7e Conférence Internationale de MOdélisation et SIMulation - MOSIM'08 - du 31 mars au 2 avril 2008 – Paris –France
« Modélisation, Optimisation et Simulation des Systèmes : Communication, coordination et coopération »

A STOCHASTIC PROGRAMMING APPROACH FOR SAWMILL PRODUCTION PLANNING

MASOUMEH KAZEMI ZANJANI[a], DAOUD AIT-KADI[b], MUSTAPHA NOURELFATH[c]

Université Laval, Faculty of science and engineering, Department of mechanical engineering, Interuniversity Research Centre on Enterprise Networks, Logistics and Transportation (CIRRELT), Québec, Canada, G1K 7P4

[a] masoumeh.kazemi-zanjani.1@ulaval.ca ,
[b]Daoud.Aitkadi@gmc.ulaval.ca
[c] Mustapha.Nourelfath@gmc.ulaval.ca

ABSTRACT: This paper investigates a sawmill production planning problem where the non-homogeneous characteristics of logs results in random yields. A two-stage stochastic linear programming approach is proposed to address this problem. The random yields are modeled as scenarios with discrete probability distribution. The solution methodology is based on the sample average approximation method. Confidence intervals are constructed for the optimality gap of several candidate solutions based on common random number streams. A computational study involving a real sawmill is presented to highlight the significance of using the stochastic model instead of the mean-value deterministic linear programming model which is the traditional production planning tool in sawmills.

KEYWORDS: production planning, random yield, sawmill, stochastic programming, sample average approximation

1. INTRODUCTION

Most production environments are characterized by multiple types of uncertainties. Random characteristic of raw materials is a common issue in manufacturing environments that process the natural resources, namely refineries, sawmills, etc. This randomness, as a consequent, can cause random yields of the production processes. The presence of random yield causes the fraction of the quantity actually processed, which turns out to be usable, to be uncertain.

The goal of this work is to address multi-period multi-product (MPMP) production planning in sawmills where possible combinations of log classes and cutting patterns can produce simultaneously different mix of lumbers with random yield. Raw materials (logs) in sawmills are classified based on some attributes namely: diameter class, species, length, taper, etc. Logs are broken down into different pieces of lumbers (products) by means of different cutting patterns. We define a production process in a sawing unit as a combination of a log class and a cutting pattern. Due to non homogeneity in quality of logs, each cutting pattern yields a random quantity of corresponding products after processing a known quantity of each log class. In the production line, whenever a log from a special class enters into a cutting pattern, after some preliminary activities, it passes though an X-ray scanner. The result of the scanning is transferred to a log sawing optimizer which decides about the optimal mix of lumbers with their quantity that should be yielded by that cutting pattern. The objective of the optimizer is to maximize value/volume of the yielded products for each log. Production planning in a sawing unit is to decide about the optimal quantity of log consumption from different classes and selection of corresponding cutting patterns in each period of the planning horizon, in order to fulfill customer demand regarding machine capacities and log inventory. The objective is to minimize log consumption, as well as product inventory/backorder costs.

Two different approaches have been already proposed in the literature to address sawmill production planning. In the first approach, the randomness of the process yields is ignored and their expected value is considered in a MPMP linear programming (LP) model (Gaudreault *et al.*, 2004). However, the production plans issued by these models result usually in extra inventory of products with lower quality and price while backorder of products with higher quality and price. The second approach is focused on combined optimization type solutions linked to real-time simulation sub-systems (Maness and Norton, 2002), (Maness and Adams, 1991), (Mendoza *et al.*, 1991). In this approach, the stochastic characteristic of logs is taken into account by assuming that all the input logs are scanned through an X-ray scanner, before planning. Maness and Norton, (2002) have developed an integrated multi-period production planning model which is the combination of an LP model and a log sawing optimizer (simulator). The LP model acts as a coordinating problem that allocates limited resources. A series of dynamic programming sub-problems, titled in the literature as "log sawing optimization models" are used to generate activities (columns) for the coordinating LP based on the products' shadow prices. The log sawing optimization model is a sawing algorithm for lumber grade, based on data collected from the X-ray scanner. Although the stochastic characteristics of logs are considered in the second approach, they include the following drawbacks: logs, needed for the next planning horizon, are not always available in the sawmill to be scanned before planning. Furthermore, to implement this method, the logs should be processed in production line in the same order they have been simulated, which is not an easy practice.

