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OPTIMIZATION OF SELECTIVE MAINTENANCE FOR MULTI-MISSION SERIES-PARALLEL SYSTEMS

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RÉSUMÉ : *This paper presents a selective maintenance optimization model for a multi-mission series-parallel system. Such a system experiences several missions with breaks between successive missions. The reliability of each system component is characterized by its hazard function. To maintain the reliability of the system, preventive maintenance actions are performed during breaks. Each preventive maintenance action is characterized by its ability to reduce the age of system components. The selective maintenance problem consists on finding an optimal sequence of maintenance actions the cost of which minimizes the total maintenance cost while providing the desired system reliability level for each mission. As a solution technique, an optimization method is proposed on the basis of the extended great deluge algorithm. An application example with numerical results are given for illustration.*

MOTS-CLÉS : *Multi-mission systems, Preventive maintenance, Minimal repair, Reliability, Optimization, Extended great deluge algorithm*

1 INTRODUCTION

A variety of mathematical models appeared in the literature for the design of optimal maintenance policies for reparable systems (for a survey, see for example (Cho and Parlar 1991, Dekker 1996)). Nevertheless, most of these models do not take into account the limitations on the resources required to perform maintenance actions. this drawback has motivated the development of a relatively new concept called the selective maintenance. The objective of the selective maintenance consists on finding, among all available maintenance actions, the set of appropriate actions to be performed, under some operation constraints, so that to maximize the system reliability or either to minimize the total maintenance cost or the total maintenance time.

Selective maintenance, as a maintenance policy, is relevant to systems that are required to operate a sequence of missions such that, at the end of a mission, the system is put down for a finite length of time to provide an opportunity for equipment maintenance. Such systems may include for example manufacturing systems, computer systems and transportation systems.

Dealing with selective maintenance, the first work is reported in (Rice, Cassady and Nachlas 1998). In this work, Rice *et al.* consider a series-parallel system which operates a series of missions. Replacement of failed components is performed during breaks between successive missions. Since breaks are of finite length, it may be difficult to replace all failed components. In (Rice et al. 1998), to determine the optimal replacement to be performed, a maintenance optimization problem is derived to maximize the system reliability for the next mission. To solve this problem, an heuristic-based method is proposed to overcome the limitations of total enumeration method. The work of Rice *et al.* (Rice et al. 1998) is extended by Cassady *et al.* (Cassady, Pohl and Murdock 2001) to allow other reliability structures. Three different selective maintenance problem are addressed and illustrated by using combinations of series, parallel and bridge reliability structures. Cassady *et al.* (Cassady, Murdock and Pohl 2001) propose another extension of the work of *Rice et al.* by considering a series-parallel system where the lifetime of each component follows a Weibull distribution. Three maintenance actions are allowed, namely the minimal repair of failed components, replacement of failed components and preventive replacement of functioning components. Chen *et al.* (Chen, Mend and Zuo 1999), addresses selective maintenance problem in multi-state systems setting. Each component and the system itself may be in one of the $(K+1)$ states. A maintenance optimization problem is then derived to minimize the total maintenance cost while providing a given required system reliability level for the next mission. To solve this problem, Chen *et al.* propose a procedure based on the short path method. More recently, Khatab *et al.* (Khatab, Ait-Kadi and Nourelfath 2007b) studied the selective maintenance for a series-parallel system where components in parallel are possibly different. A maintenance optimization problem is formulated to maximize the system reliability for the next mission under both time and budget constraints. As an optimization technique, two heuristic-based methods, adapted from those applied within redundancy allocation problem (Aggrawal 1976) (Gopal, Aggrawal and Gupta 1978) (Misra 1972) (see also (Kuo and Zuo 2002)), are then proposed. The authors in (Khatab et al. 2007b) compare these two heuristics on the basis of large test problems generated randomly. In khatab *et al.* (Khatab, Ait-Kadi and Nourelfath 2007a), a simulated annealing, as a local search metaheuristic, is successfully applied to solve the selective maintenance optimization problem addressed in (Khatab et al. 2007b).

