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TWO-STAGE FLEXIBLE SUPPLY CONTRACT WITH PAYBACK AND INFORMATION UPDATE

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ABSTRACT : *In this paper we consider a two-stage supply contract model for advanced reservation of capacity, with payback option at the beginning of the selling horizon. Between the two decision stages, an external information is collected that serves to update the demand forecast and permits to adjust the decisions of the first stage by exercising options or by returning some units to the supplier. This type of contracts apply in the case of products with short life cycle. For this type of products, the demand occurs during a single selling period (season). At the end of the period, the remaining inventory, if any, is sold (or returned to the supplier) at a salvage value that is usually less than the initial unit production/procurement cost. During the selling season, any satisfied demand is charged with a unit selling price, and any unsatisfied demand is lost and a penalty shortage cost is paid. The objective of the model is to determine the quantities to be ordered before the beginning of the selling season which can be interpreted as the amount of capacity to be reserved, in order to satisfy optimally the demand.*

KEYWORDS : *Supply contract; information update; payback; newsvendor; capacity reservation; dynamic programming.*

1. INTRODUCTION

The *style-goods* type products are characterized by a short product-life. Demand uncertainty management has long been regarded as crucial. Since it is impossible to have a perfect knowledge about the demand before the selling season, and both understocking and overstocking are undesirable, policies that advocate the use of market information to improve stocking decisions have been proposed (Choi, 2007). Therefore, many firms are recognizing opportunities to collect information permitting the improvement of the demand forecast. This information may have different sources: advanced custom forecasts, early season demand, advanced bookings or sales of a pre-seasonal product. In fashion industry (Fisher and Raman, 1996) and in the computer industry (Padmanabhan, 1999) the early season information collected by firms constitutes a strong indicator for the total season demand. Many studies have been proposed in the literature on the use of information for inventory problems (Scarf, 1959), (Murray and Silver, 1966), (Azoury and Miller, 1984), (Azoury, 1985) and (Lovejoy, 1990).

The literature of the supply contracts is very rich (see for example Tsay, et al. 1999). Many factors distinguish the models of supply contracts in the literature. The most important are the form (e.g. options-futures, quantity flexibility or backup agreement) and the number of period in the planning horizon (one period, two periods or multiple periods)

The first factor that distinguishes the contract models is the structure of the contract. The first structure can be the options-futures contract ((Brown and Lee, 1998, a), (Barnes-Schuster, et al. 1998) and (Cachon and Lariviere, 1997)). In this case the firm has two decision stages. At the first stage two decisions are fixed: the number of futures (a non-refundable and unchangeable commitment) and the number of options (a flexible commitment). At the second decision stage the firm can exercise a number or the totality of the prescribed options by paying an exercise cost. The second contract type is the backup contract (Eppen and Iyer, 1997). In this type of contracts, the firm makes an initial order decision, and at the final decision point, a part of the initial order can be cancelled, up to a certain predefined percentage. The third contract type is the quantity flexibility contract ((Bassok and Anupindi, 1995) and (Tsay, 1999)). In this type of contracts, the firm makes an initial order decision and can later revise this order decision within certain upside and downside percentages.

The second factor that distinguishes contracts is the number of periods. (Bassok and Anupindi, 1995) analyse a rolling horizon flexibility contract without information updates. (Tsay and Lovejoy, 1999) considers the contract of (Bassok and Anupindi, 1995) and allows for forecast updates. The multiple period models are difficult to solve and no analytical insights can be obtained.

Most of the supply contract papers study two-period models, which permits some analytical insights. (Milner and Rosenblatt, 1997) consider a two-period model in which demands are assumed to be independent like (Bassok and Anupindi, 1995). Most papers allow demand updates using an external information((Eppen and Iyer, 1997), (Donohue, 1996 and 2000), (Tsay, 1999), (Brown and Lee, 1998, a), (Brown, 1999)).

In this paper we assume that the demand is characterized by a probability density function that is updated before the beginning of the selling season using an external market information. We assume also that the demand and the information are jointly distributed using a bivariate normal distribution. The external information could be, for example, the sales of a preseasonal product whose demand is closely related to our product demand. The information could also be a completely external information about an external condition (like the weather forecasts).

In our paper, we develop a new type of contracts that is more flexible than the models existing in the literature: in addition to the classical contract process, we assume that at the beginning of the selling period, a certain quantity can be returned to the supplier. In general, the manufacturer (supplier) has many retailer or many retail channels. Therefore, if any of these retailers decides to return a certain quantity to this supplier (in application of the contract), then he could resell it to another retailer before the beginning of the selling horizon, or he can resell it via another retail channel. This important option permits the retailer to profit from the external information that is collected between the two decision stages, and to adjust his previous decisions. More precisely, if the updated demand distribution is relatively low, then a part of the already ordered quantity is returned to the supplier, and if the updated demand distribution is relatively high, then a part of the already reserved capacity is used. The return option, does not exist, to our knowledge, in any of the existing contracts papers.

In this paper the decision process is divided into two stages: in the first stage a first group of decisions about the production and the capacity reservation (options) is made. In the second stage an exogenous information, that is correlated with the demand distribution, is collected, which permits to update the demand forecast. After updating the demand forecast, an other decision group about the production and about the return of a certain quantity, from the quantity which was already received, is made.

The remaining part of the paper is structured as follows: in the following section we define the model, the parameters and assumptions, the objective function and the optimal policy for the two decision stages. In Section (3), we study a particular case that represents the perfect information case in which the correlation between the information and the demand is perfect. In section (4), we provide a numerical study and in Section (5) we give conclusions and perspectives.

In an extended version of this paper, we assume stochastic cost parameters for the second decision stage. We also study and additional particular case with worthless information and we compare it with the perfect information case.

