

AVAILABILITY OPTIMIZATION FOR RANDOMLY FAILING SYSTEMS SUBJECTED TO IMPERFECT MAINTENANCE

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ABSTRACT: *This paper deals with the modeling of a preventive maintenance strategy applied to a system subject to random failures. According to this policy, the system is subjected to a perfect preventive maintenance action at times kT ($k = 1, 2, 3, \dots$). If a failure occurs within a cycle $](k-1)T, kT[$ ($k = 1, 2, 3, \dots$) an imperfect corrective maintenance action is undertaken. Considering the average durations of the maintenance actions and the inefficiency extent of the corrective actions, a mathematical model is developed in order to study the evolution of the system stationary availability according to the period of preventive maintenance, T , and to determine the optimal period T^* which maximizes the system availability. The modeling of the imperfection of the corrective maintenance actions requires the knowledge of the quasi renewal function. A new expression approximating this function is proposed for the case of systems whose times to first failure follow a Gamma distribution. Numerical results are obtained and discussed.*

KEYWORDS: *Maintenance strategies, Imperfect maintenance, Quasi-renewal Process, Gamma Distribution.*

1. INTRODUCTION

In the current context of an increasingly wild competition, the constraints of time, quality and cost imposed on the industrial companies put them in front of the obligation to ensure a maximum availability of their production equipment at the lowest cost. Under these conditions, the implementation of preventive maintenance strategies proves to be inevitable.

A maintenance strategy is defined as a decision rule which establishes the sequence of actions to be undertaken according to the state of the system. With each maintenance action one associates, a cost, a duration and a certain quality or effectiveness. The performance of a strategy is then evaluated in terms of the average total cost on a given horizon or in terms of the system stationary availability. Other performance criteria are proposed in the literature.

A great number of publications propose several maintenance policies, implying different types of preventive and corrective actions, such as inspections, replacements by new or used identical equipment, overhauls or repair to the as good as new state, the minimal repair which consists in bringing the system back to the same operating state as just before failure (as bad as old state), etc.

Basic maintenance strategies have been proposed in (Barlow and Proschan, 1965). A great number of publications on the subject followed with maintenance policies based on complex mathematical models using

the renewal theory and many other stochastic processes. There are several reviews of the literature on the subject summarizing the various models and gathering them in various classes. Among these reviews, we find those of (Valdez-Flora et al., 1989), and (Wang, 2002).

Various maintenance models consider that maintenance actions are perfectly executed. Actually, the effectiveness of maintenance actions is generally between the two extreme limits ('as good as new' and 'as bad as old'), what is generally called imperfect maintenance. Several models of imperfect maintenance were proposed in the literature. They can be classified in two categories: models based on an arithmetic reduction of the age of the system (Malik, 1979), (Kijima et al., 1988), (Kijima, 1989), and models of reduction of the intensity function (Chan et al., 1993), (Shin I. et al. 1996).

With regard to the reduction of age, the improvement of the state of the system, following a maintenance action, is equivalent to an arithmetic reduction of its age. For the models of reduction of the intensity of failures, the improvement of the state of the system, following a maintenance action, is equivalent to a reduction of its failure rate of a quantity proportional to its value right before maintenance. A literature review on imperfect maintenance can be found in (Pham and al., 1996). Let's also quote a recent work of (Shey-Huei and al. 2005) on optimal maintenance strategies with imperfect maintenance actions.

In this work, we consider an industrial context where the planned preventive maintenance actions are generally carried out in a perfect way; they are generally allocated the most qualified resources, the adequate tools and original spare parts. In opposition to the corrective actions which must generally be carried out in emergency following unexpected failures. We do not always find the best technicians nor the most suitable tools or spare parts to carry out these repairs. Hence, they are generally imperfect repairs. We consider in this context a periodic preventive maintenance strategy applied to a system prone to random failures. This strategy consists in subjecting the system to an action of perfect preventive maintenance at instants kT ($k = 1,2,3,\dots$). If the system fails between consecutive preventive actions, an imperfect repair is undertaken. Considering the average durations of the maintenance actions, as well as the imperfect nature of the repairs following failures, we study the evolution of the system stationary availability according to the preventive maintenance period, T , and we determine the optimal period T^* which maximizes the system availability. In order to model the imperfection or the inefficiency of the repair actions, we use the imperfect maintenance model developed by (Wang and Pham., 1996, 1997) which is based on the quasi-renewal process.

