

MULTIPLE RETAILER COOPERATION SYSTEM ANALYSIS

A. El Omri, A. Ghaffari, and Z. Jemai

Laboratoire Génie Industriel, Ecole Centrale Paris
Grande Voie des Vignes, Châtenay-Malabry, France
{adel.omri, asma.ghaffari, zied.jemai@ecp.fr}

ABSTRACT: *In this paper, we analyze a new cooperative situation applied to supply chain management. Specifically, we study a distribution system where a set of retailers may order a single product from a unique supplier to satisfy a deterministic and constant rate demand of final customers. Each retailer, when ordering a quantity of a product, has to pay a fixed ordering cost which is the same for all the retailers. In addition to the ordering cost, every retailer has to pay a specific cost; this cost may include transportation and all additional charges that depend on the retailer's characteristics. Retailers may choose to cooperate by grouping their orders as a big order. In this case, they realise some cost saving benefits by reducing the sum of ordering cost to only one cost paid by the coalition. In case of cooperation we suppose that there is a decision maker or a warehouse that coordinates the decisions. We suppose that the delivered products are stored in local warehouses where every retailer has to support his own holding cost. In this paper, major differences with existent inventory cooperation situations are highlighted. We use a set of empirical tests to corroborate some useful insights and to raise some relevant questions to address in a future work. Further, an efficient algorithm is proposed to address the difficult question of the coalition generation.*

KEYWORDS: *Supply chain coordination; coalition generation; economic order quantity.*

1 INTRODUCTION

The economic and industrial context has become very complex, making it more important than ever for supply chains to be effective. Indeed, supply chains are becoming complex networks with a large number of entities. Each entity may have its own objectives and makes its management decisions so as to maximize its own profits. Obviously, such a decentralized structure often leads to a loss of efficiency for the chain as a whole. Centralized structures, however, where decisions are made to maximize the global profit of the whole system, prove to be more efficient. This observation has given raise to a tremendous interest for research on coordination-based mechanisms. Existent research in this field generally aims at identifying inefficiencies of a part or a whole of a classical decentralized supply chain and developing coordination mechanisms that permit to align the individual benefits with those realizable in a centralized framework. Detailed reviews of some research on this topic may be found in (Cachon, 2003) and (Li and Wang, 2007).

Many recent publications focused on modelling and analyzing the coordination in the supply chain through the concepts of game theory, namely the cooperative game theory, see for example (Cachon and Zipkin, 1999), (Sarmah et al, 2006), (Bylka, 2003), (Meca et al, 2004), (Meca et al, 2003) and (Guardiola et al, 2007). Indeed, beyond the fact that cooperative game theory is a suitable tool for modelling many cooperation

mechanisms, it is namely useful to determine the benefit that any entity, taking part in a cooperation mechanism, would realize.

In this paper, we are concerned with the analysis of a cooperation mechanism that may arise between some retailers. These retailers have to replenish their inventory from a unique supplier to satisfy a deterministic demand. When ordering individually, each retailer has to pay an ordering cost and a delivery cost. The former is fixed and the latter depends on the retailer's location and some other characteristics as his holding resources. The retailer also charges the holding cost generated by the delivered inventory until its selling. When cooperating, a set of retailers may join their orders and place one big order instead, reducing the total fixed cost to only one ordering cost. This kind of cooperation situation is commonly called in the literature as an inventory centralization situation.

In the existent literature, similar cooperation situations implying some inventory centralization are modeled using cooperative game theory. Thus, (Ôzen et al, 2004) consider a set of retailers, each facing a newsvendor problem. The retailers may choose to join their orders. The ordered quantity is assumed to be available after a lead time, during which the individual needs become more accurate. Thus, the cooperating retailers can increase their expected joint profits by making the distribution of the big order, only when it becomes available, according to the individual needs. The authors of (Ôzen and Sosic, 2006) extend this model by assuming that the allocation of the inventory is based on the observation of

a demand signal that updates the demand information. They discuss three kinds of cooperation situations. Retailers' cooperation, manufacturer- retailers' cooperation and finally the coordination of the entire supply chain through a buyback contract. The authors of (Hartman et al. 2000) also consider an inventory centralization situation bringing together some retailers experiencing stochastic, generally correlated demand. These retailers may choose to realize some cost saving benefits by participating in a centralized inventory ordering arrangement, sharing the cost of running the central storage facility.