2. THE MODEL

2.1. Model parameters

The stochastic demand of a mono-product is defined with an exogenous information by bivariate normal probability density function. The random exogenous information becomes deterministic and completely known between two different stages of the decision process. To satisfy the demand, the retailer can order a first time with a low cost production mode and he can reserve a certain amount of capacity of his supplier (options), but the available information about the demand is not accurate. Once the exogenous information is known the demand forecast is updated. In the case of a sufficiently high correlation between the collected information and the demand distribution, the variability of the demand decreases, and the retailer has an accurate demand distribution that permits him to adjust his previous decisions. The retailer could either exercise a certain number of the options bought from his supplier, or he could choose to return a part of the order that he had previously bought from its supplier. After the end of the single period horizon, the unsatisfied demands are lost and the remaining inventory is returned to the supplier or sold in a parallel market.

Define the following model parameters:

- D : the stochastic demand,
- I : the stochastic exogenous information collected between the two stages of the decision process,

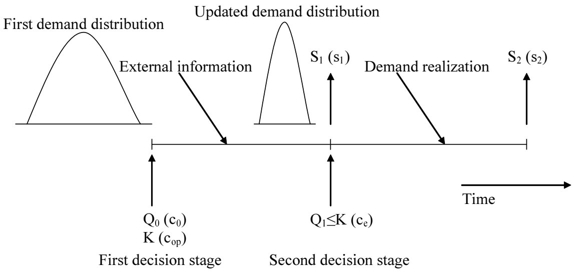

Figure 1: Decision process

- $f(I, D)$: the bivariate normal probability density function of the demand D and the information I,
- μ_0 and θ_0 : the means of the bivariate normal distribution with respect to D and I, respectively,
- σ_0 and δ_0 : the standard deviations of the bivariate normal distribution with respect to D and I respectively,
- ρ : the correlation coefficient between D and I,
- $g(I)$ and $G(I)$: the marginal probability density and cumulative distribution functions of the information I, respectively,
- $h(D|I)$ and $H(D|I)$: the conditional probability density and cumulative distribution functions of the demand D conditionally to the given the information I, respectively,
- $\Phi(\cdot)$: is the standard normal cumulative distribution function,
- Q_0 : quantity ordered at the first decision stage,
- K : amount of capacity reserved (or number of options bought) at the first decision stage,
- Q_T : the maximal number of units that might be ordered until the second decision stage ($Q_T = Q_0 + K$),
- Q_1 : quantity ordered at the second decision stage (number of exercised options). ($Q_1 \leq K$),
- S_1 : quantity returned to the supplier or salvaged in a parallel market at the beginning of the selling horizon (at the second decision stage),
- c_0 : unit order cost for the quantity Q_0,
- c_{op} : unit cost for the capacity reservation,
- c_{Moy} : defined as $c_{Moy} = c_{op} + c_e$,
- s_1 : unit price (salvage value) of the returned quantity at the beginning of the selling horizon S_1,
- p : unit selling price during the selling horizon,
- b : unit shortage penalty cost at the end of the horizon,
- s_2 : unit salvage value at the end of the horizon.

Note that the reserved capacity (K) at the first decision stage, plays two essential roles. The fact that the retailer reserves a certain amount of capacity, paying a reservation cost, permits to the supplier to prepare his raw material and production facilities to probably satisfy the demand corresponding to this reservation in the second decision stage. For the retailer, that permits to exploit the information updating and to order an additional quantity at the second stage in order to optimally satisfy the demand.

Since $f(D, I)$ is a bivariate normal distribution, then it is well known that $(D|I)$ is normally distributed (Bickel and Doksum, 1977), with mean μ_1 and standard deviation σ_1, given by:

$$\mu_1 = \mu_0 + \rho\frac{(I-\theta_0)\sigma_0}{\delta_0} \quad \text{and} \quad \sigma_1 = \sigma_0\sqrt{1-\rho^2}. \quad (1)$$

To develop the optimal policy, we will begin by determining the optimal policy of the second stage of the decision process, then using the dynamic programming, we will use the optimal policy of the second stage in order to develop the optimal policy of the first decision stage.

2.2. Model parameters assumptions

Some assumptions on the model parameters are made in order to avoid classes of specific cases for which the optimal solution is trivial and doesn't permit any interesting insight.

The first one is relative to the selling price: we assume that $Max(c_0, c_{op} + c_e) < p$.

A second assumption define the ranges of the salvage and the unit payback values and that gives sense to the payback option at the beginning of the selling horizon: $s_2 < s_1$.

We assume also that $s_1 < c_e$. This assumption aims at avoiding situations in which it will be profitable to exercise options at the beginning of the second decision stage, and to receive articles from the supplier, and the return these articles immediately to the same supplier.

An other intuitive assumption is taken into account and is relative to the payback value: $Min(c_0, c_{op} + c_e) > s_1$.

2.3. Second Decision Stage Subproblem

2.3.1 Objective Function

Let $E_X(\cdot)$ be the expectation taken over random variable X, $x^+ = \max(0, x)$, $x \wedge y = \min(x, y)$.

Define the objective function of the second stage by the following equation :

$$\begin{aligned}&\Pi_1(Q_1, S_1|I, Q_0, Q_T) = \qquad (2)\\ &p\left((Q_0 + Q_1 - S_1) \wedge D\right) - b(D - Q_0 - Q_1 + S_1)^+\\ &+s_2(Q_0 + Q_1 - S_1 - D)^+ - c_e Q_1 + s_1 S_1.\end{aligned}$$

Where Q_1 and S_1 are the decision variables of the second stage.

