The author shows that there are some cases to consider. Among them, when demand during the lead time is less than the replenishment level *r* and when the demand during the lifetime is less than *r* minus lead time demand, then *Q* units will outdate if demand during the lifetime is satisfied wholly out of the oldest units. The case where these oldest units perish before they are demanded is not considered by Chiu's model. For this reason, we suggest an approximation on the outdating units of the inventory level just after order arrival. The major difference between the two approximations is that we compute the expression of expected outdating units for the total on hand inventory just after order arrival, however the Chiu's approximation is allowed for the current order size *Q*. We derive after that, the expressions of the expected cycle length, the expected shortage units and the expected on hand inventory. We obtain an *(r, Q)* inventory policy with positive lead time. The optimal parameters *r* and *Q* are calculated by minimizing the total expected average cost. Numerical studies confirm that our model considerably outperforms the classical *(r, Q)* policy and Chiu's model. Our paper ends with conclusions.

2. THE MODEL

Assumptions

We study a single perishable product with fixed lifetime of *m* period. This product is controlled with the continuous review system *(r, Q)*; an order of size *Q*>0, is placed whenever the inventory level drops to the replenishment level *r*. We assume that the aging of products begins just at the time orders arrive (units arrive fresh in stock) and there is no decrease in value of products during their usable lifetime. We suppose that there is a positive replenishment lead time L. The inventory is depleted according to an FIFO issuing policy and all unmet demands are backlogged. The demand in unit time, z, is a nonnegative random variable following a distribution with mean D, pdf $f(z)$ and cdf $F(d)$.

It is also assumed that the density function of the sum ($d_n = z_1 + z_2 + ... + z_n$) of n mutually independent variables z_i have a common density $f(z_i)$, is the n-fold convolution of $f(z_i)$ with it self and it is denoted by $f_n(d_n)$ with mean *nD*. The cumulative probability distribution is noted by $F_n(d_n)$. We assume finally that incoming backorders do not exceed the level *Q*.

Notations

C = purchase cost per unit of product
h = holding cost per unit held in stock per unit time
K = fixed ordering cost per order
P = shortage cost per unit
W = outdate cost per unit that perishes in stock
m = lifetime of product

$E[O]$ = The expected outdated quantity of the on hand inventory just after order arrival
$E[T]$ = The expected cycle length
$E[S]$ = The expected shortage quantity
$E[I]$ = The expected inventory level
$\bar{F}_m(.) = 1 - F_m(.)$

2.1. Expected outdating approximation

In order to evaluate the expected quantity of outdated products, we assume that (i) there is one age category of units available in stock before arrival of the order (ii) we suppose that no outdating occurs during the lead time, then $(r+Q-d_L)$ is the net inventory after order arrives. The total expected outdating of the on hand inventory level just at the moment where an order arrives can be written as:

$$E[O] = \int_{d_L=0}^{d_L=r+Q} \int_{d_m=0}^{d_m=r+Q-d_L} (r+Q-d_L-d_m) f_m(d_m) f_L(d_L) dd_m dd_L \quad (2)$$

Equation (2) is established by considering all cases where outdating may occur:
Case 1: $-Q < r - d_L \le 0$ corresponds to the situation where the current order is used partially to fill the backlog. The outdated units for *m* periods of the net inventory are:

$\max(r+Q-d_L-d_m, 0) = (r+Q-d_L-d_m)^+$.

Case 2: When $r - d_L > 0$ and $d_m \le r - d_L$, there is no backlog $(r-d_L>0)$ and the demand during *m* periods is satisfied by using the quantity $r-d_L$. If d_m is satisfied totally from the oldest units $(r-d_L)$, then the order *Q* will be outdated totally. If d_m is satisfied partially from the oldest units, only one part of the current order will be outdated. For the two subcases, the outdated units for m periods of the net inventory are:

$\max(r+Q-d_L-d_m, 0) = (r+Q-d_L-d_m)^+$

Case 3: the case where $r - d_L > 0$ and $r+Q-d_L \ge d_m > r - d_L$ is similar to case 2 except that d_m is greater than $r-d_L$. We get $r-d_L+Q-d_m$ outdated units of the net inventory.
Thus the expected outdating of the on hand inventory level is:

$$E[O] = \int_{d_L=r}^{d_L=r+Q} \int_{d_m=0}^{d_m=r+Q-d_L} (r+Q-d_L-d_m) f_m(d_m) f_L(d_L) dd_m dd_L$$
$$+ \int_{d_L=0}^{d_L=r} \int_{d_m=0}^{d_m=r-d_L} (r+Q-d_L-d_m) f_m(d_m) f_L(d_L) dd_m dd_L$$
$$+ \int_{d_L=0}^{d_L=r} \int_{d_m=r-d_L}^{d_m=r+Q-d_L} (r+Q-d_L-d_m) f_m(d_m) f_L(d_L) dd_m dd_L$$