Closer to our work, are the papers of (Heuvel et al., 2007), (Meca et al., 2004) and (Guardiola et al., 2007). (Heuvel et al., 2007) analyze a new class of cooperative games: the so-called Economic Lot Sizing games (ELS). In this game, a set of retailers selling the same product are facing a deterministic demand for a fixed number of periods. They order the product from the same manufacturer, who charges them some ordering and production costs. By joining their orders, both ordering and production costs may be reduced. A similar situation is considered in (Meca et al., 2004). But unlike the model of (Heuvel et al., 2007), retailers in (Meca et al., 2004) are assumed to pay ordering and holding costs. The authors of (Tijds et al., 2005) consider a holding game, where only holding costs are taken into account. In their model, the retailers may cooperate by sharing a storage capacity for their inventories. In (Guardiola et al., 2007), the authors studied the coordination of actions and the allocation of profits in a supply chain where the supplier offers wholesale prices to induce the retailers to make big orders. Two kinds of cooperation situations are studied: In the first, the supplier is not considered as a cooperating agent, while in the second situation, the supplier may cooperate with the retailers. This may be the first paper where the supplier takes part of a cooperation situation. In this same paper, the authors of (Guardiola et al. 2007) showed that it is preferable to include the supplier as a player in the cooperative game. They demonstrated that the profit in this case is larger than an exclusive retailers' cooperation situation.

The remainder of the paper is organized as follows: In section 2, we expose the model considered in this work and its main assumptions. The optimal replenishment policies in the case of the decentralized and the centralized system are then analyzed. In section 3, we focus on analyzing the cooperative system; we compare both of decentralized and cooperative situations. Moreover, we comment numerical tests and we introduce the complexity of the coalition problem generation. In section 4, we propose a coalition generation approach based on profit thresholds. In section 5, we conclude and give some insights for the future development.

2 THE MODEL

In this section, we first introduce a new model for a cooperation situation. Then, we analyse the classic situation where each retailer of the distribution system is operating independently and the cooperation situation where a group of retailers choose to align their ordering policies.

In the framework of a distribution system, the goal of any retailer is to keep a high customer service level with a minimum cost. This problem reduces to the determination of the lowest cost ordering policy. Of course, a retailer can operate separately and follow the optimal ordering policy minimizing his own costs. This situation corresponds to the classic decentralized or competitive system. But, we also assume that a group of retailers may cooperate by joining their orders as a single big order, in order to reduce some fixed cost.

When only inventory related costs are considered, this cooperation situation is modelled as an inventory game introduced in (Meca et al., 2004) and (Guardiola et al., 2007).

The model studied in this work may be considered as an extension of the inventory game introduced in these papers. Indeed, besides inventory related costs, we also consider a new cost component that corresponds to individual charges. Thus, the cost structure consists of three components: an ordering cost, a holding cost and an individual cost. The ordering cost is a fixed cost charged for a single order (corresponding for example to administrative expenditure), depending neither on the ordered quantity nor on the ordering retailer.

The holding cost is the cost of storing goods, which is the financial charge of keeping one unit of good in stock for one time unit. This cost is assumed to be constant through time. Since we suppose that any retailer keeps his products in his own warehouse, this cost is assumed to be paid by every retailer.

The individual cost represents the additional financial charges yielded by the delivery and any eventual handling operation of the products. We assume that this cost depends on the retailer (his location, his handling resources, etc...).

In the following subsections, we consider the problem of determining the optimal ordering policy for both decentralized and cooperative systems using the EOQ (Economic order quantity) model as an optimization method. The following notations are used in the remainder of the paper.

N : The set of retailers, $N := \{1, \dots, n\}$. $\{0\}$: corresponds to the supplier.

D_i : The deterministic demand of retailer $i \in N$.

Q_i : The order size of retailer $i \in N$.

a : The ordering cost.

h_i : The holding cost for retailer $i \in N$.

G_i : The individual cost for retailer $i \in N$.

$C(i)$: The average total cost per time unit for retailer $i \in N$.

T_i : The ordering cycle length for retailer $i \in N$.

m_i : The ordering frequency for retailer $i \in N$.