Selective maintenance optimization problems handled by the above mentioned works are limited to the treatment of only one next mission. However, since the system is required to perform a sequence of missions, while requiring short development schedules and very high reliability, it becomes increasingly important to develop appropriate approaches to manage selective maintenance decisions when the planning horizon considers more than a single mission. At the best of our knowledge, the only work which deals with selective maintenance with *several next* missions is proposed by Maillart *et al.* (Maillart, Cassady, Rainwater and Schneider 2005). In (Maillart et al. 2005), the authors consider a series-parallel system where each subsystem is composed of identical components whose time to failure is exponentially distributed. The system is assumed to operate a sequence of identical missions such that breaks between two successive missions are of equal durations. At the end of a given mission, the only available maintenance action is the replacement of failed components. At a given time, the average number of successful missions remaining in the planning horizon is defined. To maximize such a number, a stochastic programming model is then proposed. Numerical experiments are conducted to perform and compare the results obtained for three maintenance optimization problems, namely the classical one-mission problem, the two-mission problem and the infinite-mission problem. Nevertheless, the approach proposed in (Maillart et al. 2005) merely relies on a series-parallel system with few subsystems each composed of components of identical constant failure rates. Furthermore, replacement of failed components is the only available maintenance action, missions are of identical time interval and breaks are also of identical durations.

This paper proposes a selective maintenance optimization model for a multi-mission series-parallel system. The system is composed of series of subsystems each composed of parallel, and possibly, different components the lifetime of which are generally distributed. The system operates a sequence of missions with possibly different durations such that nonidentical breaks are allowed between successive missions. During a given mission, a component that fail undergoes minimal repair while at the end of a mission several preventive maintenance actions are available. Each preventive maintenance action is characterized by its ability to affect the effective age of system components. In this paper, the proposed selective maintenance optimization problem consists on finding an optimal sequence of preventive maintenance actions the cost of which minimizes the total maintenance cost while providing the desired system reliability level for each mission. To solve this problem, we present an optimization method inspired from the extended great deluge metaheuristic (Burke, Bykov, Newall and Petrovic 2004) . This method has the advantage, over other methods, to be simple and requires less implementation effort.

The remainder of this paper is organized as follows. The next section gives some notations and definitions related to the studied multi-mission series-parallel system. Section 3 addresses the proposed selective maintenance optimization model and the problem formulation. The optimization method is presented in Section 4, and an application example with numerical results are given in Section 5. Conclusion is drawn in Section 6.

2 MULTI-MISSION SERIES-PARALLEL SYSTEM DESCRIPTION

Consider a series-parallel system $\mathcal{S}$ composed of n subsystems S_i $(i = 1, \ldots, n)$. Each subsystem S_i

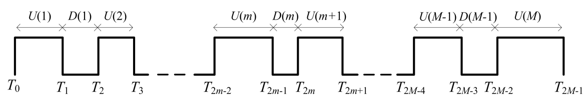

Figure 1: Mission and break durations profile in the planning horizon.

is composed of N_i independent, and possibly, non-identical components C_{ij} $(j = 1, \ldots, N_i)$. Components, subsystems and the system are assumed to experience only two possible states, namely functioning state and failure state.

Assume that the system is initially new and required to perform a sequence of M missions each with known duration $U(m)$, $m = 1, \ldots, M$. Between two successive missions there are breaks of known length of time $D(m, m+1)$ for $m = 1, \ldots, M-1$. Namely, the system operates according to two successive states: *Up state* $\longrightarrow$ *Down state* $\longrightarrow$ *Up state* ....In the *Up state* the system is operating while in the *Down state* the system is not operating, but available for any maintenance actions. Such a scenario may arise for systems that operate for some time per a day and then put into the down state for the rest of the day. According to Figure **??**, durations $U(m)$ and $D(m, m+1)$ are given by the following equations:

$$U(m) = T_{2m-1} - T_{2m-2}; \quad m = 1, \ldots, M; \tag{1}$$
$$D(m, m+1) = T_{2m} - T_{2m-1}; \quad m = 1, \ldots, M-1, \tag{2}$$

where $T_0 = 0$.