2.2. Expected cycle length

We define a cycle as the time separating two successive instances that the inventory level hits the replenishment level *r*. Assuming that $Q > r$, as shown on figure 1, there are three possibilities for the realization of a cycle and the cycle length may be equal to *m+L* or less than *m+L*, according to the evolution of the inventory level. Let T_1 denote a cycle length equal to *m+L* and T_2 the cycle length which is less than *m+L*. Then we can write:

$$T = \begin{cases} T_1 = m+L & if \quad d_m \le Q-r \\ T_2 \ or \ T_1 & if \quad Q-r < d_m < Q-d_L \\ T_2 & if \quad d_m \ge Q-d_L \end{cases} \quad (3)$$

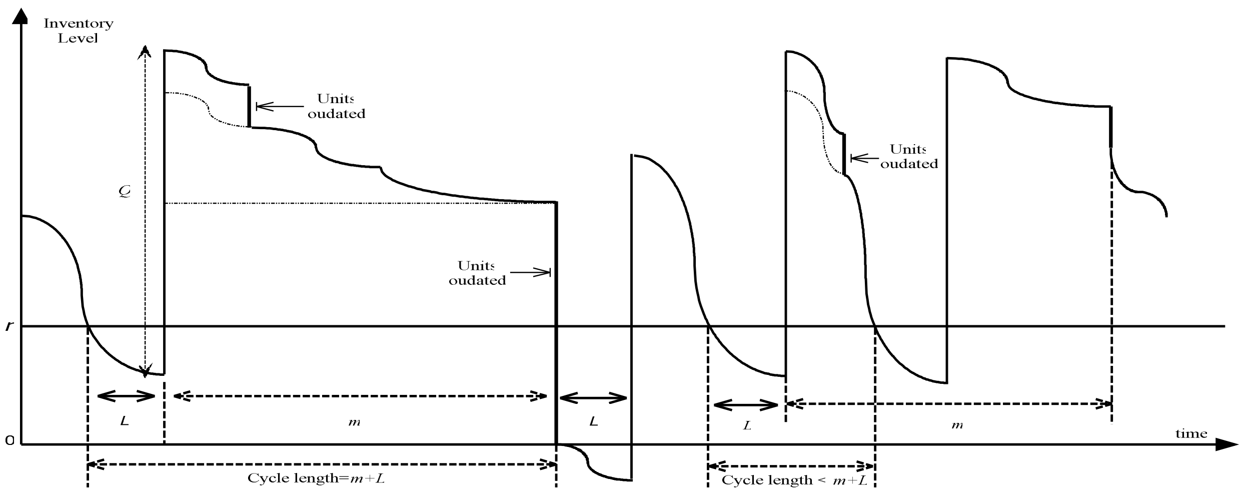

Figure 1. An *(r, Q)* perishable inventory model

We note that if $d_m < Q - r$ then the cycle length is equal to T_1. This is easy to understand because if we suppose that $r - d_L$ is positive and if the $r - d_L$ units perished before they are demanded, then we have a cycle length equal to T_1 for $d_m < Q - r$. For the third case, it is clear that whenever $d_m > Q - d_L$ then the cycle will be T_2. In the second case of equation (3), it is not easy to determine exactly if the cycle length is equal to T_1 or T_2. This complexity is caused by the fact that if $r - d_L$ is positive, then it is difficult to know the remaining time to consume all or a partial part of these oldest units and consequently, deduce the outdating units of $r - d_L$. The information about the remaining time is sufficient enough to point out the nature of cycle. For the convenience of this study, we assume that if $d_m > Q - d_L$ then the cycle length is T_2. The expression of T_2 is assumed to be given as follows:

$$If\ d_m > Q - r\ then\ T_2 = \frac{2Q - E[O]}{2D}$$

Proof

Consider a cycle denoted by (i) for which $d_m > Q - d_L$. Let $x(i)$ be the expected outdating units in cycle (i) and $y(i)$ the expected outdating units of the current order size in cycle (i+1). Then the total expected outdating units of the net inventory level is:

$$E[O] = x(i) + y(i) \tag{4}$$

The expression of T_2 depends on type of the cycle (i-1) and (i+1):