Sawmill production planning problem can be considered as the combination of several classical production planning problems in the literature which have been modeled by linear programming. Product mix problem and a special case of process selection problem (Johnson and Montgomery, 1974); (Sipper and Bulfin, 1997) are the two main building blocks of this problem. Most of the work in the literature for including uncertainty in production planning models is focused on considering random product demand in the models. In (Bakir and Byrune, 1998), demand uncertainty in a MPMP production planning model has been studied. They have developed a demand stochastic LP model based on the two-stage deterministic equivalent problem. In (Escudero *et al.*, 1993) a multi-stage stochastic programming approach has been used for solving a MPMP production planning model with random demand. In (Leung *et al.*, 2006) the uncertain data for almost all the model parameters have been considered in an aggregate production planning problem. They have developed a robust optimization model to introduce production plans which are less sensitive to the change in the uncertain data. In (Kazemi *et al.*, 2007) three approaches have been proposed as the potential methodologies to address MPMP production planning in a manufacturing environment with random yield. These approaches include stochastic programming, robust optimization and fuzzy linear programming.

In this paper, a new approach is proposed for sawmill production planning while considering random characteristics of logs and consequently random process yields. We develop a two-stage stochastic program with recourse (Kall *et al.*, 1994 and 2005), (Birge and Louveux, 1997) to address this problem. The random yields are modeled as scenarios with discrete probability distribution. Due to astronomic number of scenarios for random yields in the two-stage stochastic model, a Monte-Carlo sampling strategy, the sample average approximation (SAA) method (cf. Shapiro et al., 1998; Mak et al, 1999; Shapiro et al., 2000) is implemented to solve the stochastic model. The confidence intervals on the optimality gap for the candidate solutions are constructed based on common random number (CRN) streams (Mak *et al.*, 1999). Our computational results involving one real sawmill indicate that the proposed approach can be served as a viable tool for sawmill production planning.

The remainder of this paper is organized as follows. In the next section, we provide a theoretical framework for two-stage stochastic linear programming (LP). In section 3 we describe a two-stage stochastic linear program for sawmill production planning under uncertainty of process yields. In section 4, we discuss about some challenges involved in developing a solution strategy for the model and we develop the solution methodology, we also explain the SAA technique with the sampling technique based on the common random numbers. In section 5, we present the implementation results of the stochastic model and solution methodology for a real sawmill. We also compare the quality of solutions resulted from the new approach with those of the mean-value deterministic LP model. Our concluding remarks are given in section 6.

2. A THEORRTICAL FRAMEWORK FOR TWO-STAGE STOCHASTIC LP

Linear programming is a fundamental planning tool. When one or more of the parameters in a linear program is represented by a random variable, a stochastic linear program (SLP) is resulted. Model (1)-(3) is an example of a stochastic LP.

$$\min \quad c^T x, \tag{1}$$

Subject to

$$Ax = b, \tag{2}$$

$$T(\xi)x \geq h^T(\xi), \tag{3}$$

$$x \geq 0.$$

Where $T(\xi)$ and $h(\xi)$ are the random parameters. In the above model, constraint (2) and (3) represent the set of deterministic and stochastic constraints, respectively.

In two-stage stochastic models, we explicitly classify the decision variables according to whether they are implemented before or after an outcome of the random variable is observed. In other words, we have a set of decisions to be taken without full information on the random parameters. These decisions are called first-stage decisions, and are usually represented by a vector (x). Later, full information is received on realizations (scenarios) of some random vector ξ. Then, second-stage or recourse actions (y) are taken. These second-stage decisions allow us to model a response to each of the observed outcomes (scenarios) of the random variable, which constitutes our recourse. In general, this response will also depend upon the first-stage decisions. In mathematical programming terms, this defines the so-called two-stage stochastic program with recourse of the form:

$$\min \quad c^T x + E_\xi Q(x,\xi), \tag{4}$$

Subject to

$$Ax = b, \tag{5}$$
$$x \geq 0.$$

where $Q(x,\xi) = \min\left\{q^T(\xi)y \mid Wy = h^T(\xi) - T(\xi)x\right\}$, W is the recourse matrix, $q^T(\xi)$ is the vector of penalty cost of second-stage (recourse) variables, ξ is the random vector formed by the components of $q^T(\xi)$, $h^T(\xi)$, $T(\xi)$, and E_ξ denotes mathematical expectation with respect to ξ.