Let $A_{ij}(m)$ and $B_{ij}(m)$ be the ages of component C_{ij}, respectively, at the beginning and at the end of a given mission m $(m = 1, \ldots, M)$. Clearly, one may write $B_{ij}(m)$ as:

$$B_{ij}(m) = A_{ij}(m) + U(m). \tag{3}$$

If X_{ij} denotes the lifetime of component C_{ij}, then the reliability $R_{ij}(m)$ of component C_{ij} to survive mission m is given such that:

$$\begin{aligned} \mathcal{R}_{ij}(m) &= \Pr(X_{ij} > B_{ij}(m) \,|X_{ij} > A_{ij}(m)) \\ &= \frac{\Pr(X_{ij} > B_{ij}(m))}{\Pr(X_{ij} > A_{ij}(m))} \\ &= \frac{R(B_{ij}(m))}{R(A_{ij}(m))}, \end{aligned} \tag{4}$$

where R is the survival time distribution function of the random variable X_{ij}.

If component C_{ij} is characterized by its corresponding hazard function $h(t)$, then the conditional reliability $\mathcal{R}_{ij}(m)$ can be written as:

$$\mathcal{R}_{ij}(m) = \exp\left(\int_0^{A_{ij}(m)} h_{ij}(t)dt - \int_0^{B_{ij}(m)} h_{ij}(t)dt\right) \tag{5}$$
$$= \exp\left(H_{ij}(A_{ij}(m)) - H_{ij}(B_{ij}(m))\right),$$

where $H_{ij}(t) = \int_0^t h_{ij}(x)dx$ is the cumulated hazard function of component C_{ij}.

From the above equation, it follows that the reliability of subsystem S_i and that of the system $\mathcal{S}$ are respectively denoted by $\mathcal{R}_i(m)$ and $\mathcal{R}(m)$ and given as:

$$\mathcal{R}_i(m) = 1 - \prod_{j=1}^{N_i} (1 - \mathcal{R}_{ij}(m)), \text{ and} \tag{6}$$
$$\mathcal{R}(m) = \prod_{i=1}^{n} \mathcal{R}_i(m). \tag{7}$$

3 SELECTIVE MAINTENANCE MODEL AND PROBLEM FORMULATION

In this paper two types of maintenance are considered, namely corrective maintenance (CM) and preventive maintenance (PM). CM by means of minimal repair is carried out upon components failures during a given mission while PM is a planned activity conducted at the end of missions (i.e. within breaks) to improve the overall system mission reliability. It is assumed that component failure is operational dependent and the time in which a given component undergoes minimal repair is negligible if compared to the mission duration.

Each component C_{ij} of the system is characterized by its hazard function $h_{ij}(t)$ and its minimal repair cost cmr_{ij}. The preventive maintenance model is given on the basis of the age reduction concept initially introduced by Nakagawa (Nakagawa 1988). According to this concept, the age of a given component is reduced when PM action is performed on this component. In this paper, the vector $\mathbf{VPM} = [a_1, \ldots, a_p, \ldots, a_P]$ represents the P PM actions available for a given multi-mission system. For each PM action a_p $(p = 1, \ldots, P)$ is assigned the cost $cpm(a_p)$ and the time duration $dpm(a_p)$ of its implementation, the age reduction coefficient $\alpha(a_p) \in [0, 1]$ and the set $Comp(a_p)$ of components that may undergoes PM action a_p. Regarding the values taken by a given age reduction coefficient $\alpha(a_p)$, two particular cases may be distinguished. The first case corresponds to $\alpha(a_p) = 1$ which means that the PM action a_p has no effect on the component age (the component status becomes *as bad as old*), while the second case is $\alpha(a_p) = 0$ and corresponds to the case where the component age is reset to the null value (i.e. the component status becomes *as good as new*).

Selective maintenance model attempts to specify a PM action that should be performed, on which component and at the end of which mission it has to be performed. To construct such a model, the following decision variable is introduced:

$$a_p(C_{ij}, m) = \begin{cases} 1 \text{ if component } C_{ij} \text{ undergoes} \\ \quad \text{PM } a_p \text{ at the end of} \\ \quad \text{mission } m, \\ 0 \text{ otherwise.} \end{cases} \quad (m = 1, \dots, M-1) \tag{8}$$

In this paper, the selective maintenance problem consists in finding an optimal sequence of maintenance actions the cost of which minimizes the total maintenance cost while providing the desired system reliability level for each mission. The total maintenance cost is composed of minimal repair cost $CMR_{ij}(m)$ induced by the repair of each component C_{ij} during each mission m, and the preventive maintenance cost $CPM_{ij}(m)$ of each component C_{ij} that undergoes preventive maintenance at the end of mission m.