Case 1: *The cycle (i) is preceded and followed by a cycle T_1*

In this case there is no outdating occurs because the order size received in (i-1) is already perished. Hence, no oldest units remained in cycle (i) and the expected length of cycle (i) is:

$$T_2^i = \frac{Q}{D}$$

Case 2: *The cycle (i) is preceded by a cycle T_1 and followed by a cycle T_2*

From the same reasoning given in case 1, we found that $x(i) = 0$ and $T_2{}^i = \frac{Q}{D}$. From equation 4, we notice that:

$$T_2^{i+1} = \frac{Q - y(i)}{D} = \frac{Q - E[O]}{D}$$

Then

$$T_2^i + T_2^{i+1} = 2T_2 = \frac{Q}{D} + \frac{Q - E[O]}{D} \Rightarrow T_2 = \frac{2Q - E[O]}{2D}$$

Case 3: *The cycle (i) is preceded and followed by a cycle T_2*

In this case the perishability in cycle (i) could happen because the order size received in cycle (i-1) has not totally perished. Thus we have:

$$T_2^i = \frac{Q - x(i)}{D} \text{ and } T_2^{i+1} = \frac{Q - y(i)}{D}$$

$$E\left[\frac{Q - x(i)}{D}\right] = E\left[\frac{Q - y(i)}{D}\right]$$

$$\Rightarrow E\left(Q - x(i)\right) = E\left(Q - y(i)\right)$$

$$\Rightarrow E\left(x(i)\right) = E\left(y(i)\right) = \frac{E[O]}{2} \Rightarrow T_2 = \frac{2Q - E[O]}{2D}$$

Case 4: *The cycle (i) is preceded by a cycle T_2 and followed by a cycle T_1*

As in case 3, we can write:

$$T_2^{i-1} = \frac{Q - x(i-1)}{D} \text{ and } T_2^i = \frac{Q - y(i-1)}{D}$$

$$E(Q - x(i-1)/Q) = E(Q - y(i-1)/Q)$$

$$\Rightarrow E\left(x(i-1)\right) = E\left(y(i-1)\right) = \frac{E[O]}{2} \Rightarrow T_2 = \frac{2Q - E[O]}{2D}$$

End of proof

From the discussion above, the expected cycle length can be approximated by:

$$E[T] = T_1 F_m(Q - r) + T_2 \bar{F}_m(Q - r) \tag{5}$$

$$\Rightarrow E[T] = (m + L) F_m(Q - r) + \left(\frac{2Q - E[O]}{2D}\right) \bar{F}_m(Q - r)$$

2.3. Expected shortage quantity

A stockout occurs when the total lead time demand exceeds the replenishment level r. An order is triggered either by the perishabilility or by the demand. If the order is triggered by the perishability then all demands during the lead time are backordered and $d_m \le Q - r$. Denote by S_1 the expected shortage quantity in this case, then:

$$S_1 = \int_0^\infty d_L f_L(d_L)\, dd_L = DL \tag{6}$$

If the order was cause by demand, then a partial part of demands during the lead time is backordered. Let S_2 be the expected shortage quantity, then:

$$S_2 = \int_r^{\infty} (d_L - r)\, f_L(d_L)\, dd_L \tag{7}$$

Form (6) and (7), the total expected shortage quantity will be:

$$E[S] = (S_1) F_m(Q-r) + (S_2)\bar{F}_m(Q-r) \tag{8}$$

2.4. Expected inventory level

The expected inventory level can be approximated by considering separately the expected average inventory level during the lead time and the expected average inventory level from the time an order is received to the time where the next reorder is placed. Denote by I_L the expected average inventory level during the lead time. The expression of I_L is given by the following equation:

$$I_L = L\int_0^r \frac{1}{2}\big(r + (r - d_L)\big) f_L(d_L) dd_L$$
$$+ L\int_r^{\infty} \frac{1}{2}(r+0) f_L(d_L) dd_L$$
$$\Rightarrow I_L = \frac{L}{2}\left(r + \int_0^r (r - d_L) f_L(d_L) dd_L \right)$$

The equation above can be simplified to:

$$\Rightarrow I_L = \frac{(S_2 - DL)}{2} L + rL \tag{9}$$

The expected average inventory level after order reception until the next reorder depends on the type of the cycle. Let I_{T_1} and I_{T_2} be the expected average inventory level after order arrival until the next reorder for a cycle of type T_1 and T_2 respectively.