In the case of continuous distribution for random variables in model (4)-(5), the calculation of the expected value $E_\xi Q(x,\xi)$ requires the calculation of multiple integrals with respect to the measure describing the distribution of ξ. However the computational effort increases with the dimension of the stochastic variables vector and this leads to tremendous amount of work. On the other hand, if ξ has a finite discrete distribution $\left\{(\xi^i, p^i), i = 1,...,n\right\}$, then (4)-(5) can be transformed into its *deterministic equivalent* which is an ordinary linear program as follows.

$$\min \quad c^T x + \sum_{i=1}^{n} p^i Q(x,\xi^i), \tag{6}$$

Subject to

$$Ax = b, \tag{7}$$
$$x \geq 0.$$

where, $Q(x,\xi^i) = \min\left\{q^{iT}(\xi)y^i \mid Wy^i = h^{iT}(\xi) - T^i(\xi)x\right\}$, y^i, $q^{iT}(\xi)$, $h^i(\xi)$ and $T^i(\xi)$ represent the ith scenarios for y, $q^T(\xi)$, $h^T(\xi)$ and $T(\xi)$, respectively. Model (6)-(7) can be solved by the LP solvers. Although this model can become (very) large in scale, its particular block structure is amenable to specially designed algorithms.

3. PROBLEM DESCRIPTION

In this section we first describe the deterministic linear program (LP) formulation for sawmill production planning considered in this paper. Then we develop the proposed stochastic model to address the problem by considering the uncertainty of process yields.

3.1. The deterministic LP model for sawmill production planning

Consider a sawing unit with a set of products "P", a set of classes of logs "C", a set of production processes "A", a set of resources (machines) "R", and a planning horizon consists of "T" periods. We define a production process in a sawing unit as a combination of a log class and a cutting pattern. As it is mentioned before, each process produces a mix of lumbers with different dimensions. However, due to random quality of input logs the quantity of products (yield of the processes) is a random variable. Figure 1 is a schematic illustration of sawing process in sawmills.

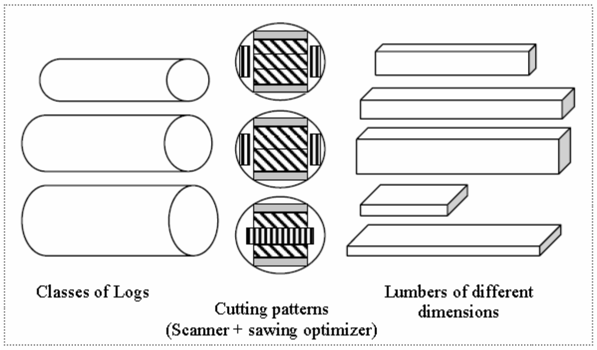

Figure 1. Sawing process in sawmills

To state the deterministic linear programming model for the sawmill production planning problem, the following notations are used.

3.1.1. Notations

<u>*Indexes*</u>

p: product
t: period
c: log class
a: production process
r: resource (machine)

<u>*Parameters*</u>

i_{pt} : Inventory cost per unit of product p in period t
b_{pt} : Backorder cost (lost opportunity and goodwill) per unit of product p in period t
m_{ct} : Log cost per unit of log class c in period t
I_{c0} : The inventory of log class c at the beginning of planning horizon

I_{p0} : The inventory of product *p* at the beginning of planning horizon
s_{ct} : The quantity of material of class *c* supplied at the beginning of that period
d_{pt} : Demand of product *p* in period *t*
ϕ_{ac} : The units of log class *c* consumed by process *a* (consumption factor)
ρ_{ap} : The units of product *p* produced by process *a* (yield of process *a*)
δ_{ar} : The capacity consumption of resource *r* by process *a*
M_{rt} : The capacity of resource *r* in period *t*

Decision variables

X_{at} : The number of times each production process *a* should be run in each period *t*
I_{ct} : Inventory level of log class *c* by the end of period *t*
I_{pt} : Inventory level of product *p* by the end of period *t*
B_{pt} : Back order level of product *p* by the end of period *t*

3.1.2. The LP model

$$\min\ Z = \sum_{p\in \mathrm{P}}\sum_{t=1}^{T}[i_{pt}I_{pt} + b_{pt}B_{pt}] + \sum_{c\in \mathrm{C}}\sum_{t=1}^{T}\sum_{a\in A} m_{ct}\phi_{ac}X_{at}, \qquad (8)$$

Subject to

Material inventory constraint

$$I_{ct} = I_{ct-1} + s_{ct} - \sum_{a\in A}\phi_{ac}X_{at}, \qquad t = 1,...,T,\ c\in \mathrm{C}, \qquad (9)$$

Product inventory constraint

$$I_{p1} - B_{p1} = I_{p0} + \sum_{a\in \mathrm{A}}\rho_{ap}X_{a1} - d_{p1},$$

$$I_{pt} - B_{pt} = I_{pt-1} - B_{pt-1} + \sum_{a\in \mathrm{A}}\rho_{ap}X_{at} - d_{pt}, \qquad t = 2,...,T;\ p\in \mathrm{P}, \qquad (10)$$