The cost induced by minimal repairs is function of components failure rates. Following the work of Boland (Boland 1982), for a given component C_{ij}, the expected minimal repair cost in an interval $[0, t]$ is:

$$cmr_{ij} \int_0^t h_{ij}(x)dx. \tag{9}$$

According to the above equation, the minimal repair cost $CMR_{ij}(m)$ induced by component C_{ij} during mission m is such that:

$$CMR_{ij}(m) = cmr_{ij} \int_{A_{ij}(m)}^{B_{ij}(m)} h_{ij}(x)dx, \tag{10}$$

where $A_{ij}(m)$ and $B_{ij}(m)$ represent the ages of component C_{ij}, respectively, at the beginning and at the end of a given mission m $(m = 1, \dots, M)$ and $A_{ij}(1) = 0$ by definition. If component C_{ij} undergoes preventive maintenance action a_p $(p = 1 \dots, P)$ at the end of mission m, then the value of the component age $B_{ij}(m)$ is reduced by the age reduction coefficient $\alpha(a_p)$. In this case, the minimal repair cost $CMR_{ij}(m)$ assigned to C_{ij} becomes:

$$CMR_{ij}(m) = cmr_{ij} \left(\int_{A_{ij}(m)}^{g(\alpha(a_p)) \times B_{ij}(m)} h_{ij}(x)dx \right), \tag{11}$$

where the function g is related to the value taken by the decision variable $a_p(C_{ij}, m)$ and defined to be such that:

$$g(\alpha(a_p)) = \begin{cases} \alpha(a_p) \text{ if } a_p(C_{ij}, m) = 1, \\ 1 \text{ otherwise.} \end{cases} \tag{12}$$

According to the above equation, the total minimal repair cost CMR_{ij} assigned to C_{ij} which undergoes preventive maintenance actions at the end of missions $1, \dots, M-1$ is given such that:

$$CMR_{ij} = cmr_{ij} \int_{A_{ij}(M)}^{B_{ij}(M)} h_{ij}(x)dx + cmr_{ij} \sum_{p=1}^{P} \sum_{m=1}^{M-1} \left(\int_{A_{ij}(m)}^{g(\alpha(a_p)) \times B_{ij}(m)} h_{ij}(x)dx \right). \tag{13}$$

By using components accumulated hazard rates, Equation (13) may be written as:

$$CMR_{ij} = cmr_{ij} \left(\Delta H_{ij}(M + \sum_{p=1}^{P} \sum_{m=1}^{M-1} \Delta H_{ij}(m, p) \right), \tag{14}$$

where $\Delta H_{ij}(M) = H_{ij}(B_{ij}(M)) - H_{ij}(A_{ij}(M))$ and $\Delta H_{ij}(m, p) = H_{ij}(g(\alpha(a_p)) \times B_{ij}(m)) - H_{ij}(A_{ij}(m))$.

From Equation (14), it follows that the total cost CMR of minimal repair, induced by all components during missions, is given by:

$$CMR = \sum_{i=1}^{n} \sum_{j=1}^{N_i} CMR_{ij}. \tag{15}$$

The total preventive maintenance cost CPM_{ij} assigned to component C_{ij}, which undergoes preventive maintenance actions at the end of missions, is given by:

$$CPM_{ij} = \sum_{p=1}^{P} \sum_{m=1}^{M-1} cpm(a_p) \times a_p(C_{ij}, m). \tag{16}$$

It follows, from the above equation, that the total preventive maintenance cost CPM induced by all system components is:

$$CPM = \sum_{i=1}^{n} \sum_{j=1}^{N_i} CPM_{ij}. \tag{17}$$

Finally, the total maintenance cost C_{total} to be minimized is given from Equations (15) and (17) such that:

$$C_{total} = CMR + CPM. \tag{18}$$

To complete the selective maintenance optimization problem, let note that, due to the limited time (break) between missions, it may be not possible that all preventive maintenance actions be performed at the end of a given mission. Therefore, time between missions should be taken into account as an operation constraint. The total duration $DPM(m)$ spent by preventive maintenance actions at the end of a given mission m is given by the following formula:

$$DPM(m) = \sum_{p=1}^{P} \sum_{i=1}^{n} \sum_{j=1}^{N_i} dpm(a_p) \times a_p(C_{ij}, m). \tag{19}$$