Let's also define K_i and H_i respectively as the fixed cost and the holding cost for a retailer i , i.e.:

$$K_i = a + G_i, \text{ and } H_i = \frac{1}{2} \cdot h_i \cdot D_i.$$

2.1 The decentralized system

Under the economic order quantity model, a retailer i orders an amount Q_i of goods every T_i units of time to face a deterministic demand D_i . The frequency m_i denotes the number of orders placed by time unit, that is $m_i = 1/T_i = D_i/Q_i$. When ordering a quantity Q_i , a retailer i has to pay a total fixed and delivery cost equal to

$$(a + G_i)m_i = (a + G_i)/T_i = \frac{K_i}{T_i} \text{ u.m (monetary unit)}$$

Since an order is placed only when the stock is null, the average size of the inventory is $\frac{1}{2}(Q_i + 0) = \frac{Q_i}{2}$. The average holding cost per time unit is then: $h_i \cdot \frac{Q_i}{2} = H_i T_i$.

Finally, the average total cost per time unit, paid by any retailer i , is the sum of the ordering cost, the delivery cost and the holding cost, i.e.:

$$C(T_i) = \frac{K_i}{T_i} + H_i T_i \quad (1)$$

This total cost is minimum for an optimal cycle length

$$T_i^* \text{ such that } T_i^* = \sqrt{\frac{K_i}{H_i}}.$$

As a result, the minimum total cost that any retailer, operating independently, has to pay, is given by:

$$C(T_i^*) = 2 \cdot \sqrt{H_i \cdot K_i} \quad (2)$$

2.2 The cooperative system

Above, we have determined the optimal replenishment policy for any retailer operating by his own. Now, let us focus on the case of cooperating retailers. Consider a non-empty coalition of retailers $S \subset N$. The retailers in S cooperate by making their orders jointly. Thus, they have equal cycle times. Let's denote the common cycle time of a non-empty coalition $S \subset N$ by T_S . So, $T_S = T_i, \forall i \in S$.

Let $i \in S$ be any retailer. Expressing Q_i as a function of T_S leads to: $Q_i = D_i T_S$.

When joining their orders, the retailers realize some cost savings by reducing the ordering cost. The total fixed cost induced by the joint order is then given by: $K_S = a_S + G_S = a + \sum_{i \in S} G_i$. While the total holding cost is given as the sum of the individual holding costs, i.e.: $H_S = \sum_{i \in S} H_i = 1/2 \sum_{i \in S} h_i D_i$.

As a result, the average total cost for a coalition S , $C(S)$, is given by:

$$\begin{aligned} C(S) &= \frac{(a + \sum_{i \in S} G_i)}{T_S} + \sum_{i \in S} \frac{h_i \cdot Q_i}{2} = \frac{(a + \sum_{i \in S} G_i)}{T_S} + \left(\sum_{i \in S} \frac{h_i \cdot D_i}{2} \right) T_S \\ &= \frac{K_S}{T_S} + H_S T_S \end{aligned} \quad (3)$$

This total cost is minimum for an optimum common cycle time T_S^* equal to:

$$T_S^* = \sqrt{\frac{K_S}{H_S}} \quad (4)$$

By substituting T_S^* by its expression in equation (3), we get:

$$C^*(S) = 2 \cdot \sqrt{H_S \cdot K_S} \quad (5)$$

A coalition S may be characterised by the corresponding total cost $C^*(S)$. The whole cooperative system of N retailers is then characterized by a global cost $C(N)$, such that $C(N) = \sum_{S \subset N} C^*(S)$ where the set of the selected coalitions ($S \subset N$) is a partition in N , called the coalition structure.

Intuitively, a retailer may be interested in joining a coalition if the corresponding total cost is promising regarding the total costs of its members when operating in a decentralized way.

In the following section, we use these results together with some numerical tests to look deeper into the attractivity aspect of any cooperative system.

In the remainder of the paper, for simplification reasons, $C(S)$ will refer to the optimal cost of the coalition S , $C^*(S)$, described in (5). Also $C(i)$ will refer to the optimal cost, $C^*(i)$, of retailer i described in (2).