As shown in figure 1, I_{T_1} has the following form:

$$I_{T_1} = Om + \frac{Dm^2}{2} \tag{10}$$

Using the same reasoning for I_{T_2} leads to:

$$I_{T_2} = \left[r + \frac{1}{2}\left(Q - DL - \frac{E[O]}{2} \right) \right](T_2 - L)$$
$$+ \frac{E[O]}{2}(m - T_2) \tag{11}$$

Let $I^1_{T_1}$ and $I^2_{T_2}$ be total expected average inventory level for a cycle T_1 and T_2 respectively. Then we have:

$$I^1_{T_1} = I_L + I_{T_1} \tag{12}$$

$$I^2_{T_2} = I_L + I_{T_2} \tag{13}$$

Clearly, equation (12) must be weighted by $\big(1/(m+L)\big)$ and equation (13) by $\big(2D/(2Q-O)\big)$ to obtain the total expected average inventory level per unit time.

$$E[I] = \frac{I^1_{T_1}}{m+L} F_m(Q-r) + \frac{2DI^2_{T_2}}{2Q-O} \bar{F}_m(Q-r) \tag{14}$$

Now we can formulate the total expected average cost per unit time as:

$$TC(r,Q) = \frac{K + CQ + PE[S] + WE[O]}{E[T]} + hE[I] \tag{15}$$

3. EVALUATION OF THE PERFORMANCE

Due to the complex form of (15), investigating analytical properties of the cost function turns out to be impractical. We were unable to prove analytically that $TC(r,Q)$ is a convex function, nevertheless several numerical examples indicate that $TC(r,Q)$ is convex in Q for any given value of r and convex in r for a fixed value of Q as shown on figure 2.

In this section, we conduct a comprehensive numerical analysis. We evaluate (15) by using Matlab's non-linear solver FMINCON. The optimizer converges to a local minimum of the optimal parameters r^*, Q^* and the optimal cost TC^*.

In this section, we focus on 2 comparisons:

- The expected total cost pertaining to the model we propose vs. the expected total cost pertaining to the classical *(r, Q)* inventory policy that does not take into account the perishability of products.
- The expected total cost pertaining to the model we propose vs. the expected total cost of Chiu's model.

The comparison is made for a sample of 24 settings (cf. Table 1).