Production capacity constraint

$$\sum_{a\in \mathrm{A}}\delta_{ar}X_{at} \le M_{rt}, \qquad t = 1,...,T, r\in \mathrm{R}, \qquad (11)$$

Non-negativity of all variables

$$X_{at}\ge 0, I_{ct}\ge 0, I_{pt}\ge 0, B_{pt}\ge 0, \qquad (12)$$
$$t = 1,...,T,\ p\in \mathrm{P},\ c\in \mathrm{C}, a\in \mathrm{A}.$$

The objective function (8) is a linear cost minimization equation. It consists of total inventory and backorder cost for all products and log cost for all classes in the planning horizon. Constraint (9) ensures that the total inventory of log of class *c* at the end of period *t* is equal to its inventory in the previous period plus the quantity of material of class *c* supplied at the beginning of that period (s_{ct}) minus its total consumption in that period. It should be noted that the total consumption of each class of log in each period is calculated by multiplying material consumption factor of each process (ϕ_{ac}) by the number of times that process is executed in that period. Constraint (10) ensures that the sum of inventory (or backorder) of product *p* at the end of period *t* is equal to its inventory (or backorder) in the previous period plus the total production of that product in that period minus the product demand for that period. Total quantity of production for each product in each period is calculated as the sum of the quantities yielded by each of the corresponding processes regarding the yield (ρ_{ap}) of each process. Finally, constraint (11) requires that the total production do not exceed the available production capacity. In other words, the sum of capacity consumption of a machine *r* by corresponding processes in each period should not be greater than the capacity of that machine in that period.

3.2. The Two-stage stochastic model for sawmill production planning

To include the random nature of process yields in MPMP production planning we expand the model (8)-(12) to a two-stage stochastic linear program with recourse. It is assumed that the probability distribution of random yields is known. We represent the random yield vector by ξ, where $\xi = \{\rho_{ap} \mid a\in \mathrm{A}, p\in \mathrm{P}\}$. We also represent each realization of random process yields by $\rho_{ap}(\xi)$. It should be emphasized that the stages of the two-stage recourse problem do not refer to time units. They correspond to steps in the decision making. In other words, in the first-stage, the decision maker does not have any information about the process yields due to lack of complete information on the characteristic of logs. However, the production plan should be determined before the complete information is available. In the second stage when the realized yields are available, based on the first-stage decision, the recourse actions (inventory or backorder levels) can be computed. The objective of the second-stage problem is to minimize the inventory and backorder costs (recourse action costs). The resulting formulation is as follows.

First-stage model

$$\min\ Z = \sum_{c\in \mathrm{C}}\sum_{t=1}^{T}\sum_{a\in A} m_{ct}\phi_{ac}X_{at} + E_{\xi}[Q(X_{at},\xi)], \qquad (13)$$

Subject to

$$I_{ct} = I_{ct-1} + s_{ct} - \sum_{a\in A}\phi_{ac}X_{at}, \qquad t = 1,...,T,\ c\in \mathrm{C}, \qquad (14)$$

$$\sum_{a\in A}\delta_{ar}X_{at}\le M_{rt}, \qquad t=1,\dots,T,\ r\in R, \tag{15}$$

$$X_{at}\ge 0, I_{ct}\ge 0, \qquad a\in A, c\in C, t=1,\dots,T. \tag{16}$$

where $Q(X_{at},\xi)$ is the optimal value of the following problem:

Second-stage model

$$\min\ Q(X_{at},\xi)=\sum_{p\in P}\sum_{t=1}^{T}[i_{pt}I_{pt}+b_{pt}B_{pt}], \tag{17}$$

Subject to

$$I_{p1}-B_{p1}=I_{p0}+\sum_{a\in A}\rho_{ap}(\xi)X_{a1}-d_{p1},$$

$$I_{pt}-B_{pt}=I_{pt-1}-B_{pt-1}+\sum_{a\in A}\rho_{ap}(\xi)X_{at}-d_{pt},$$

$$t=2,\dots,T,\ p\in P, \tag{18}$$

$$I_{pt}\ge 0, B_{pt}\ge 0, \qquad p\in P, t=1,\dots,T. \tag{19}$$

Note again that ξ is a random vector corresponding to different scenarios for the uncertain process yields, and the optimal value $Q(x,\xi)$ of the second-stage problem (17)-(19) is the function of the first-stage decision variable X_{at} and a realization (or a scenario) of the uncertain yield ($\rho_{ap}(\xi)$). The expectation in (13) is taken with respect to the probability distribution of ξ which is supposed to be known.

4. SOLUTION METHODOLOGY

In this section we first discuss about some difficulties to solve the stochastic model and then we explain the sample average approximation (SAA) method which is used to solve this model.