In next section we define the strategy and the working assumptions, and we establish the expression of the system stationary availability under the considered maintenance policy. This expression calls upon the quasi-renewal function whose analytical expression is very difficult even impossible to obtain. In section 3 we will remind the general formulation of the quasi-renewal function enunciated by (Wang and Pham., 1996). We will then derive a new expression of this quasi-renewal function in the particular case of a Gamma distribution and we will discuss the related numerical calculations. Section 4 will be dedicated to the study of the evolution of the system stationary availability when submitted to the suggested maintenance policy. We will establish the conditions of existence of an optimal strategy. A numerical example will be treated in section 5. The obtained results will be presented and interpreted. The paper is concluded in section 6.

2. STRATEGY DEFINITION AND MATHEMATICAL MODEL

We consider a randomly failing system with a known lifetime probability distribution. A perfect preventive maintenance action (overhaul or replacement by a new identical one) is periodically performed on the system at instants kT ($k = 1,2,3,\dots$). If it fails between successive preventive maintenance actions, it undergoes an imperfect repair action at the end of which the system lifetime is reduced by a factor 'a' between 0 and 1. This factor 'a' is considered as a characteristic of the corrective maintenance action efficiency. The extreme cases represent a minimal repair for $a = 0$, and a perfect

repair bringing the system at the as good as new state for $a = 1$.

The reduction factor 'a' can be deterministic or random. We consider in this study the deterministic case. The preventive and corrective maintenance actions have respectively constant average durations T_p and T_c . Under these conditions, the system stationary availability is expressed as follows:

$$\left\{ \begin{array}{ll} SA(T) = 1 - \frac{T_p + T_c \cdot Q(T)}{T} & \text{for } T \geq T_p \\ SA(T) = 0 & \text{for } T \leq T_p \end{array} \right\} \quad (1)$$

$Q(T)$ stands for the average number of system restarts following failure during period T . It is called the quasi-renewal function. It has been studied by (Wang and Pham., 1996) in the context of a quasi-renewal process. In order to study the system stationary availability given by equation (1), we will consider in next section the quasi renewal-process as defined by (Wang and Pham., 1996). We will derive a new expression approximating the quasi-renewal function in the particular case of a system whose time to first failure follows a Gamma distribution.

3. THE QUASI-RENEWAL PROCESS

3.1. Definition

(Wang and Pham., 1996, 1997) define the quasi-renewal process as follows:

Observe the sequence of non-negative random variables $\{T_1, T_2, \dots, T_n\}$, the counting process $\{N(t), t > 0\}$ is said to be a quasi-renewal process with parameter a and the first inter-arrival time T_1 , if $T_1 = Z_1$; $T_2 = aZ_2$; $T_3 = a^2Z_3$; ...; $T_n = a^{n-1}Z_n$, where the Z_i are independent and identically distributed and $a > 0$ is a constant.

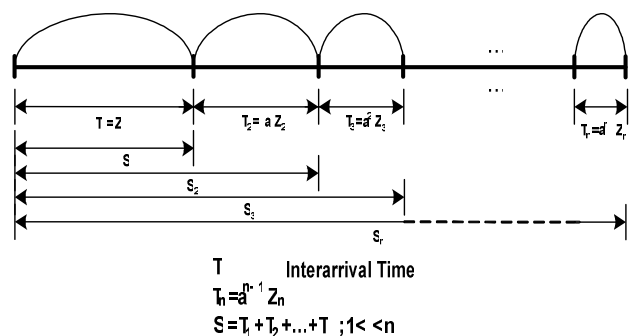


Figure 1. The quasi-renewal process model

Since we consider imperfect repair throughout the paper, the constant 'a' will be taken between 0 and 1 as previously mentioned in section 2.