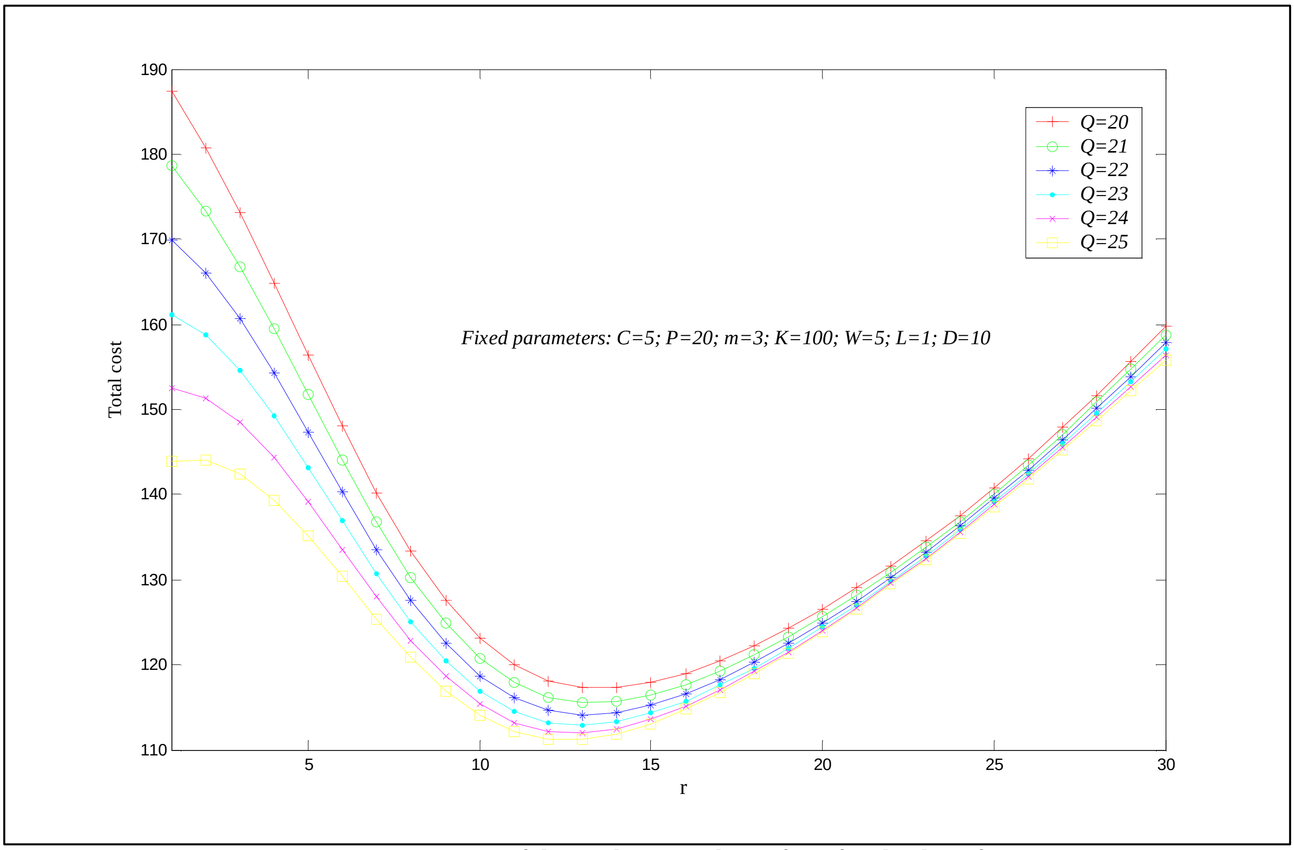

Figure 2. Convexity of the total expected cost for a fixed value of Q

3.1. Comparison with the classical *(r, Q)* policy

In this subsection, we compare the proposed model to the classical model which can be written as:

$$TC_{classic}(r,Q) = \frac{K + CQ + PE[S]}{E[T]} + hE[I]$$

Where:

$$E[S] = \int_r^{\infty} (r - d_L)\, dd_L$$

$$E[I] = r - DL + \frac{Q}{2} + DL\frac{E[S]}{2Q}$$

And $E[T] = \frac{Q}{D}$

We note that the expression of $E[I]$ is the Wagner approximation which is more accurate than the known $r - DL + Q/2$ expected inventory level (Wagner, 1975).
The results of comparison are summarised in Table 1 for a Poisson demand distribution with mean *D=10*. We set *L=1*, *m=3* and *h=1*. Our model is not Markovian. However, we used the Poisson demand distribution in order to facilitate for the solver FMINCON to converge more rapidly than a continuous distribution demand.
Results of Table 1 are as expected intuitively: The model we propose outperforms considerably the classical *(r, Q)* policy for the entire range of considered parameter values. It observed that the optimal ordering quantity Q^* increases as K increases for both the proposed model and the classical model. However, this increase is much higher for the classical policy as it is for the proposed model. This can be explained intuitively by the fact that perishable products call for a smaller order quantity.

On the other hand, we notice that in our model r^* is slightly small than the classical model. The reason is because of the assumption of non perishability during the lead time.
We observe also that the objective function value $TC(r,Q)$ is more sensitive to the ordering cost K than $TC_{classic}(r,Q)$ and the percentage difference between the two policies $(TC\% = 100(TC_{classic} - TC)/TC_{classic})$ is significantly large. This is easy to understand since there is no outdating cost to consider in the classical model.
We note finally that the expected average total cost is highly sensitive to the product lifetime (m) as shown in table 2. When m increases the proposed model tends to converge to the classical one.

3.2. Comparison with Chiu's model

Table 3 illustrates the inventory control parameters obtained from the proposed model, Chiu's model and the simulated model. Results of the simulated average total costs are taken from (Chiu, 1995). Each simulated cost is the average of ten simulation runs and each simulation run is done when the simulated time exceed 1000 time units.