Let $\mathcal{R}_0$ denote the required reliability level of the system at the beginning of each mission m ($m = 2, \ldots, M$). The selective maintenance problem is then formulated as follows: from the vector **VPM** find the optimal sequence of PM actions which minimizes the total maintenance cost C_{total} while providing the desired reliability level $\mathcal{R}_0$. To derive the mathematical programming model corresponding to such a problem, let the vector $\mathbf{S} = [s_1, \ldots, s_K]$ be the sequence of PM actions performed so that to keep the system reliability at the desired level $\mathcal{R}_0$. Roughly speaking, the vector $\mathbf{S}$ is of dimension $K \leqslant P$ and composed of elements of the vector **VPM**. At the end of a given mission, if preventive maintenance is required, then the first PM action to be performed corresponds to the first element s_1 of $\mathbf{S}$. Whenever, action s_1 is not sufficient to guaranty the system reliability level, in this case PM actions s_1 and s_2 should be performed simultaneously, and so on. Therefore, if we assume that the system has just achieved the first mission and will operate the remaining missions, the mathematical programming model corresponding to the selective maintenance optimization problem is:

$$\text{Minimize } C_{total}(\mathbf{S}) = CMR(\mathbf{S}) + CPM(\mathbf{S}), \tag{20}$$

Subject to:

$$\mathcal{R}(m+1) \geqslant \mathcal{R}_0, \tag{21}$$

$$DPM(m) \leqslant D(m, m+1), \tag{22}$$

$$\sum_{p=s_1}^{s_K} a_p(C_{ij}, m) \leqslant 1, \tag{23}$$

$$a_p(C_{ij}, m) \in \{0, 1\}, \tag{24}$$

$$i = 1, \ldots, n;\ j = 1, \ldots, N_i;\ p = s_1, \ldots, s_K, \tag{25}$$

$$m = 1, \ldots, M-1; \text{ and } K \leqslant P. \tag{26}$$

where constraint (22) stands that PM actions undertaken at the end of a given mission should be completed within the allotted time, constraint (23) imposes the fact that each component may receive almost one PM action at the end of each mission, while constraint(24) is a $\{0, 1\}$-integrality constraint.

```
1. Set the initial solution s
2. Set the value of ΔB
3. Calculate the objective function f(s)
4. Set B = f(s)
5. While the stopping criteria is not satisfied do
    Randomly select a feasible solution s* ∈ N(s)
    If f(s) ≥ f(s*) or f(s) ≤ B, then accept accept s*
      and set s = s*
    Set B = B − ΔB.
```

Figure 2: An overview of the extended great deluge algorithm.

4 OPTIMIZATION METHOD

4.1 The extended great deluge

The extended great deluge is a local search metaheuristic recently introduced by Burke *et al.* (Burke et al. 2004). Like other local search methods, the extended great deluge iteratively repeats the replacement of a current solution s by a new one s^*, until some stopping condition has been satisfied. The new solution is selected from a neighborhood $\mathcal{N}(s)$. The mechanism of accepting or rejecting the candidate solution from the neighborhood is different of other methods. In extended great deluge approach, the algorithm accepts every solution whose objective function is more or equal (for the maximization problems) to the upper limit $\mathcal{B}$, which is monotonically increased during the search by $\Delta\mathcal{B}$. Figure 2 presents an overview of the extended great deluge algorithm (Burke et al. 2004).