3.2. The quasi-renewal function $Q(t)$

The quasi-renewal function represents the average number of the system restarts following a repair during a time interval t . Considering $F_{T_i}(\cdot)$ as the probability distribution of the system times to the i^{th} failure T_i ($i=1, 2, \dots, n$), (Wang and Pham. 1996) expressed the quasi-renewal function as follows:

$$Q(t) = \sum_{n=1}^{\infty} \underset{i=1}{\overset{n}{Y}} F_{T_i}(t) \quad (2)$$

where $\underset{i=1}{\overset{n}{Y}} F_{T_i}(t)$ stands for the convolution Product of

$$F_{T_i}(\cdot) \text{ n times and } F_{T_i}(t) = F_{T_i}\left(\frac{1}{a^{i-1}}t\right)$$

It clearly appears that the exact analytical formulation of the quasi-renewal function is very difficult even impossible to obtain. (Rehmer, 2000) uses Laplace transforms to compute an estimate of the quasi-renewal function for the Exponential and Gamma distributions. In the case of a Normal distribution $F(\cdot)$ of the time to fist failure with mean μ and standard deviation σ , the quasi-renewal function can be calculated relatively straightforward (Rehmer, 2000):

$$Q(t) = \sum_{n=1}^{\infty} F\left(\frac{1-a^n}{1-a}\mu, \frac{1-a^{2n}}{1-a^2}\sigma^2\right); 0 < a < 1 \quad (3)$$

3.2.1 The case of Gamma distribution

We develop in what follows a new expression which approximates the quasi-renewal function for systems whose time to first failure follows a Gamma distribution with shape parameter α and scale parameter β .

$$f_{T_i}(t) = \frac{1}{\beta^\alpha \Gamma(\alpha)} t^{\alpha-1} \exp\left(-\frac{t}{\beta}\right) \quad (4)$$

$$\text{with } \Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x} dx$$

The quasi-renewal function associated with this system is given by:

$$Q(t) = \sum_{n=1}^{\infty} n \Pr[N(t) = n] \quad (5)$$

$$\text{Given that } S_n = \sum_{i=1}^n T_i = Z_1 + a Z_2 + a^2 Z_3 + \dots + a^{n-1} Z_n$$

(see figure 1), we have:

$$\Pr[N(t) = n] = \Pr[S_{n+1} > t] - \Pr[S_n > t]$$

Let $G_{S_n}(t)$ be the probability distribution function associated to the random variable S_n .

Hence,

$$\Pr[N(t) = n] = [1 - G_{S_{n+1}}(t)] - [1 - G_{S_n}(t)]$$

$$\Pr[N(t) = n] = G_{S_n}(t) - G_{S_{n+1}}(t) \quad (6)$$

Let's find an approximation of $G_{S_n}(t)$.

In the general case, (Wang et Pham. 1996) showed that:

$$f_{T_i}(t) = \frac{1}{a^{i-1}} f_{T_1}\left(\frac{t}{a^{i-1}}\right) \text{ for } i=1,2,\dots,n$$

Thus, if the system times to first failure T_i follow a Gamma distribution: $\text{Gamma}(\alpha, \beta)$, it is clear that the times T_i to the i^{th} failure follow a Gamma distribution: $\text{Gamma}(\alpha, \beta a^{i-1})$.

Given that the Z_{is} are independent, S_n is the sum of n independent T_{is} , which means the sum of n independent random variables respectively distributed according to the Gamma law: $\text{Gamma}(\alpha, \beta a^{i-1})$.

The distribution of this sum of random variables, each one distributed according to a Gamma law, can be validly approached by a Gamma distribution (see Morschopoulos, 1985 and Pierrat et al, 2006).

Let $G_{S_n}(\Phi, \Psi)$ be this distribution function of S_n : a Gamma distribution with shape parameter Φ and scale parameter Ψ .

We use in what follows the two first moments of $G_{S_n}(\Phi, \Psi)$ to determine its two parameters Φ et Ψ .