We measure the performance of the proposed model and Chiu's model by employing the percentage difference defined as follows:

$$TC_1\% = 100\left(TC_{sim} - TC\right)/TC_{sim}$$

$$TC_2\% = 100\left(TC_{sim} - TC_{Chiu}\right)/TC_{sim}$$

The results support the following statements:

- For the twenty four test problems (except test problem 3 and 6) the proposed model outperforms Chiu's model with an absolute value of *TC1%* less than *TC2%*. For the test problem 3 and 6, Chiu's model has a very short deviation from the simulated total cost than the proposed model. To explain this finding, we compare each term of equation (15) to the simulation results appeared in (Chiu, 1995).

We found that the underestimation of our total cost is attributed to upper estimation of the expected cycle length *E[T]*. For test problem number 3, we get:

$E[T] = 2{,}65;\ E[I] = 14{,}47;\ E[S] = 0{,}83;\ O = 1{,}72$

Simulation results show that:

$E[T] = 2{,}45;\ E[I] = 12{,}59;\ E[S] = 1{,}19;\ O = 0{,}93$

Hence, we have:

$K / E[T] >> PE[S] / E[T] >> WO / E[T]$

The upper estimation of the expected cycle length is due to the assumption: whenever $d_m > Q - r$ then the cycle length is equal to $(2Q - O)/2D$

- The average total costs for the proposed model and Chiu's model are slightly different when *K=10*, but the TC of the proposed model are positively higher than the simulated one and the TC of Chiu's model are negatively higher than the simulated costs.

Test problems					Our model			Classical policy			*TC%*
Cost parameters					Optimal parameters			Optimal parameters			
	C	*P*	*K*	*W*	*r*	*Q*	*TC*	*r*	*Q*	$TC_{classic}$	
1	5	20	10	5	14,48	14,48	71,15	15,00	15,57	70,57	-0,83
2	5	20	50	5	13,00	22,00	92,65	13,00	33,63	86,65	-6,93
3	5	20	100	5	11,06	27,06	113,36	12,00	47,10	99,09	-14,40
4	5	40	10	5	15,01	15,01	72,64	16,00	16,00	71,64	-1,41
5	5	40	50	5	14,00	21,00	95,03	15,00	32,92	87,92	-8,09
6	5	40	100	5	13,00	25,00	117,05	14,00	46,38	100,38	16,61
7	15	20	10	5	14,00	14,00	171,35	15,00	15,57	170,57	-0,46
8	15	20	50	5	13,00	20,11	194,01	13,00	33,64	186,65	-3,94
9	15	20	100	5	11,02	25,02	215,97	12,00	7,46	199,09	-8,48
10	15	40	10	5	15,00	15,00	173,05	16,00	16,00	171,64	-0,82
11	15	40	50	5	14,00	20,00	196,57	15,00	32,92	187,92	-4,60
12	15	40	100	5	13,00	23,36	220,03	14,00	46,38	200,38	-9,80
13	5	20	10	15	14,00	14,00	71,53	15,00	15,57	70,57	-1,36
14	5	20	50	15	12,44	20,56	94,98	13,00	33,63	86,65	-9,62
15	5	20	100	15	11,01	24,01	117,84	12,00	47,10	99,09	-18,92
16	5	40	10	15	15,00	15,00	73,46	16,00	16,00	71,64	-2,54
17	5	40	50	15	14,00	19,00	97,70	15,00	32,92	87,92	-11,13
18	5	40	100	15	13,00	22,00	122,12	14,00	46,38	100,38	-21,65
19	15	20	10	15	14,00	14,00	171,70	15,00	15,57	170,57	-0,66
20	15	20	50	15	12,03	19,96	195,70	13,00	33,64	186,65	-4,85
21	15	20	100	15	11,00	23,00	219,24	12,00	7,46	199,09	-10,12
22	15	40	10	15	15,00	15,00	173,86	16,00	16,00	171,64	-1,30
23	15	40	50	15	14,00	18,00	198,64	15,00	32,92	187,92	-5,70
24	15	40	100	15	13,00	22,00	223,80	14,00	46,38	200,38	-11,69

Table 1. Comparison of the proposed model and the classical *(r, Q)* policy

m	Our model	Classical model	*TC%*
2	132,1985	99,0922	-33,4096
3	113,3625	99,0922	-14,4010
4	104,6367	99,0922	-5,5953
5	100,8570	99,0922	-1,7810
6	99,4583	99,0922	-0,3695
7	99,1255	99,0922	-0,0336
8	99,0933	99,0922	-0,0011
9	99,0922	99,0922	-0,0000
10	99,0922	99,0922	0
11	99,0922	99,0922	0
Fixed parameters: C=5; P=20; K=100; W=5; L=1; D=10			