The expected value of the objective function with respect to the demand will be

$$\begin{aligned}&E_{D|I}\left[\Pi_1(Q_1, S_1|I, Q_0, Q_T)\right] =\\ &p\int_0^{Q_0+Q_1-S_1} Dh(D|I)dD\\ &+p\int_{Q_0+Q_1-S_1}^{\infty} (Q_0 + Q_1 - S_1)h(D|I)dD\\ &-b\int_{Q_0+Q_1-S_1}^{\infty} (D - Q_0 - Q_1 + S_1)h(D|I)dD\\ &+s_2\int_0^{Q_0+Q_1-S_1} (Q_0 + Q_1 - S_1 - D)h(D|I)dD\\ &-c_e Q_1 + s_1 S_1 \qquad (3)\end{aligned}$$

Lemma 1 *The expected objective function $E_{D|I}\left[\Pi_1(Q_1, S_1|I, Q_0, Q_T)\right]$ is jointly concave with respect to the decision variables Q_1 and S_1.*

Proof 1 *See (Cheaitou and al., 2007).* □

2.3.2 Optimization problem

After proving the concavity of the expected objective function of the second stage decision subproblem, we can now write the optimization problem of that stage as follows

$$\max_{0\leq Q_1 \leq K,\ \ 0\leq S_1} E_{D|I}\left[\Pi_1(Q_1, S_1|I, Q_0, Q_T)\right] \qquad (4)$$

Therefore, (Cheaitou et al, 2007) showed that the optimal policy of the unconstrained problem ($K = \infty$) is a modified newsvendor solution that can be completely characterized by two threshold levels, and that are given by

$$Y_1(I) = H^{-1}\left(\frac{p + b - c_e}{p + b - s_2}|I\right)$$
$$\text{and } \ Y_2(I) = H^{-1}\left(\frac{p + b - s_1}{p + b - s_2}|I\right), \qquad (5)$$

with, from the model assumptions (Section (2.2)), are related by:

$$Y_1(I) \leq Y_2(I). \qquad (6)$$

These two threshold levels could be expressed differently as follows:

$$\begin{aligned}Y_1(I) &= \mu_0 + \rho(I - \theta_0)\frac{\sigma_0}{\delta_0}\\ &+(\sigma_0\sqrt{1-\rho^2})\Phi^{-1}\left(\frac{p + b - c_e}{p + b - s_2}\right), \qquad (7)\end{aligned}$$

and

$$\begin{aligned}Y_2(I) &= \mu_0 + \rho(I - \theta_0)\frac{\sigma_0}{\delta_0}\\ &+(\sigma_0\sqrt{1-\rho^2})\Phi^{-1}\left(\frac{p + b - s_1}{p + b - s_2}\right). \qquad (8)\end{aligned}$$

2.3.3 Optimal policy of the constrained problem

Using the concavity of the expected objective function of the second decision stage, with respect to the decision variables Q_1 and S_1, the optimal policy of the second decision stage can be defined as a function of the threshold levels $Y_1(I)$ and $Y_2(I)$, as follows, :

- if $Y_2(I) < Q_0$ then $Q_1^* = 0$ and $S_1^* = Q_0 - Y_2(I)$,
- if $Y_1(I) \leq Q_0 \leq Y_2(I)$ then $Q_1^* = S_1^* = 0$,
- if $Q_0 < Y_1(I) < Q_T$ then $Q_1^* = Y_1(I) - Q_0$ and $S_1^* = 0$,
- if $Q_T < Y_1(I)$ then $Q_1^* = Q_T - Q_0$ and $S_1^* = 0$.

This solution is simply a modified newsvendor solution using the updated demand distribution $H(D|I)$, constrained by the initial decisions Q_0 and Q_T of the first decision stage.

2.4. First Decision Stage Subproblem

First of all, we make the assumption that increasing values of realization of I denote increasing forecasts of demand. In other words, D is positively correlated with I. Mathematically, we can express this using the concept of stochastically larger (Ross, 1983). Consider two realizations of I: i_1 and i_2. Then, $i_1 > i_2$ implies that $D|i_1$ is stochastically larger than $D|i_2$, expressed $D|i_1 \geq_{st} D|i_2$. This stochastic relationship implies that $H(D|i_1) \leq H(D|i_2)$ for all D (Brown and Lee, 1998, b).

Since D is stochastically increasing in I, $H(D|I)$ is decreasing in I for all D. Thus, $H^{-1}\left(\frac{p+b-c_e}{p+b-s_2}|I\right)$

and $H^{-1}\left(\frac{p+b-s_1}{p+b-s_2}|I\right)$ are increasing in I, so $Y_1(I)$ and $Y_2(I)$ are increasing in I. Because of this monotonic behavior, we can express the optimal policy of the second stage as a function of the observed external information as follows,

- if $I < U_2(Q_0)$ then $Q_1^*(I,Q_0) = 0$ and $S_1^*(I,Q_0) = Q_0 - Y_2(I)$,
- if $U_2(Q_0) \leq I \leq U_1(Q_0)$ then $Q_1^*(I,Q_0) = S_1^*(I,Q_0) = 0$,
- if $U_1(Q_0) < I < V_1(Q_T)$ then $Q_1^*(I,Q_0) = Y_1(I) - Q_0$ and $S_1^*(I,Q_0) = 0$,
- if $V_1(Q_T) < I$ then $Q_1^*(I,Q_0) = Q_T - Q_0$,

with $U_1(Q_0)$, $V_1(Q_T)$ and $U_2(Q_0)$ are the values of I so that $Y_1(I) = Q_0$, $Y_2(I) = K$ and $Y_2(I) = Q_0$ respectively. $U_1(Q_0)$, $V_1(K)$and $U_2(Q_0)$ are given by the following equations

$$U_1(Q_0) = \theta_0 + \frac{\delta_0}{\rho\sigma_0}\left[Q_0 - \sigma_0\sqrt{1-\rho^2}\Phi^{-1}\left[\frac{p+b-c_e}{p+b-s_2}\right] - \mu_0\right], \tag{9}$$

$$V_1(Q_T) = \theta_0 + \frac{\delta_0}{\rho\sigma_0}\left[Q_T - \sigma_0\sqrt{1-\rho^2}\Phi^{-1}\left[\frac{p+b-c_e}{p+b-s_2}\right] - \mu_0\right], \tag{10}$$

and

$$U_2(Q_0) = \theta_0 \quad \frac{\delta_0}{\rho\sigma_0}\left[Q_0 - \sigma_0\sqrt{1-\rho^2}\Phi^{-1}\left[\frac{p+b-s_1}{p+b-s_2}\right] - \mu_0\right]. \tag{11}$$