4.1. The difficulties to solve the stochastic model

To solve the two-stage stochastic model (13)-(19), at the first step different scenarios for random yields ($\rho_{ap}(\xi)$) with their probability distribution should be determined based on historical data in industry. In sawmill case, respecting the limited volume of logs and dimensions of different products, a discrete distribution for random yields of processes can be considered. Consequently, the probability distribution of scenarios for the two-stage model (13)-(19) will also have a discrete distribution with a known probability for each scenario. Therefore the expected value $E_\xi[Q(X_{at},\xi)]$ in (13) can be written as $\sum_{i=1}^{n}p^iQ(X_{at},\xi^i)$, where n denotes total number of scenarios, ξ^i denotes the *i*th scenario, and p^i denotes the probability of scenario *i*. Finally, the first and second-stage problems (13)-(19) can be summed in its deterministic equivalent as follows.

First-stage model

$$\min\ Z=\sum_{c\in C}\sum_{t=1}^{T}\sum_{a\in A}m_{ct}\phi_{ac}X_{at}+\sum_{i=1}^{n}p^iQ(X_{at},\xi^i) \tag{20}$$

Subject to
Constraints (14)-(16)

Second-stage model

Model (17)-(19)

It is evident that, the LP model (20) can be solved by the linear programming solvers. However, in the case of huge number of scenarios solving this model would be far beyond the present computational capacities. For example in sawmill case, due to vide variety of logs in the same class, the number of scenarios for each process and thus for the stochastic model is very large. In order to have a better idea on the size of scenario set in the stochastic model for a sawmill, let us consider the following example. Suppose that there are 3 log classes and 5 cutting patterns; and thus 15 processes in the sawmill. If each process yields 10 products and the quantity of each product can take *k* different values, then we are facing with k^{150} scenarios, which in the modest case of *k=5* the size of scenario set would be equal to $5^{150}\approx 8\times 10^{69}$. Thus, we are facing with an astronomic number of scenarios in the two-stage model (13)-(19) or in its deterministic equivalent (20). In such situations, we can use Monte Carlo sampling techniques, which consider only randomly, selected subsets of the set $\{\xi^1,\xi^2,\dots,\xi^n\}$ to obtain approximate solutions. Sampling techniques can be applied in different ways. One approach uses sampling in an "interior" fashion; among such algorithms we can mention the stochastic decomposition method of Higle and Sen, 1996. A second fundamental approach which uses sampling is the average approximation (SAA) scheme (cf. Shapiro et al., 1998; Mak et al, 1999; Shapiro et al., 2000) which is an "exterior approach".

4.2. The sample average approximation (SAA) scheme

In the SAA scheme, a random sample of *n* realizations (scenarios) of the random vector ξ is generated and the expectation $E_\xi[Q(X_{at},\xi)]$ is approximated by the sample average function $\frac{1}{n}\sum_{i=1}^{n} Q(X_{at},\xi^i)$. In other words, the "true" problem (20) is approximated by the sample average approximation (SAA) problem (21).

First-stage model

$$\min\ \hat{Z} = \sum_{c\in C}\sum_{t=1}^{T}\sum_{a\in A} m_{ct}\phi_{ac}X_{at} + \frac{1}{n}\sum_{i=1}^{n} Q(X_{at},\xi^i) \qquad (21)$$

Subject to
Constraints (14)-(16)

Second-stage model

Model (17)-(19)

It is possible to be shown that under mild regularity conditions, as the sample size *n* increases, the optimal solution vector $\hat{X}_n$ and optimal value $\hat{Z}_n$ of the SAA problem (21) converge with probability one to their true counterparts, and moreover $\hat{X}_n$ converges to an optimal solution of the true problem with probability approaching one exponentially fast (Shapiro and Homem-de-Mello., 1998 and 2000). This convergence analysis suggests that a fairly good approximate solution to the true problem (20) can be obtained by solving an SAA problem (21) with a modest sample size. It can easily be verified that the mentioned regularity conditions are true for our problem, especially regarding the discrete distribution of random yields.

In practice, the SAA scheme involves repeated solutions of the SAA problem (21) with independent samples. Statistical confidence intervals are then derived on the quality of the approximate solutions (Mak et al., 1999). According to the work of Mak et al. (1999), an obvious approach to testing solution quality for a candidate solution ($\bar{X}$) is to bound the optimality gap, defined as $Ef(\bar{X},\tilde{\xi}) - z^*$ using standard statistical procedures, where $f(\bar{X},\tilde{\xi})$ and z^* are the true objective value for $\bar{X}$ and the true optimal solution to the problem (20), respectively. In our work a sampling procedure based on common random numbers (CRN) is used to construct the optimality gap confidence interval which provides significance variance reduction over naive sampling as it has been proposed in Mak et al., 1999. This approach is described next.