The extended great deluge algorithm is an extension of the *great deluge* method which was introduced as an alternative to simulated annealing. Extended great deluge and simulated annealing algorithms share the characteristic that they may both accept worse candidate solutions than the current one. The difference is in the acceptance criterion of worse solutions. The simulated annealing method accepts configurations which deteriorate the objective function only with a certain probability. The extended great deluge algorithm incorporates both the worse solution acceptance (of the "great deluge" algorithm) if the solution fitness is less than or equal to some given upper limit $\mathcal{B}$, i.e. ($f(s^*) \geq \mathcal{B}$), and the well-known hill climbing rule ($f(s^*) \geq f(s)$). The introduction of the dynamic parameter has an important effect on the search. As explained in (Burke et al. 2004), the decreasing of $\mathcal{B}$ may be seen as a control process, which drives the search towards a desirable solution. Note finally that extended great deluge algorithm has the advantage to require only one parameter ($\Delta\mathcal{B}$) to be tuned.

4.2 Extended great deluge for the selective maintenance optimization problem

In this paper, the extended great deluge algorithm is used as an optimization technique to solve the selective maintenance optimization problem. The solution representation is inspired from that of (Levitin and Lisnianski 2000) (see also (Nahas, Khatab, Ait-Kadi and Nourelfath 2007)). The element of the vector **VPM** of available PM actions are numbered from 1 to P. The maintenance plan, as a solution, is represented by a vector $\mathbf{S} = [s_1, \ldots, s_K]$ with finite length $K \leq P$ and where $s_p \in \{1, 2, \ldots, P\}$, for $p = 1, \ldots, K$. The length of a given solution depends on its feasibility. The initial feasible solution is derived on the basis of the following procedure (Nahas et al. 2007).

4.2.1 *Initial solution construction*

1. Set the length of **S** to a constant number $K_{\max}$
2. Generate the elements of **S** from a random permutation of the set $\{1, \ldots, P\}$
3. Set $K = 1$
4. Calculate the objective function and the constraint values by using the K first elements (i.e. PM actions) of **S**
5. if ($K = K_{\max}$) and (**S** is not feasible) then return to step 2
6. If ($K < K_{\max}$) and (**S** is not feasible) then set $K = K + 1$ and proceed from step 4

To define the appropriate neighborhood, several structures were investigated. The following procedure provides the neighbor solution.

4.2.2 *Neighboring solution construction*

1. Generate randomly a number x from the interval $[0, 1]$
2. If ($x \geq 0.5$) then choose randomly two elements $\mathbf{S}(i)$ and $\mathbf{S}(j)$ such that $1 \leq i \leq K$ and $K+1 \leq j \leq K_{\max}$; and exchange the contents of $\mathbf{S}(i)$ and $\mathbf{S}(j)$.
3. If ($x < 0.5$) then choose randomly two elements $\mathbf{S}(i)$ and $\mathbf{S}(j)$ with $1 \leq i \leq K$ and $K_{\max} + 1 \leq j \leq P$, and exchange the contents of $\mathbf{S}(i)$ and $\mathbf{S}(j)$.

It is worth noticing that in order to ensure the feasibility of a given solution, one needs to evaluate the objective function and the constraints of the optimization problem. To this end, a procedure is developed for the feasibility test of a given solution vector $\mathbf{S} = [s_1, \ldots, s_K]$.

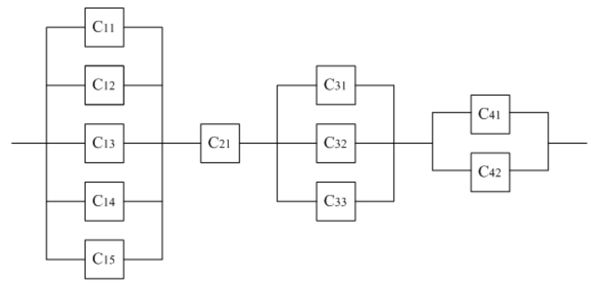

Figure 3: The series-parallel system for the application example.

5 APPLICATION EXAMPLE

The test problem used in this paper is based on a series-parallel system composed of $n = 4$ subsystems S_i ($i = 1, \ldots, 4$) as shown in Figure 3. Subsystems S_1, S_2, S_3 and S_4 are, respectively, composed of $N_1 = 5$, $N_2 = 1$, $N_3 = 3$ and $N_4 = 2$ components. The reliability function of each component is given by a Weibull hazard function $h(t) = \beta^\gamma \gamma (t)^{\gamma - 1}$, where β and γ are, respectively, the scale and the shape parameters. The accumulated hazard function is then $H(t) = (\beta t)^\gamma$.