2.4.1 Objective function

Using the obtained results in the previous sections, we can write the expected objective function of the first decision stage as follows

$$\Pi_0(Q_0,Q_T) = -c_{op}(Q_T - Q_0) - c_0 Q_0 + E_I\left[E_{D|I}\left[\Pi_1^*(Q_1^*,S_1^*|I,Q_0,Q_T)\right]\right]. \tag{12}$$

Define the optimal expected (with respect to $D|I$) objective function of the second period $E_{D|I}\left[\Pi_1^*(Q_1^*,S_1^*|I,Q_0,Q_T)\right]$ as follows:

$$E_{D|I}\left[\Pi_1^*(Q_1^*,S_1^*|I,Q_0,Q_T)\right] = \tag{13}$$

$$\begin{cases} E_{D|I}\left[\Pi_{11}^*(Q_1^*,S_1^*|I,Q_0,Q_T)\right] & if\ I < U_2(Q_0) \\ E_{D|I}\left[\Pi_{12}^*(Q_1^*,S_1^*|I,Q_0,Q_T)\right] & if\ U_2(Q_0) < I < U_1(Q_0) \\ E_{D|I}\left[\Pi_{13}^*(Q_1^*,S_1^*|I,Q_0,Q_T)\right] & if\ U_1(Q_0) < I < V_1(Q_T) \\ E_{D|I}\left[\Pi_{14}^*(Q_1^*,S_1^*|I,Q_0,Q_T)\right] & if\ V_1(Q_T) < I \end{cases}$$

with

$$\begin{aligned} E_{D|I}\left[\Pi_{11}^*(Q_1^*,S_1^*|I,Q_0,Q_T)\right] = \\ p\int_0^{Y_2(I)} Dh(D|I)dD + p\int_{Y_2(I)}^{\infty} Y_2(I)h(D|I)dD - \\ b\int_{Y_2(I)}^{\infty} (D - Y_2(I))h(D|I)dD + \\ s_2\int_0^{Y_2(I)} (Y_2(I) - D)h(D|I)dD + s_1(Q_0 - Y_2(I)), \end{aligned} \tag{14}$$

$$\begin{aligned} E_{D|I}\left[\Pi_{12}^*(Q_1^*,S_1^*|I,Q_0,Q_T)\right] = p\int_0^{Q_0} Dh(D|I)dD \\ +p\int_{Q_0}^{\infty} Q_0 h(D|I)dD - b\int_{Q_0}^{\infty} (D - Q_0)h(D|I)dD \\ +s_2\int_0^{Q_0} (Q_0 - D)h(D|I)dD, \end{aligned} \tag{15}$$

$$\begin{aligned} E_{D|I}\left[\Pi_{13}^*(Q_1^*,S_1^*|I,Q_0,Q_T)\right] = p\int_0^{Y_1(I)} Dh(D|I)dD \\ +p\int_{Y_1(I)}^{\infty} Y_1(I)h(D|I)dD - \\ -b\int_{Y_1(I)}^{\infty} (D - Y_1(I))h(D|I)dD \\ +s_2\int_0^{Y_1(I)} (Y_1(I) - D)h(D|I)dD \\ -c_e(Y_1(I) - Q_0), \end{aligned} \tag{16}$$

and

$$\begin{aligned} E_{D|I}\left[\Pi_{14}^*(Q_1^*,S_1^*|I,Q_0,Q_T)\right] = -c_e(Q_T - Q_0) \\ +p\int_0^{Q_T} Dh(D|I)dD + p\int_{Q_T}^{\infty} Q_T h(D|I)dD \\ -b\int_{Q_T}^{\infty} (D - Q_T)h(D|I)dD \\ +s_2\int_0^{Q_T} (Q_T - D)h(D|I)dD. \end{aligned} \tag{17}$$

Then the expected (with respect to $D|I$ and I) optimal objective function of the second period is given by:

$$E_I\left[E_{D|I}\left[\Pi_1^*(Q_1^*,S_1^*|I,Q_0,Q_T)\right]\right] = \int_0^{U_2(Q_0)}\left[E_{D|I}\left[\Pi_{11}^*(Q_1^*,S_1^*|I,Q_0,Q_T)\right]\right]g(I)dI + \int_{U_2(Q_0)}^{U_1(Q_0)}\left[E_{D|I}\left[\Pi_{12}^*(Q_1^*,S_1^*|I,Q_0,Q_T)\right]\right]g(I)dI + \int_{U_1(Q_0)}^{V_1(Q_T)}\left[E_{D|I}\left[\Pi_{13}^*(Q_1^*,S_1^*|I,Q_0,Q_T)\right]\right]g(I)dI + \int_{V_1(Q_T)}^{\infty}\left[E_{D|I}\left[\Pi_{14}^*(Q_1^*,S_1^*|I,Q_0,Q_T)\right]\right]g(I)dI$$

Lemma 2 *The expected objective function $\Pi_0(Q_0,Q_T)$ is concave with respect to the decision variables Q_0 and Q_T.*

Proof 2 *The different parts that define the expected optimal objective function of the second decision stage, given in Equation(13) are jointly concave with respect to the decision variables Q_0 and Q_T and the information I.*

Indeed, let us prove the concavity of the $E_{D|I}\left[\Pi_{11}^*(Q_1^*,S_1^*|I,Q_0,Q_T)\right]$ and therefore the concavity of the other three functions could be easily shown using the same methodology.