The SAA algorithm (with common random number streams)

Step 1- Generate n_g i.i.d. batches of samples each of size *n* from the distribution of $\tilde{\xi}$, i.e., $\left\{\xi_j^1, \xi_j^2, ..., \xi_j^n\right\}$ for *j*=1,…, n_g. For each sample solve the corresponding SAA problem (21). Let $\hat{Z}_n^j$ and $\hat{X}_n^j$, *j*=1,…,n_g, be the corresponding optimal objective value and an optimal solution, respectively.

Step 2- Compute

$$\bar{Z}_{n,n_g} = \frac{1}{n_g}\sum_{j=1}^{n_g}\hat{Z}_n^j \qquad (22)$$

$$s^2_{\bar{Z}_{n,n_g}} = \frac{1}{n_g(n_g-1)}\sum_{j=1}^{n_g}(\hat{Z}_{n_g}^j - \bar{Z}_{n,n_g})^2 \qquad (23)$$

It is well known that the expected value of $\hat{Z}_n$ is less than or equal to the optimal value z^* of the true problem (see e.g., Mak et al., 1999). Since $\bar{Z}_{n,n_g}$ is an unbiased estimator of $E[\hat{Z}_n]$, we obtain that $E[\bar{Z}_{n,n_g}] \le z^*$. Thus $\bar{Z}_{n,n_g}$ provides a lower statistical bound for the optimal value z^* of the true problem and $s^2_{\bar{Z}_{n,n_g}}$ is an estimate of the variance of this estimator.

Step 3- Choose a candidate feasible solution $\bar{X}$ of the true problem, for example, a computed $\hat{X}_{n'}^j$ by using a sample size (n') larger than used for lower bound estimation (n). Estimate the true objective function value $f(\bar{X})$ for all batches of samples (*j*=1,…,n_g) as follows.

$$\tilde{f}_n^j(\bar{X}) = \sum_{c\in C}\sum_{t=1}^{T}\sum_{a\in A} m_{ct}\phi_{ac}\bar{X}_{at} + \frac{1}{n}\sum_{i=1}^{n} Q(\bar{X},\xi_j^i) \qquad (24)$$

Step 4- Compute the observations of the optimality gap G_n^j for the candidate solution $\bar{X}$ for all *j*=1,…,n_g as follows.

$$G_n^j = \tilde{f}_n^j(\bar{X}) - \hat{Z}_n^j \qquad (25)$$

It has been shown in Mak et al., 1999 that

$$E\underbrace{\left[\tilde{f}_n(\bar{X}) - \hat{Z}_n\right]}_{G_n} \ge E[f(\bar{X},\xi)] - z^*$$

where $f(\bar{X},\xi)$ and z^* are the true objective value for $\bar{X}$ and the true optimal solution to the problem (20), respectively and ($E[f(\bar{X},\xi)] - z^*$) is the true optimality gap for the candidate solution $\bar{X}$. We also have:

$$\sqrt{n_g}\left[\bar{G}_{n_g} - EG_n\right] \Rightarrow \mathrm{N}(0,\sigma_g^2) \quad \text{as } n_g \to \infty$$

where σ_g^2=var G_n

Step 5- Compute the sample mean and sample variance for the optimality gap G_n^j as follows.

$$\bar{G}_{n_g} = \frac{1}{n_g}\sum_{j=1}^{n_g} G_n^j \qquad (26)$$

$$s_{G_n^j}^2 = \frac{1}{n_g\,(n_g-1)}\sum_{j=1}^{n_g}(G_n^j - \bar{G}_{n_g})^2 \qquad (27)$$

Step 6- Compute the approximate $(1-\alpha)$-level confidence interval for the optimality gap for $\bar{X}$ as $\left[0, \bar{G}_{n_g} + \tilde{\varepsilon}_g\right]$ where $\tilde{\varepsilon}_g = \dfrac{t_{n_g-1,\alpha}\, s_{G_n^j}}{\sqrt{n_g}}$.

5. COMPUTATIONAL RESULTS

In this section, we describe the numerical experiments using the proposed approach to solve a real sawmill production planning problem. We first describe the characteristics of the test problem and some implementation details, and then comment on the quality of the stochastic model solution in comparison to that obtained using the mean-value deterministic model.