3 THE ATTRACTIVENESS OF A COOPERATIVE SYSTEM

In this section, we first emphasize some of the specificities of the studied model, namely regarding the coalition structure of the most attractive structure of a cooperative system. Precisely, we try to characterize the attractiveness of a cooperative system at different levels: a global level, a coalition level and an individual level. Sufficient conditions are provided to ensure the attractiveness both at the global and the coalition levels.

3.1 The global attractiveness

In the previous section, we roughly linked the interest that a retailer may have to join a coalition to the cost reduction the group in the coalition should be able to realize. To characterize more precisely this interest, we first distinguish the global attractiveness of a given coalition structure of a cooperative system.

A coalition structure, or equivalently a partition, P of a cooperative system of N retailers may be defined as a set of coalitions in the system, $P = \{S_1, S_2, \dots, S_p\}$, such that: $S_i \cap S_j = \emptyset$ for any $i \neq j$ and $\cup_{i=1,p} S_i = N$.

A partition P is said to be globally attractive if it allows a global cost reduction regarding the optimized decentralized system costs.

More precisely, P is an attractive partition of N if:

$$C(P) = \sum_{S \in P} C(S) \leq \sum_{i \in N} C(i)$$

Obviously, for the model we deal with in this work as well as related models in the literature, there always exists at least one globally attractive partition P of the set N . Indeed, the single coalition partition, which corresponds to the whole group of retailers, is always attractive since it allows the reduction of the ordering cost. A less trivial question is to find out the most attractive partition P^* of a set N . Dealing with this question reduces to solve a well known combinatory problem, the so-called set partitioning problem. One possible formulation of the set partitioning problem is given by the following linear program (K):

$$\text{Min} \sum_{i=1}^m C(S_i) \quad (6)$$

$$\bigcup_{i=1}^m S_i = N \quad (7)$$

$$S_i \cap S_j = \emptyset \quad \forall i, j \in \{1, \dots, m\}; \quad i \neq j \quad (8)$$

$$S_j \in M, j \in \{1, \dots, m\} \quad (9)$$

Where $N = \{1, \dots, n\}$ denotes the set of retailers and M is the set of all sub-sets in N . So, $\text{Card}(M) = m = 2^n - 1$.

The objective function (6) is then to minimize the total cost of the selected coalitions under the constraints (7, 8, 9) that these coalitions constitute a partition in N .

The set partitioning problem consists in identifying, among the $(2^n - 1)$ possible coalitions, the set of coalitions constituting the most attractive partition which minimizes the total cost. The number of partitions grows in $O(n^n)$ with the size of the set N (Sandhom et al 1999). Although this is a difficult but crucial problem, it has never been dealt with in the literature related to the application of the cooperative game theory to supply chain management problems (Nagarajan and Sobic, 2007). Indeed, all the existent models, and especially the inventory games (Meca et al., 2004, Meca et al., 2003, Guardiola et al., 2007, Heuvel et al., 2007,...), fulfill the superadditivity condition. A superadditive cooperative game refers to games where any pair of coalitions is better off merging into one (Bezalel and Sudhölter, 2003). In such games, the problem of optimally allocating the players to coalitions is trivial because the grand coalition, where all the players are grouped, is always the most attractive (Larson, 2000).

However, as one can intuitively suspect, in the model considered by the present work, the set partitioning

problem is relevant. Indeed, the most attractive partition for a set N may correspond to a set of disjoint coalitions. The following numerical tests give an empirical corroboration for this intuitive result and some additional insights.

3.1.1. Numerical tests

We use a set of numerical tests to appreciate the influence of the ordering cost a on the structure of the most attractive partition for a given set of retailers. These tests were generated for a number of retailers $N = 6, 8$ and 10 . For each case, the demand characterizing each retailer is generated randomly in the interval $[10, 1000]$. The corresponding holding cost h is generated randomly in the interval $[1, 20]$. The individual cost G is generated randomly in the interval $[10, 100]$. A set of K tests is then generated for each value of a in $\{0, 1, 5, 10, 20, \dots, 100\}$. For each test, we first determine the most attractive partition P^* by solving the mathematical model corresponding to the set partitioning problem described above. We then calculate the whole system decentralized cost $\sum_{i \in N} C(i)$, the partition cost $C(P^*)$ and the corresponding ratio

$$\text{of cost reduction given by } \pi = \frac{\sum_{i \in N} C(i) - C(P^*)}{\sum_{i \in N} C(i)}$$