For each component C_{ij} ($i = 1, \ldots, 4; j = 1, \ldots, N_i$), the scale β_{ij} and shape γ_{ij} parameters and minimal repair cost cmr_{ij}, are reported in Table 1. The vector **VPM** of available PM actions each with its corresponding characteristics is given by Table 2. This vector is borrowed from (Levitin and Lisnianski 2000) where we have add the duration taken by each PM action according to the following rule:

$$dpm(a_p) = \frac{0.1 \times cpm(a_p)}{1 + \alpha(a_p)}, \; p = 1, \ldots, P. \tag{27}$$

Component C_{ij}	β_{ij}	γ_{ij}	cmr_{ij}
C_{11}	0.01	1.98	0.9
C_{12}	0.01	1.98	0.9
C_{13}	0.01	1.98	0.9
C_{14}	0.014	1.32	0.8
C_{15}	0.002	1.65	0.5
C_{21}	0.002	1.98	2.4
C_{31}	0.004	1.98	1.3
C_{32}	0.0016	2.20	0.4
C_{33}	0.0040	2.31	0.7
C_{41}	0.0068	1.76	1.2
C_{42}	0.0016	2.09	1.9

Table 1: Parameters of system components

PM a_p	$Comp(a_p)$	$\alpha(a_p)$	$cmp(a_p)$	$dpm(a_p)$
1	C_{11}	1.00	2.2	0.11
2	C_{11}	0.56	2.9	0.19
3	C_{11}	0.00	4.1	0.41
4	C_{12}	1.00	2.2	0.11
5	C_{12}	0.56	2.9	0.19
6	C_{12}	0.00	4.1	0.41
7	C_{13}	1.00	2.2	0.11
8	C_{13}	0.56	2.9	0.19
9	C_{13}	0.00	4.1	0.41
10	C_{14}	0.76	3.7	0.21
11	C_{14}	0.00	5.5	0.55
12	C_{15}	1.00	7.3	0.37
13	C_{15}	0.60	9.0	0.56
14	C_{15}	0.00	14.2	1.42
15	C_{21}	0.56	15.3	0.98
16	C_{21}	0.00	19.0	1.90
17	C_{31}	0.75	4.3	0.25
18	C_{31}	0.00	6.5	0.65
19	C_{32}	0.80	5.0	0.28
20	C_{32}	0.00	6.2	0.62
21	C_{33}	1.00	3.0	0.15
22	C_{33}	0.65	3.8	0.23
23	C_{33}	0.00	5.4	0.54
24	C_{41}	1.00	8.5	0.43
25	C_{41}	0.70	10.5	0.62
26	C_{41}	0.00	14.0	1.40
27	C_{42}	1.00	8.5	0.43
28	C_{42}	0.56	12.0	0.77
29	C_{42}	0.00	14.0	1.40

Table 2: Parameters of preventive maintenance actions

Vectors **U** and **D** corresponding, respectively, to durations of missions and breaks are given in Table 3, while for each mission the required reliability level is fixed to $\mathcal{R}_0 = 0.95$.

Mission m	$U(m)$	$D(m, m+1)$
1	52	5
2	48	6
3	48	6
4	40	7
5	52	8
6	52	9
7	52	5
8	44	9
9	52	8
10	48	9
11	44	5
12	48	7
13	44	4
14	52	10
15	44	9
16	52	5
17	40	8
18	40	7
19	52	8
20	44	–

Table 3: Durations of missions and breaks between successive missions

The proposed algorithm is implemented by using MATLAB software tool on a 1.86 GHz Intel Core Duo processor. It was tested for several values of the parameter $\Delta\mathcal{B}$. The most appropriate value is found to be $\Delta\mathcal{B} = 0.01$. The length assigned to the solution vector **S** is set to $K_{\max} = 15$, while the stopping criterion is reached when either the number of iterations (i.e. number of evaluated solutions) is equal to 30000 or the value of $\mathcal{B}$ is less than or equal to $0.9 \times C^*_{total}$, where C^*_{total} stands for the current best total maintenance cost.