The hessian of $E_{D|I}\left[\Pi_{11}^*(Q_1^*,S_1^*|I,Q_0,Q_T)\right]$ with respect to Q_1, S_1 and I is given by

$$\nabla^2 E_{D|I}\left[\Pi_{11}^*(Q_1^*,S_1^*|I,Q_0,Q_T)\right] = -\frac{p+b-s_2}{\delta_0^2}\rho^2\sigma_0^2 h\left(Y_2(I)\right)\begin{bmatrix}0&0&0\\0&0&0\\0&0&1\end{bmatrix}$$

For each vector $\mathbf{V} = \begin{pmatrix}V_1\\V_2\\V_3\end{pmatrix}$, where $(V_1;V_2;V_3) \in I\!R^3$, we find

$$V^T\left\{\nabla^2 E_{D|I}\left[\Pi_{11}^*(Q_1^*,S_1^*|I,Q_0,Q_T)\right]\right\}V = -\frac{p+b-s_2}{\delta_0^2}\rho^2\sigma_0^2 h\left(Y_2(I)\right)V_3^2. \tag{18}$$

From the model assumptions (Section (2.2)), one has $p+b>s_2$, which means that

$$V^T\left\{\nabla^2 E_{D|I}\left[\Pi_{11}^*(Q_1^*,S_1^*|I,Q_0,Q_T)\right]\right\}V \le 0,$$

which proves that the matrix $\nabla^2 E_{D|I}\left[\Pi_{11}^*(Q_1^*,S_1^*|I,Q_0,Q_T)\right]$ is semi-definite negative. Consequently, the objective function $E_{D|I}\left[\Pi_{11}^*(Q_1^*,S_1^*|I,Q_0,Q_T)\right]$ is jointly concave with respect to Q_1, S_1 and I.

Note that if the functions $E_{D|I}\left[\Pi_{11}^*(Q_1^*,S_1^*|I,Q_0,Q_T)\right]$, $E_{D|I}\left[\Pi_{12}^*(Q_1^*,S_1^*|I,Q_0,Q_T)\right]$, $E_{D|I}\left[\Pi_{13}^*(Q_1^*,S_1^*|I,Q_0,Q_T)\right]$ and $E_{D|I}\left[\Pi_{14}^*(Q_1^*,S_1^*|I,Q_0,Q_T)\right]$ are all defined for each I value, then an alternative way to define $E_{D|I}\left[\Pi_1^*(Q_1^*,S_1^*|I,Q_0,Q_T)\right]$ is (see Bassok and Anupindi 1997)

$$E_{D|I}\left[\Pi_1^*(Q_1^*,S_1^*|I,Q_0,Q_T)\right] = \max_{1\le i\le 4}\left(E_{D|I}\left[\Pi_{1i}^*(Q_1^*,S_1^*|I,Q_0,Q_T)\right]\right) \tag{19}$$

Theorem 4.13 in (Avriel 1976) ensures that a function that is defined as the maximum (pointwise) of several concave functions is concave. This proves that $E_{D|I}\left[\Pi_1^*(Q_1^*,S_1^*|I,Q_0,Q_T)\right]$ is jointly concave with respect to Q_0, Q_T and I.

It is well known that the weighted non-negative sum (or integral) of concave functions is a concave one (Boyd and Vandenberghe, 2004).

We conclude then that the function $\Pi_0(Q_0,Q_T)$ is concave with respect to Q_0 and Q_T. □

Lemma 3 *The optimal values of the decision variables Q_0 and Q_T are characterized by a system of two independent equations given by*

$$\frac{\partial\Pi_0(Q_0,Q_T)}{\partial Q_T} = 0 \text{ and } \frac{\partial\Pi_0(Q_0,Q_T)}{\partial Q_0} = 0 \tag{20}$$

where the two partial derivatives are defined in the following equations:

$$\frac{\partial\Pi_0(Q_0,Q_T)}{\partial Q_T} = -c_{op} - \int_{V_1(Q_T)}^{\infty}\left[c_e - p - b + (p+b-s_2)H(K|I)\right]g(I)dI + \frac{\delta_0}{\rho\sigma_0}g(V_1(Q_T))\Big[(c_e-p-b)(Q_T - Y_1(V_1(Q_T))) - (p+b-s_2)\Big[(Y_1(V_1(Q_T)) - Q_T)H(Y_1(V_1(Q_T))) + \int_0^{Q_T} Dh(D|I=V_1(K))dD - \int_0^{Y_1(V_1(Q_T))} Dh(D|I=V_1(K))dD\Big]\Big] \tag{21}$$

and

$$\begin{aligned}\frac{\partial \Pi_0(Q_0,Q_T)}{\partial Q_0} &= -c_0 + c_{op} + \\ & s_1 G(U_2(Q_0)) - c_e G(U_1(Q_0)) + c_e + \\ & \int_{U_2(Q_0)}^{U_1(Q_0)} \left[(s_2 - b - p)H(Q_0|I) + p + b\right] g(I)dI \\ & -\frac{\delta_0}{\rho\sigma_0} g(U_1(Q_0)) \Bigg[(c_e - b - p)(Q_0 - Y_1(U_1(Q_0))) \\ & +(p+b-s_2)\Big[(Q_0 - Y_1(U_1(Q_0))) \\ & H(Y_1(U_1(Q_0))|I = U_1(Q_0))\Big] \\ & -\int_0^{Q_0} Dh(D|I = U_1(Q_0))dD \\ & + \int_0^{Y_1(U_1(Q_0))} Dh(D|I = U_1(Q_0))dD \Bigg] \\ & -\frac{\delta_0}{\rho\sigma_0} g(U_2(Q_0)) \Bigg[(s_1 - b - p)(Q_0 - Y_2(U_2(Q_0))) \\ & +(p+b-s_2)[(Q_0 - Y_2(U_2(Q_0))) \\ & H(Y_2(U_2(Q_0))|I = U_2(Q_0))] \\ & -\int_0^{Q_0} Dh(D|I = U_2(Q_0))dD \\ & + \int_0^{Y_2(U_2(Q_0))} Dh(D|I = U_2(Q_0))dD \Bigg]\end{aligned} \tag{22}$$

These equations characterize the optimal policy of the first decision stage, but they are very complicated to be handled in order to get simple formulae that determine this optimal policy. For this reason, we will develop in the following sections some special cases in which we will provide closed form formulae of the optimal policy.

3. PERFECT INFORMATION SPECIAL CASE

In this section we will develop the optimal policy of the entire problem in the case where the correlation between the external information and the demand is perfect ($\rho = 1$). The other special case where $\rho = 0$ is obvious and will be discussed in section(4.2).