The number of coalitions in the set P^* , $\text{card}(P^*)$, is also given. We choose to show only the results corresponding to 10 retailers' case (see Table 1).

a	P^*	$C(P^*)$	$\sum_{i \in N} C(i)$	π (%)	$\text{card}(P^*)$
0	{{1} {2} {3} {4} {5} {6} {7} {8} {9} {10}}	5880,3	5880,3	0,0	10
1	{{1 3 4 7} {2 5 6 10} {7} {8} {9}}	5922,4	5947,5	0,4	5
5	{{1 3 4 7} {2 5 6 10} {8 9}}	6046,4	6208,3	2,6	3
10	{{1 3 4 7} {2 5 6 10} {8 9}}	6156,4	6518,6	5,6	3
20	{{1 3 4 7} {2 5 6 10} {8 9}}	6341,6	7096,3	10,6	2
30	{{1 3 4 7} {2 5 6 10} {8 9}}	6461,9	7628,7	15,3	2
40	{{1 3 4 7} {2 5 6 10} {8 9}}	6580,1	8125,3	19,0	2
50	{{1 3 4 7} {2 5 6 10} {8 9}}	6696,2	8592,6	22,1	2
60	{{1 3 4 7} {2 5 6 10} {8 9}}	6810,3	9035,3	24,6	2
70	{{1 3 4 7} {2 5 6 10} {8 9}}	6922,5	9457	26,8	2
80	{{1 3 4 7} {2 5 6 10} {8 9}}	7032,9	9860,5	28,7	2
90	{{1 3 4 7} {2 5 6 10} {8 9}}	7141,6	10248	30,3	2
100	{1 2 3 4 5 6 7 8 9 10}	7214,5	10621	32,1	1

Table 1. Numerical tests' results for a set of 10 retailers

In the figure 1, using the data of 6 numerical tests with 10 retailers, we represent the mean value of the number

of coalitions and the ratio of cost reduction corresponding to the optimal partition.

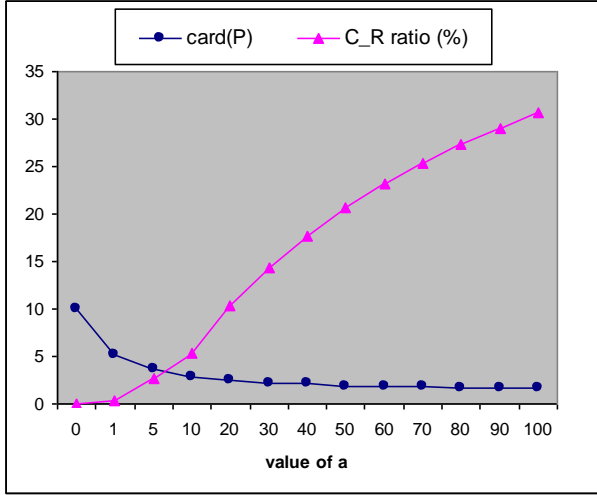


Figure 1: The mean value of coalition's number and cost reduction ratio of the optimal partition.

3.1.2. Interpretation and insights

From the results above, one observation may be immediately done. The structure of P^* depends on the relative importance of the ordering cost regarding the other cost components.

Further, one may notice that, as the ordering cost increases, the cost reduction ratio of the optimal partition increases in a nonlinear way. The number of coalitions in the optimal solution decreases until reaching the value 1 referring to the grand coalition. In other words, the growth of the ordering cost induces some coalitions to merge together until reducing to only one coalition where all retailers cooperate together. Then, the grand coalition remains the optimal partition of the system whatever the value of the ordering cost.

To study this phenomenon we examine the cooperative behaviour of retailers. We remark that the merging of the coalitions is likely to follow some neighborhood logic. In other words, for the same set of data, if a variation affects the ordering cost, the coalitions that are likely to merge are those with comparable cost data.

3.2 The coalition attractiveness

For a given partition, that is globally attractive, the coalitions may have different performances in terms of cost reduction. In what follows, we define the attractiveness of a coalition.