Table 2. Variation of the expected total cost with *m*

Test problem N°					Our model			Chiu's model			Simulation	$TC_1\%$	$TC_2\%$
Cost parameters					Optimal parameters			Optimal parameters					
	C	*P*	*K*	*W*	*r*	*Q*	*TC*	*r*	*Q*	TC_{Chiu}	TC_{sim}		
1	5	20	10	5	14,48	14,48	71,15	14,52	13,76	71,12	73,12	2,69	-2,73
2	5	20	50	5	13,00	22,00	92,65	12,56	21,03	93,60	92,17	-0,53	1,56
3	5	20	100	5	11,06	27,06	113,36	11,36	24,63	116,06	116,03	2,30	0,02
4	5	40	10	5	15,01	15,01	72,64	15,76	13,13	72,52	74,78	2,86	-3,03
5	5	40	50	5	14,00	21,00	95,03	14,04	19,92	96,28	94,63	-0,43	1,75
6	5	40	100	5	13,00	25,00	117,05	13,04	23,25	120,14	119,15	1,76	0,84
7	15	20	10	5	14,00	14,00	171,35	14,43	13,00	171,47	171,85	0,29	-0,23
8	15	20	50	5	13,00	20,11	194,01	12,41	19,68	195,37	192,18	-0,95	1,66
9	15	20	100	5	11,02	25,02	215,97	11,14	22,96	219,19	216,24	0,12	1,37
10	15	40	10	5	15,00	15,00	173,05	15,64	12,37	172,99	173,93	0,51	-0,54
11	15	40	50	5	14,00	20,00	196,57	13,91	18,45	198,41	194,72	-0,95	1,90
12	15	40	100	5	13,00	23,36	220,03	12,91	21,53	223,84	219,91	-0,06	1,79
13	5	20	10	15	14,00	14,00	71,53	14,43	13,00	71,47	74,22	3,62	-3,71
14	5	20	50	15	12,44	20,56	94,98	12,41	19,68	95,37	94,68	-0,32	0,73
15	5	20	100	15	11,01	24,01	117,84	11,19	22,91	119,19	118,35	0,43	0,71
16	5	40	10	15	15,00	15,00	73,46	15,71	12,26	72,99	76,26	3,68	-4,29
17	5	40	50	15	14,00	19,00	97,70	13,91	18,45	98,41	97,36	-0,35	1,08
18	5	40	100	15	13,00	22,00	122,12	12,88	21,55	123,84	122,02	-0,08	1,49
19	15	20	10	15	14,00	14,00	171,70	14,42	12,50	171,73	172,16	0,26	-0,25
20	15	20	50	15	12,03	19,96	195,70	12,37	18,73	196,58	193,31	-1,23	1,69
21	15	20	100	15	11,00	23,00	219,24	11,09	22,01	221,31	218,51	-0,33	1,28
22	15	40	10	15	15,00	15,00	173,86	15,61	11,87	173,34	179,19	2,97	-3,26
23	15	40	50	15	14,00	18,00	198,64	13,80	17,66	199,87	196,42	-1,13	1,76
24	15	40	100	15	13,00	22,00	223,80	12,77	20,59	226,35	222,19	-0,73	1,87

Table 3. Comparison between the proposed model and Chiu's model

4. CONCLUSION

The problem of inventory control for perishable products with fixed lifetime is known to be difficult. As mentioned by (Schmidt and Nahmias, 1985), it is unlikely to find or to use an optimal and exact policy. Hence, research has been shifted on the way to develop a heuristics approach. Our contribution is therefore to provide a comprehensive heuristic approach dealing with a large wide range of perishable products with fixed lifetime and positive replenishment lead time.

In this paper, we have proposed a continuous review inventory model for perishable products with finite lifetime operating under a positive lead time. Allowing the backordering case, a modified approximation of (Chiu, 1995) model was presented. The performance of the proposed model was assessed by using Matlab's non-linear solver FMINCON. Our heuristic performs better than the conventional *(r, Q)* policy especially for a product that has a short lifetime. The results of the comparison with Chiu's model demonstrate that our

model performs better than the one of Chiu since our results are closer to the results obtained from simulation.
Possible further research can be addressed on the same problem by relaxing some assumptions. An interesting extension of our work will be based on the possibility of having perishability during the lead time. Another perspective would be to extend the model by giving a more accurate expression of the expected cycle length or considering the lost sales case.

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