When $\rho = 1$, the conditional demand distribution will be deterministic with value $\mu_0 + \rho(I - \theta_0)\sigma_0/\delta_0$. The conditional demand will be $D|I = \mu_0 + \rho(I - \theta_0)\sigma_0/\delta_0$.

First of all we will begin by determining the optimal policy of the second decision stage and then using the dynamic programming we determine that of the first decision stage.

3.1. Second decision stage optimal policy

The problem of the second decision stage is deterministic. Once the information I is known, the demand D becomes deterministic and get the value of $D|I = \mu_0 + \rho(I - \theta_0)\sigma_0/\delta_0$. In this case whatever the value of the couple $(c_e; s_1)$ is, the optimal values of the decision variables Q_1 and S_1 will not be influenced. Therefore depending on the value of the realized information I, we can have one of the following three cases:

- if $Q_0 \leq Q_T \leq D|I$ then $Q_1 = Q_T - Q_0$ and $S_1 = 0$ and the optimal objective function is $\Pi_1^*(Q_1^*, S_1^*|I) = pQ_T - b(D|I - Q_T) - c_e(Q_T - Q_0)$,
- if $Q_0 \leq D|I \leq Q_T$ then $Q_1 = D|I - Q_0$ and $S_1 = 0$ with the following expected optimal objective function $\Pi_1^*(Q_1^*, S_1^*|I) = pD|I - c_e(D|I - Q_0)$,
- if $D|I \leq Q_0 \leq Q_T$ then $Q_1 = 0$ and $S_1 = Q_0 - D|I$ with $\Pi_1^*(Q_1^*, S_1^*|I) = pD|I + s_1(Q_0 - D|I)$.

3.2. First decision stage optimal policy

Using dynamic programming we can write the expected objective function of the first decision stage as follows:

$$\begin{aligned}\Pi_0(Q_0,Q_T) &= \\ & -c_{op}(Q_T - Q_0) - c_0 Q_0 + E_I\left[\Pi_1^*(Q_1^*, S_1^*|I)\right] \\ & = -c_{op}(Q_T - Q_0) - c_0 Q_0 \\ & + \int_{\delta_0/\sigma_0(Q_T-\mu_0)+\theta_0}^{\infty} \Big[pQ_T - c_e(Q_T - Q_0) \\ & -b(D|I - Q_T)\Big] g(I)dI \\ & + \int_{\delta_0/\sigma_0(Q_0-\mu_0)+\theta_0}^{\delta_0/\sigma_0(Q_T-\mu_0)+\theta_0} \left[pD|I - c_e(D|I - Q_0)\right] g(I)dI \\ & + \int_0^{\delta_0/\sigma_0(Q_0-\mu_0)+\theta_0} \left[pD|I + s_1(Q_0 - D|I)\right] g(I)dI\end{aligned} \tag{23}$$

To derive the optimal values of the decision variables Q_0 and Q_T, we will use the first order optimality criterion.

The optimal values of Q_0 and Q_T will be the solutions of the following equations:

$$\begin{aligned}\frac{\partial \Pi_0(Q_0,Q_T)}{\partial Q_0} &= -c_0 + c_{op} + c_e \\ & -(c_e - s_1) G\left[\delta_0/\sigma_0(Q_0 - \mu_0) + \theta_0\right] \\ & = 0,\end{aligned} \tag{24}$$

and

$$\frac{\partial \Pi_0(Q_0, Q_T)}{\partial Q_T} = p + b - c_e - c_{op}$$
$$-(p + b - c_e)G\left[\delta_0/\sigma_0(Q_T - \mu_0) + \theta_0\right]$$
$$= 0. \qquad (25)$$

These two equations gives the following optimal values of the decision variables:

$$Q_0^* = \frac{\sigma_0}{\delta_0}\left[G^{-1}\left[\frac{c_e + c_{op} - c_0}{c_e - s_1}\right] - \theta_0\right] + \mu_0, \quad (26)$$

and

$$Q_T^* = \frac{\sigma_0}{\delta_0}\left[G^{-1}\left[\frac{p + b - c_{Moy}}{p + b - c_e}\right] - \theta_0\right] + \mu_0. \quad (27)$$

4. NUMERICAL ANALYSIS

In this section we provide some numerical applications to show the impact of each of our model parameters on the structure of the optimal policy.

First of all, we define nominal numerical set of parameters that will be used as a base example, in order to vary one or more parameters to plot figures and to make comparisons showing the effect of some parameters of our model on the optimal policy.

The nominal numerical values of our model parameters are defined as follows: $\mu_0 = 1000$, $\theta_0 = 1000$, $\sigma_0 = 300$, $\delta_0 = 300$, $\rho = 0.5$, $p = 100$, $b = 30$, $s_2 = 15$, $c_0 = 50$, $c_{op} = 5$, $c_e = 45$, $s_1 = 35$.

Note that in all the following numerical examples, we plot the optimal values of the decision variables of the first decision stage Q_0 and Q_T, namely Q_0^* and Q_T^*, and the expected optimal values of the decision values of the second decision stage, namely $E[Q_1^*]$ and $E[S_1^*]$. Indeed, the expectation of the optimal values of the decision variables Q_1 and S_1 is with respect to the stochastic information I.

4.1. Effect of the unit order cost c_0

In this section we show the effect of the unit order cost c_0 on the optimal policy of the first decision stage. We use nominal parameters defined above, and we vary the unit order cost of the first decision stage c_0 to show its impact on the optimal policy.