At the coalition level, a coalition $S \in P$ is attractive if:

$$C(S) \leq \sum_{i \in S} C(i), S \in P \quad (10)$$

Suppose that a retailer i have to determine his optimal replenishment policy, under the EOQ model, the minimal average cost is determined by the equation (2), in this case let's denote by T_i^* the optimal cycle length. If

the retailer deviates from the optimal solution to another cycle length, his total cost is increased.

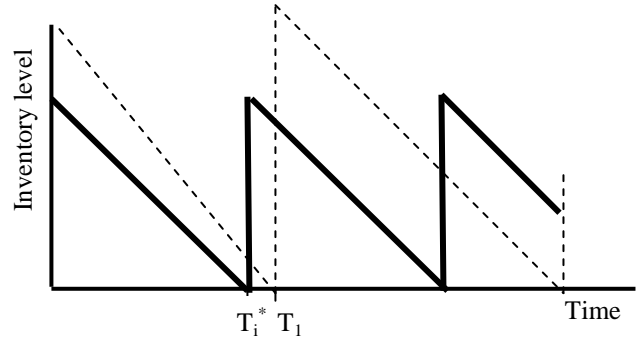


Figure 2. Unequal cycle length

Suppose that the optimal cycle length of retailer i deviates from T_i^* to T_1 (Figure 2). Let's define $\alpha_i = \frac{T_1}{T_i^*}, \alpha_i \in \mathbb{R}_+$.

As mentioned in equation (1)

$$C(T_1) = \frac{K_i}{T_1} + H_i T_1 = \frac{K_i}{\alpha_i T_i^*} + H_i \alpha_i T_i^*, \text{ by replacing } T_i^* \text{ by its expression } T_i^* = \sqrt{\frac{K_i}{H_i}} \text{ we get:}$$

$$\begin{aligned} C(T_1) &= \frac{K_i}{\alpha_i T_i^*} + H_i \alpha_i T_i^* = \frac{K_i}{\alpha_i} \cdot \sqrt{\frac{H_i}{K_i}} + H_i \alpha_i \cdot \sqrt{\frac{K_i}{H_i}} \\ &= \frac{\sqrt{K_i \cdot H_i}}{\alpha_i} + \alpha_i \cdot \sqrt{K_i \cdot H_i} = \left(\frac{1}{\alpha_i} + \alpha_i\right) \cdot \sqrt{K_i \cdot H_i} = \left(\frac{1}{\alpha_i} + \alpha_i\right) \cdot \frac{C(T_i^*)}{2} \end{aligned}$$

$$\text{In sum, } C(T_1) = \left(\frac{1}{\alpha_i} + \alpha_i\right) \cdot \frac{C(T_i^*)}{2} \quad (11)$$

In the cooperative situation, in a given coalition S , the retailers have to make order simultaneously, ie: they align their cycle time length to have a common cycle time denoted by T_S^* and calculated like in (4). In this case every retailer have to deviate form his optimal parameters when ordering alone (ordered quantity Q_i^* , cycle time length T_i^*) to adopt the collective strategy parameters (ordered quantity $Q_{S,i}^*$, cycle time length T_S^*). For the coalition S , the total cost rise is as follows:

$$\begin{aligned}\Delta_S &= \sum_{i \in S} (C(T_S^*) - C(T_i^*)) \\ &= \sum_{i \in S} \frac{(\alpha_i - 1)^2}{2\alpha_i} \cdot C(T_i^*), \quad \forall S \subseteq N \setminus \emptyset \\ \alpha_i &= \frac{T_S^*}{T_i^*}\end{aligned}\quad (12)$$

By making joint orders, the retailers save money by reducing the ordering costs they paid one ordering cost by opposition to $|S|$ ordering cost in the case of decentralized system. Indeed for a set of retailers, S , the savings is expressed as following:

$$\Pi_S = \frac{(|S|-1)a}{T_S^*}, \quad \forall S \subseteq N \setminus \emptyset \quad (13)$$

Proposition: A coalition S is attractive if the cost rise induced by the cycle time deviation is balanced by the savings of ordering costs, i.e. $\Pi_S \geq \Delta_S$. Then, equation (10) is equivalent to:

$$C(S) \leq \sum_{i \in S} C(i), \Leftrightarrow \left[\frac{(|S|-1)a}{T_S^*} - \sum_{i \in S} \frac{(\alpha_i - 1)^2}{2\alpha_i} \cdot C(T_i^*) \right] \geq 0 \quad (14)$$

3.3 The individual level attractiveness or cost allocation problem

The global attractiveness, as well as the coalition attractiveness may represent some worthwhile incentives. However, for a retailer to make a decision to undertake or not a cooperation mechanism, he still needs a more accurate appreciation concerning what would be his individual earning if joining a given coalition.