5.1. Data and implementation

Our test problem is that of the production planning for a sawing unit in a sawmill in Quebec (Canada) where 3 classes of logs with 10 feet length can be processed by 5 cutting patterns for producing 27 products (lumbers with different dimensions). Therefore, we have 15 processes all can produce 27 products with random yield. All the processes are run on two machines: Trimmer and Bull. The planning horizon consists of 30 periods (days). Products demands in each period are supposed to be deterministic and known parameters which are calculated based on the received and forecasted orders. The number of scenarios for random yields in this example can be estimated as $5^{405} \approx 1.2 \times 10^{283}$! Recall from section 4 that the SAA method calls for the solution of n_g instances of the approximating stochastic program (21), each involving *n* sampled scenarios. Statistical validation of a candidate solution is then carried out by evaluating the objective function using the same *n* sampled scenarios in each batch. In our implementation, we used *n*=60, 100, and 150; and $n_g = 30$. Our candidate solutions are computed by solving the SAA problem (21) with $n' = 100, 150, \text{ and } 250$. To illustrate the complexity of solving (21) within the SAA scheme, we present the sizes of the deterministic equivalents of the SAA problems corresponding to the different values of *n* in table 1.

n	Constraints	Variables
1	960	2160
100	81150	162540
150	121650	243540
250	202650	405540

Table 1. Deterministic equivalent size of the SAA problem

The SAA scheme was implemented in OPL Studio 3.7.1. The OPL Script is used for solving the deterministic equivalents for different instances of SAA problems as well as for calculating the true objective function value for the candidate solutions. All computations were carried out on a Pentium (R) IV 1.8 GHz PC with 512 MB RAM running Windows XP.

5.2. Quality of stochastic solutions

In this section we first present the results of applying the SAA scheme for our test problem as well the evaluation of quality of several candidate solution; afterwards we compare the solutions of the stochastic programming model to that of the deterministic optimization model (the mean-value problem) involving the mean-values of the uncertain yields. The point estimates of the lower statistical bound for the optimal value of the problem are reported in table 2. They are computed based on 30 batches of sampled scenarios with 3 different batch sizes. Table 3 displays the quality of 3 candidate solutions and contains the 95% confidence intervals on their optimality gaps based on CRN method (see section 4). The candidate solutions $\bar{X}^{100}, \bar{X}^{150}, \bar{X}^{250}$ for the RCN strategy are computed by solving the initial approximating problem that has 100, 150 and 250 scenarios. The CPU times for computing each candidate solution are also reported in table 3.

Batch size (*n*)	60	100	150
Average ($\bar{Z}_{n,n_g}$)	515829	527981	519226
SD ($s_{\bar{Z}_{n,n_g}}$)	35582	25562	22590

Table 2. Lower bound estimation results for the optimal value (30 batches)

As it can be observed from table 3, by increasing the sample size, the quality of approximate solutions improves monotonically and the tighter confidence intervals for the optimality gaps of candidate solutions are constructed.

Candidate solution	$\bar{X}^{100}$	$\bar{X}^{150}$	$\bar{X}^{250}$
Batch size (*n*)	60	100	150
No. of batches (n_g)	30	30	30
Point estimate ($\bar{G}_{n_g}$)	13253	9284	4783
Error estimate ($\alpha = 95\%$) ($\tilde{\varepsilon}_g$)	1555	1268	393
Confidence interval (95%)	[0,14808]	[0,10552]	[0, 5176]
CPU time (sec.)	45	80	198

Table 3. Optimality gaps for candidate solutions

To compare the candidate stochastic model solution with the mean-value model solution, we calculated the value of the stochastic solution (VSS) (Birge and Louveux, 1997) for three candidate solutions. The VSS indicates the difference between the expected cost of the mean-value model solution and the stochastic model one and is computed as follows.

Step 1- Solve the deterministic problem (mean-value problem) (8)-(14) by considering the expected value of process yields and find the optimal solution X^{MVP} .

Step 2 – Compute the real objective function value (the expected cost) for X^{MVP} ($\tilde{f}_n(X^{MVP})$) by (24) (see section 4).

Step 3- The value of the stochastic solution (VSS) for each candidate solution ($\bar{X}$) is calculated by:

VSS= $\tilde{f}_n(X^{MVP}) - \tilde{f}_n(\bar{X})$.

where, $\tilde{f}_n(\bar{X})$ is the objective value of the SAA problem for the solution $\bar{X}$.

The comparison between three candidate solutions $\bar{X}^{100}$, $\bar{X}^{150}$, $\bar{X}^{250}$ and X^{MVP} is reported in table 4.