On Figure (2), one can notice the existence of three different intervals of values of c_0. The first one belongs to the lower values of c_0, where the expected optimal value of the decision variable S_1 is positive. In this region, the optimal value of the decision variable Q_0 is high, due to the attractive ordering cost c_0,

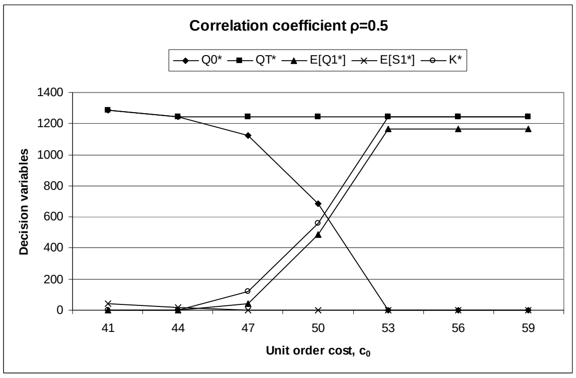

Figure 2: Effect of the unit order cost, c_0

and consequently, the expected optimal decision variable Q_1 is equal to zero. Therefore, in this region the optimal Q_T is equal to the optimal Q_0. Note that In this region, even if the salvage value s_1 is lower than the ordering cost c_0, it is optimal to order some units and to salvage them later using the payback option at the beginning of the selling period. This is due to the existence of the correlation between the information I and the demand D ($\rho = 0.5$), and to the attractive ordering cost c_0 that is lower (in this region) than the expected cost $c_{Moy} = 50$. The second region be-

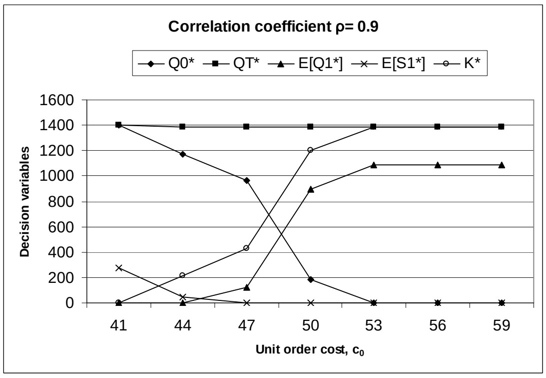

Figure 3: Effect of information quality, $\rho = 0.9$

longs to the medium values of c_0. In this region the optimal value of Q_0 is still positive but is decreasing rapidly when c_0 increases. The expected optimal value of the decision variable S_1 is equal to zero. In fact it is not profitable to order some units at the first decision stage and then to salvage them at the second decision stage. On the other hand, when c_0 increases, the optimal Q_T is still constant, and as the optimal Q_0 decreases, then the optimal K increases, to permits an increase in the number of units ordered at the second decision stage, namely Q_1.

In the third region, that corresponds to the high values of c_0, the optimal value of Q_0 is equal to zero. In this region, it is normal that $S_1^{=}0$, because there is no units able to be returned to the supplier. As $Q_0^* = 0$, then in this region the optimal policy is not affected by the increase of c_0.

4.2. Effect of the information quality

In this section, we compare the example presented in the previous section, in Figure (2), (with correlation coefficient $\rho = 0.5$) with two other examples, that have the same parameters except the correlation coefficient: the first one is with less important information, with $\rho = 0.1$ and the second one is with more important information with $\rho = 0.9$.

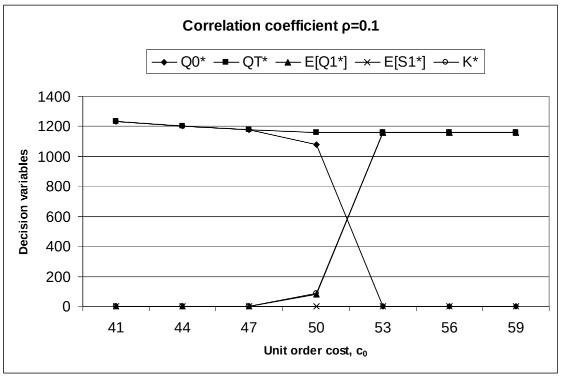

Figure 4: Effect of information quality, $\rho = 0.1$

The first remark to take into account is that when the correlation between the information and the demand decreases, the optimal policy becomes insensitive to the difference between the unit order cost c_0 and the cost c_{Moy}, in the regions where this difference is considerable, and is very sensitive in the zone where this difference is small. This implies that the width of the c_0 values interval, where both Q_0^* and $E[Q_1^*]$ are positive (the medium c_0 values) increases when ρ increases. In this region the optimal policy is to order with Q_0 and Q_1 together.

Note also that when ρ decreases, the expected returned quantity $E[S_1^*]$ decreases. Indeed, in the case where ρ is low, the quality of the information captured between the two decision stages is very bad, and it does not permit a reduction in the demand variability. As the payback value is less than the ordering value, and one knows a priori that the collected information will not change tremendously the demand distribution, then it will not be profitable to return units ordered with Q_0 with a payback value s_1, and consequently $E[S_1^*]$ decreases.

Notice that when ρ increases the optimal reserved capacity amount increases. That gives a bigger chance to profit from the information I and the variability reduction of the demand and to adjust the first ordered quantity Q_0 using a part or the totality of the reserved capacity which is implies an important ***reactivity***.

From Figures (2), (3) and (4) we can note that the effect of the information quality coupled with the unit order cost c_0 on the optimal policy can be divided into three cases.

The first case corresponds to the very low c_0 values. In this region, the difference between c_{Moy} and c_0 is very big so that the correlation coefficient ρ has no importance. Hence, the optimal K and consequently the optimal expected Q_1 are equal to zero. In this region, when the information quality increases, the optimal Q_0 increases for two reasons: if the realized I is high, which implies high demand (due to the strong correlation) then a high Q_0^* satisfies well the demand, and if the realized I is low, and due to the low difference between c_0 and s_1 in this region, a part of Q_0^* could be returned to the supplier.

The second case corresponds to the medium c_0 values. In this region, the difference between c_{Moy} and c_0 is low. Then it is the quality of the information that will determine if it is Q_0 or Q_1 that will be more profitable. Indeed, when ρ increases, K^* increases, it becomes more profitable to postpone the ordering decision to the second decision stage, in order to use better the information I. Then Q_0^* decreases and $E[Q_1^*]$ increases to compensate.