Actually, the individual participation of each member of a coalition of retailers to the global cost induced by their coalition has to be defined and approved by all the members. Indeed, defining the individual payoffs is a crucial agreement that may determine the feeling of satisfaction of each member and thus the stability of his own involvement in the cooperation mechanism.

This question may be rigorously dealt with using game theory concepts. Namely, the so-called stability conditions of a cooperative game, if satisfied by the payoffs vector, allow ensuring that no rational reason would motivate the withdrawal of any member of any coalition of the stable partition (Branzei et al., 2005). However, determining a payoffs vector that fulfills the stability conditions is sometimes a tricky problem. At the best of our knowledge, there is no paper in the literature related to the application of cooperative game theory to supply chain management that rigorously addresses this problem in coalitional environment (Nagarajan and Sossic, 2007). Most of those that deal with are only concerning superadditive environment where the stability of the grand coalition is addressed.

We are not claiming solving the cost allocation problem in this paper. But, in the next section, we propose an approach for the partition generation problem that accounts of a simplified aspect of the stability issue as defined by the cooperative game theory.

4 A COALITION GENERATION METHOD BASED ON TARGET COST REDUCTION RATES

In this section, we discuss the issue of generating coalitions of retailers under specific cost reduction requirements. Taking into account a target cost reduction rate from the coalition generation process is a novel idea that was never addressed in the related literature.

As mentioned in section 3, a group of retailers may consider forming a coalition S , if S is attractive, i.e.:

$$C(S) \leq \sum_{i \in S} C(i)$$

However, the group of retailers may have some specific requirements about the cost reduction rate their coalition should allow. For instance, the setting up of the cooperation mechanism might not be interesting if the expected cost reduction rate is under a certain economic threshold. This threshold may be estimated by taking into account some cooperation setting up costs that were not considered in the optimization process, as well as individual requirements.

To address this issue, we introduce a new parameter which represents the threshold of attractiveness of a given coalition of retailers. Let's denote it by λ_S . Thus, a coalition S is considered as attractive enough if $\pi_S \geq \lambda_S$, which is equivalent to:

$$\sum_{i \in S} C(i) - C(S) \geq \lambda_S \sum_{i \in S} C(i) \quad (15)$$

Generating such a coalition, with an a priori fixed threshold of attractiveness, reduces to solve what we call a target cost partitioning problem. A formulation of a target cost partitioning problem is obtained by adding, to the linear program of the partitioning problem (K), a constraint (15) that should be satisfied by at least one selected coalition. Let (K_λ) be the obtained linear program. A solution of (K_λ) is a partition where at least one coalition ensures the target cost reduction rate λ .

The following algorithm sums up the approach we propose to generate a feasible partition P_λ of a set of retailers, such that each coalition S of P_λ guarantees a target cost reduction rate λ_S for its members.

Algorithm 1: target cost coalition generation method

1. Set $\lambda = \lambda_0$, the maximum value for the attractiveness threshold required by the retailers of the set N .

2. Solve the target cost partitioning problem (K_λ). If (K_λ) is infeasible then no coalition, allowing a cost reduction rate greater than or equal to λ , exists. Otherwise, goto step 4.
3. Decrease λ until (K_λ) is feasible.
4. Let Q_λ be the obtained set of coalitions.
5. Set $N = N \setminus \{Q_\lambda\}$. If $N \neq \emptyset$ goto step 2.
6. Set $P_\lambda = \cup Q_\lambda$

For more comprehension of the algorithm we present below an example of a set of 10 agents taken from table 1 with an ordering cost corresponding to 20. Note that in this example, we show the maximal cost reduction rate of the selected coalitions (π_S). This rate is the upper bound of the economic threshold of attractiveness of the coalition (λ_S).