The best selective maintenance plan obtained for the mission reliability level $\mathcal{R}_0 = 0.95$ is presented in Table 4. According to the available PM actions given by Table 2, Table 4 gives components and the time at which they should receive a specific type of PM action so that to ensure the system reliability level $\mathcal{R}_0$ for each mission. Figure 4 presents the system reliability versus the number of missions. The best total maintenance cost C^*_{total}, induced by the obtained selective maintenance plan and minimal repairs, is $C^*_{total} = 460.4944$, while the execution time is about 197 sec .

End of mission m	PM action a_p	Component C_{ij}
2	$29, 16$	C_{42}, C_{21}
4	$29, 24, 16, 6$	$C_{42}, C_{41}, C_{21}, C_{12}$
5	$6, 24, 29, 23, 16$	$C_{12}, C_{41}, C_{42}, C_{33}, C_{21}$
6	$9, 29, 16$	C_{13}, C_{42}, C_{21}
7	$24, 9$	C_{41}, C_{13}
8	$16, 6, 29$	C_{21}, C_{12}, C_{42}
9	$24, 29, 9, 23$	$C_{41}, C_{42}, C_{13}, C_{33}$
10	$6, 16$	C_{12}, C_{21}
11	$9, 6, 29$	C_{13}, C_{12}, C_{42}
12	$23, 6, 16$	C_{31}, C_{12}, C_{21}
13	$16, 29, 9$	C_{21}, C_{42}, C_{13}
14	$24, 16, 9$	C_{41}, C_{21}, C_{13}
15	$9, 6, 23, 16$	$C_{13}, C_{12}, C_{33}, C_{21}$
16	$6, 29$	C_{12}, C_{42}
17	$6, 16$	C_{12}, C_{21}
18	$23, 29, 16, 24, 9$	$C_{33}, C_{42}, C_{21}, C_{41}, C_{13}$
19	$6, 29, 9$	C_{21}, C_{42}, C_{13}

Table 4: The best selective maintenance plan obtained for the required reliability level 0.95

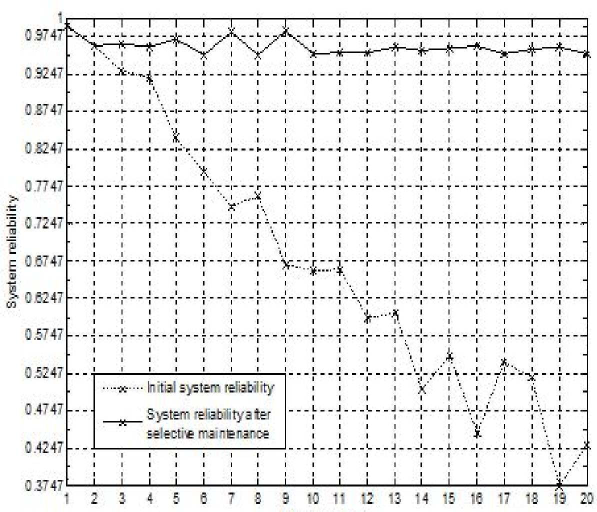

Figure 4: Mission reliability of the system in the planning horizon.

6 CONCLUSION

In this paper, we proposed a selective maintenance optimization model for a multi-mission series-parallel system. Lifetime of each system component is generally distributed. The system operates on a planning horizon composed of several missions such that between successive missions a break of finite length is allotted to perform maintenance actions. Missions as well as breaks are of possibly different durations, and during breaks a list of preventive maintenance actions are available for system components maintenance. A combinatorial optimization problem is formulated the objective of which consists in finding, during the planning horizon, an optimal sequence of preventive maintenance actions to be performed so that to minimize the total maintenance cost while providing, for each mission, the desired system reliability level. To solve this problem, an optimization method is proposed on the basis of the extended great deluge algorithm. The advantage of this method is that it requires the setting of only one parameter while it provides results within a good reasonable execution time.

Future research should address the performance of the extended great deluge algorithm as compared to that of other heuristic techniques, such as simulated annealing, genetic, tabu search, to name a few. Such a comparison should be made only on the basis of the number of evaluated solutions and the solutions quality rather than computational time. Indeed, such a comparison is more viable since it is very difficult to compare computation times using different computing systems, programming language compilers, coding techniques, etc.

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