The third case is relative to the high c_0 values. In this region it is logical to have $Q_0^* = 0$, for all ρ values, due to the high difference between c_{Moy} and c_0. When ρ increases, the optimal K^* increases also to permits a better reactivity in the case of high I realization at the second decision stage. On the other hand $E[Q_1^*]$ decreases when ρ increases as it is decided after that I is known. In fact, first of all, when ρ increases, and as $Q_0^* = 0$ for all ρ values, there is no need to postpone any orders from the first to the second decision stage or to compensate for any unordered units (with Q_0^*). In addition, for high ρ values, the collected information during the first period will be very useful to reduce the demand variability. Then after observing the information I, the variability of the demand decreases, and consequently the quantity Q_1 ordered to face this variability decreases also. Note that this results in an increase in the difference between K^* and $E[Q_1^*]$.

Note that, if $\rho = 0$ then there are only two cases: when c_0 is lower than c_{Moy} then the optimal K and consequently the optimal expected Q_1 are equal to zero. in the other case ($c_0 > c_{Moy}$), the optimal Q_0 is equal to zero.

4.3. Effect of the unit payback value s_1

In this section we study the effect of the unit payback value s_1 on the optimal policy of the two decision stages and on the expected optimal objective function. First of all we plot two numerical examples, that show the optimal policy, based on the nominal numerical data defined above except the value of s_1. In the first example shown in Figure (5) we vary s_1. Secondly, we plot an other example (Figure (6)) in which we show the effect of s_1 and of the correlation coefficient ρ on the expected optimal objective function of our model. This example (Figure (6)) is based also on the nominal numerical values defined above, the correlation coefficient ρ that we give three different values (0.1, 0.5 and 0.9) and the unit salvage value s_1 that we vary in the admissible interval of values.

For the first example, Figure (5), note that when the value of s_1 increases the expected optimal value of the returned quantity, $E[S_1^*]$ increases. This increases is accompanied with an increase in the optimal ordered quantity Q_0^*, from which some units are returned at the second decision stage. Note also that the optimal amount of reserved capacity, K^*, and the expected optimal ordered quantity at the second decision stage, $E[Q_1^*]$ increase. This means that when s_1 increases, the optimal policy tend to a policy in which one orders more at the first decision stage and less at the second decision stage, and a part of the ordered quantity at the first decision stage could be returned to the supplier.

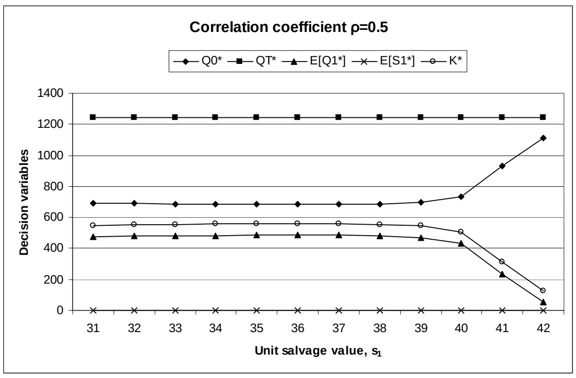

Figure 5: Effect of the high unit payback value s_1

From figure (6), the first important and rather intuitive issue that one can notice is the effect of the quality of the collected information on the expected optimal objective function. It is clear that the higher the quality of the information, the higher the expected optimal objective function. The second important issue is that the impact of the quality of the information on the expected optimal objective function increases when the unit payback value s_1 increases. This is due to the fact that, for the high ρ values, when s_1 increases, the increase in $E[S_1^*]$ is higher than the case of low ρ values.

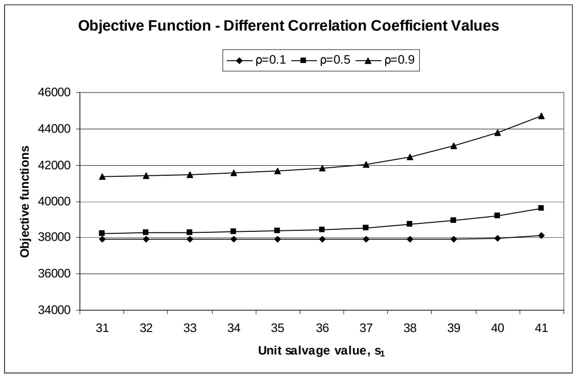

Figure 6: Effect of s_1 on the optimal expected objective function, with different ρ

5. CONCLUSION

In this paper we have presented a new production/inventory model, for short life cycle products, or *newsboy* type products. During a single period selling horizon, a single product for which the stochastic demand is characterized by a probability density function, which is distributed jointly with an exogenous market information. The decision process is divided into two stages. In the first stage, two decisions are fixed: the first concerning a first ordered quantity, and a second decision concerning a capacity reservation. In the second stage two decisions are also fixed: the first one is relative to the use of the totality or of a part of the reserved capacity (purchased options) at the first decision stage. The second one represents the quantity returned (payback) to the supplier at the beginning of the selling horizon, from the quantity already ordered at the first stage. During the first stage, the exogenous information is stochastic. Between the two decision stages, the market information is collected and the parameters of the conditional distribution of the demand are known. At the end of the selling horizon, each remaining unit, if any, is salvaged and any unsatisfied order is lost. We have provided the optimal policy of the second decision stage and the optimality equations of the first decision stage. Since there is no closed-form analytical expression for the optimal policy, we have numerically solved the first decision stage and via numerous numerical examples we have provided some insights on the optimal policy and the effect of the main model parameters on this policy. We have also solved analytically two special cases: the first one is a model with worthless information and the second one is with perfect information.

Note that in this paper we have analyzed a new type of contract from the point of view of the retailer. We have shown the increase of the expected optimal objective function of the retailer and of the total ordered quantity Q_T with the increase of the unit payback value s_1. In the perspectives of this paper we can find the study of the impact of the increase of s_1 on the optimal policy and the optimal expected objective function of the supplier, and as Q_T^* increases with s_1, we can assume that there exist a certain value s_1^* that coordinates the channel and maximizes both the supplier and retailer expected objective functions.

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