$$N = \{1,2,3,4,5,6,7,8,9,10\}$$

-Step 1:

$$S_1 = \{2,5,6,10\}$$

$$\pi_1 = \pi_{\max} = 12.89\%$$

-Step 2:

$$N = N \setminus S_1 = \{1,3,4,7,8,9\}$$

$$S_2 = \{1,3,4,7\}$$

$$\pi_2 = 9.77\%$$

- Step 3:

$$N = N \setminus S_2 = \{8,9\}$$

$$S_3 = \{8,9\}$$

$$\pi_3 = 7.35\%$$

- Step 4:

$$N = N \setminus S_3 = \emptyset$$

END

$$P_\lambda = \{\{2,5,6,10\}, \{1,3,4,7\}, \{8,9\}\}, \lambda_{\{2,5,6,10\}}=12.89\%, \lambda_{\{1,3,4,7\}}=9.77\%, \lambda_{\{8,9\}}=7.35\%. \text{ (we suppose that } \pi_S = \lambda_S)$$

This algorithm has the followings proprieties.

P1: The algorithm converges after a limited number of steps.

P2: The solution returned by the algorithm is a partition.

P3: The coalitions returned by the algorithm are stable: No sub-coalition may find an interest to leave the coalition, to which it belongs.

Prove of P1, P2:

In every step, the algorithm selects a set of disjoint coalitions. Moreover, the set of retailers is updated in every step, by eliminating the coalitions selected in the previous iterations: the set of coalitions found in different steps are then disjoint. The algorithm stops when the updated set of retailers is empty. That means that every retailer belongs to a coalition which guarantee a positive target cost reduction rate λ , or at least he

would stay in single coalition. From the preceding analysis, we can easily check that the algorithm converges after a limited number of iterations; the returned solution is then a partition.

Prove P3:

Let, S_i be a coalition in the partition found by the algorithm, π_{S_i} is the maximal cost reduction cost ensured by this coalition S_i .

Let, T be a sub-coalition in S_i ($T \subset S_i$), π_T is the maximal cost reduction cost ensured by the coalition T . The sub-coalition T would have the intension to leave the coalition S_i , if and only if it can guarantee a better cost reduction ratio to its members, i.e: $\pi_T \geq \pi_{S_i}$.

If $\pi_T > \pi_{S_i}$ the coalition T should be selected by the algorithm in the step (i-1). Else when ($\pi_T = \pi_{S_i}$) the coalition T should be selected in the step (i).

In both cases the set of retailers, T, can not be a sub-coalition belonging to a coalition selected by the algorithm. Thus, no sub-coalition can guarantee a better savings for its members. And by the way would not leave the coalition to which it belongs. We can conclude that coalitions selected by the algorithm are stable.

Nevertheless, we can not prove that no sub-coalition belonging to disjoint coalition selected by the algorithm would leave or not the system. Thus, the global stability of the system is not guaranteed.

To answer this question we should study the individual level attractiveness or the cost allocation problem. As mentioned in the sub-section (3.3) this question will be addressed in future work.

5 CONCLUSION

In this paper, we model a multiple retailer's cooperation problem. A collective of retailers selling an identical product purchased from the same supplier is considered. The retailer's dilemma is to find an optimal ordering policy with respect to three components of cost (ordering, holding and individual costs). We analyze the case where the retailers make a joint order. In this situation, they realise some cost saving benefits by reducing the sum of ordering cost to only one cost paid by the coalition.

The main part of cooperative systems developed in the supply chain literature considers stock management related costs (ordering and holding.). In those models the grand coalition is the most profitable union, in the present work, the retailer would prefer smaller coalitions. In cases where the cooperation is not profitable, they would stay in single coalition. Thus, finding the optimal solution (partition) is no longer obvious.

We use a set of empirical tests to corroborate some useful insights and to raise some relevant questions

related to retailer's cooperative behaviour. We characterized the attractiveness of a cooperative system at different levels: a global level, a coalition level and an individual level. Sufficient conditions are provided to ensure the attractiveness at the global and the coalition levels. Then after analysing and commenting the numerical tests, we highlighted the complexity of the coalition generation problem and we gave a coalition generation method based on target cost reduction rates.

The work presented in this paper has given us some insights for future developments. Current research is aiming of proposing an optimal solution and heuristic for the coalition generation problem. We will also focus on the cost allocation problem, the research task consists in developing methods that generate the optimal coalition structure and give a cost allocation scheme simultaneously.

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