Solution	X^{MVP}			$\bar{X}^{100}$ (*n*=60)	$\bar{X}^{150}$ (*n*=100)	$\bar{X}^{250}$ (*n*=150)
	n=60	*n*=100	*n*=150			
Objective function value ($\tilde{f}_n(X)$)	1735702	17135702	1704186	509108	504536	502162
VSS				1226594	1215266	1202024

Table 4. Comparison of the solutions of the stochastic model and mean-value model

It is clear that the estimated total average cost for all three candidate stochastic programming solutions are significantly smaller than that of the mean-value problem solution which reveals that the stochastic model is a more reliable production planning tool in the presence of random yields. Finally we can conclude that, by considering a moderate number of scenarios (250) among the potential enormous number of scenarios for random yields we have obtained an approximate solution in a very short time with an optimality gap of [0, 5176] which is less than 1% of the lower bound of the real optimal value (see Tables 2 and 3). Thus, this solution can be accepted as a relatively good approximation to the optimal solution regarding the high expected cost of mean-value problem solution (see table 4).

6. CONCLUSIONS

In this paper, we developed a two-stage stochastic programming model for sawmill production planning by considering random characteristics of logs. The SAA method was implemented to solve the stochastic model which provided us an efficient framework for identifying and statistically testing a variety of candidate production plans. We provided the empirical results for production planning in a real sawmill and we identified several candidate plans in a short time by solving the approximate SAA problem. Furthermore, the confidence intervals for the optimality gap of candidate solutions were constructed by common random number (CRN) streams. Our results reveal that the production plans identified by the stochastic model are superior to traditional mean-value (deterministic) problem plans regarding the high expected inventory/backorder cost of mean-value model plans.

7. ACKNOWLEDGMENTS

This work was supported by Forac research consortium, Université Laval, Québec, Canada.

8. REFERENCES

Bakir M.A., and Byrune M.D., 1998. *Stochastic Linear Optimization of an MPMP Production Planning Model.* Int. J. Production Economics 55, p. 87-96.

Birge J.R., and Louveux F., 1997. *Introduction to Stochastic Programming.* Springer.

Escudero L.F., Kamesam P.V., King A.J., and Wets R.J-B., 1993. *Production Planning via Scenario Modeling.* Annals of Operations Research 43, p. 309-335.

Gaudreault J., Rousseau A., Frayret J.M., and Cid F., 2004. *Planification Opérationnelle du Sciage.* For@c technical document.

Higle J.L., and S. Sen, 1996. *Stochastic Decomposition: An Algorithm for Two-stage Linear Programming*. Kluwer Academic Publishers, Boston, MA.

Kall P., and Wallace S.W., 1994. *Stochastic Programming.* John Wiley & sons.

Kall P., and Mayer J., 2005. *Stochastic linear programming.* Springer's International Series.

Kazemi Zanjani M., Nourelfath M., and Ait-Kadi D., 2007. *Production planning for simultaneous production of multiple products by alternative processes with random yield*. Proceedings of 7e Congrès international de génie industriel (2007), Trois-Rivières, Québec (CANADA).

Johnson L.A., and Montgomery D.C., 1974. *Operations Research in Production Planning, Scheduling, and Inventory Control.* John Wiley & Sons.

Leung S.C.H., Tsang S.O.S., W.L. Ng, and Yue Wu, 2006. *A Robust Optimization Model for Multi-site Production Planning Problem in an Uncertain Environment*. European Journal of Operational Research, doi:10.1016/j.ejor.2006.06.011.

Mak W.K., Morton D.P., and Wood R.K., 1999. *Monte Carlo bounding techniques for determining solution quality in stochastic programs*. Operations Research Letters 24, p. 47-56.

Maness T.C., and Adams D.M., 1991. *The Combined Optimization of Log Bucking and Sawing Strategies.* Wood and Fiber Science 23, p. 296-314.

Maness T.C., and Norton S.E., 2002. *Multiple-Period Combined Optimization Approach to Forest Production Planning*. Scand. J. For. Res. 17, p. 460-471.

Mendoza G.A., Meimban R.J., Luppold W.J., and Arman P.A., 1991. *Combined Log Inventory and Process simulation Models for the planning and control of sawmill operations*. Proceeding 23rd GIRP International Seminar on Manufacturing Systems, Nancy, France.

Shapiro A., and Homem-de-Mello T., 1998. *A Simulation based Approach to Two-stage Stochastic Programming with Recourse*. Mathematical Programming 81, p. 301-325.

Shapiro A., and Homem-de-Mello T., 2000. *On the Rate of Convergence of Optimal Solutions of Monte Carlo Approximations of Stochastic Programs*. SIAM J. Optim. 11(1), p. 70-86.

Sipper D., and Bulfin R., 1997. *Production: planning, control, and integration*. McGraw-Hill.