The first moment:

$$E[S_n] = E[T_1 + T_2 + T_3 + \dots + T_n]$$

$$E[S_n] = E[Z_1 + a Z_2 + a^2 Z_3 + \dots + a^{n-1} Z_n]$$

$$E[S_n] = E[Z_1] + E[a Z_2] + E[a^2 Z_3] + \dots + E[a^{n-1} Z_n]$$

$$E[S_n] = E[Z_1] + a E[Z_2] + a^2 E[Z_3] + \dots + a^{n-1} E[Z_n]$$

with $E[Z_i] = \alpha \beta$ for $i=1,2,\dots,n$ and $0 < a < 1$

$$E[S_n] = \alpha \beta + a \alpha \beta + a^2 \alpha \beta + \dots + a^{n-1} \alpha \beta = \frac{1-a^n}{1-a} \alpha \beta$$

Then we consider the variance:

$$\text{Var}[S_n] = \text{Var}[T_1 + T_2 + T_3 + \dots + T_n]$$

$$\text{Var}[S_n] = \text{Var}[Z_1 + a Z_2 + a^2 Z_3 + \dots + a^{n-1} Z_n]$$

The Z_{is} being independent, we can write:

$$\text{Var}[S_n] = \text{Var}[Z_1] + \text{Var}[a Z_2] + \dots + \text{Var}[a^{n-1} Z_n]$$

$$\text{Var}[S_n] = \text{Var}[Z_1] + a^2 \text{Var}[Z_2] + \dots + a^{2(n-1)} \text{Var}[Z_n]$$

with: $\text{Var}[Z_i] = \alpha \beta^2$ for $i=1,2,\dots,n$ and $0 < a < 1$

$$\text{Var}[S_n] = \alpha \beta^2 + a^2 \alpha \beta^2 + \dots + a^{2(n-1)} \alpha \beta^2 = \frac{1-a^{2n}}{1-a^2} \alpha \beta^2$$

Given, by definition of the Gamma distribution, that

$E[S_n] = \Psi \Phi$ et $\text{Var}[S_n] = \Psi^2 \Phi$, we obtain by identification :

$$\begin{cases} \Psi \Phi = \frac{1-a^n}{1-a} \alpha \beta \\ \Psi^2 \Phi = \frac{1-a^{2n}}{1-a^2} \alpha \beta^2 \end{cases}$$

Solving this system leads to the following result:

$$\Phi = \frac{1-a^n}{1-a} \frac{1+a}{1+a^n} \alpha \text{ et } \Psi = \frac{1+a^n}{1+a} \beta$$

The probability distribution associated with S_n is then a Gamma distribution defined as follows:

$$G_{S_n} \left(\frac{1-a^n}{1-a} \frac{1+a}{1+a^n} \alpha, \frac{1+a^n}{1+a} \beta \right); 0 < a < 1 \quad (7)$$

Hence, returning to equations (5) and (6), the quasi-renewal function in case the first time to failures follow a Gamma distribution can be expressed as follows:

$$Q(t) = \sum_{n=1}^{\infty} n \left[G_{S_n}(t) - G_{S_{n+1}}(t) \right]$$

$$Q(t) = \sum_{n=1}^{\infty} G_{S_n}(t) \quad (8)$$

with $G_{S_n}(t)$ being the Gamma distribution given by equation (7).

It can easily be shown that for the case of a perfect renewal process ($a = 1$) (Barlow and Proschan, 1965) with systems whose times to failure follow a Gamma distribution $G(\alpha, \beta)$, the renewal function is given by:

$$M(t) = \sum_{n=1}^{\infty} \text{Gamma}(n\alpha, \beta) \quad (9)$$

The formulation of the quasi-renewal function being a sum of distribution functions (equation 7), does not raise any special programming or computation problem. This in opposition to the formulation proposed by (Rehmert, 2000) which requires an inversion of a truncated infinite sum in the Laplace transform space which is very difficult and sometimes impossible to perform.

3.2.2. Calculation of the quasi-renewal function

For systems whose times to first failure are distributed according to Normal or Gamma distributions, the quasi-renewal function (equations 3 and 8) is an infinite sum of probability distribution functions. Each term of the sum represents the probability to have n imperfect repairs in a time interval $[0, t]$. The computer programming of these two quasi-renewal functions requires to truncate the corresponding sum at a certain level $n = c$. The choice of the value of c depends on the working horizon of interest t .

For illustration purpose, let us consider the case where the first time to failure follows a Normal distribution with mean $\mu = 100$ and standard deviation $\sigma = 20$, with a corrective maintenance action efficiency factor $a = 0.7$. Figure 2 shows the curves of the first three terms (distribution functions) of equation [3] as well as the curve of the sum of these three functions in case $c = 3$. Figure 3 illustrates the same thing for $c = 5$.

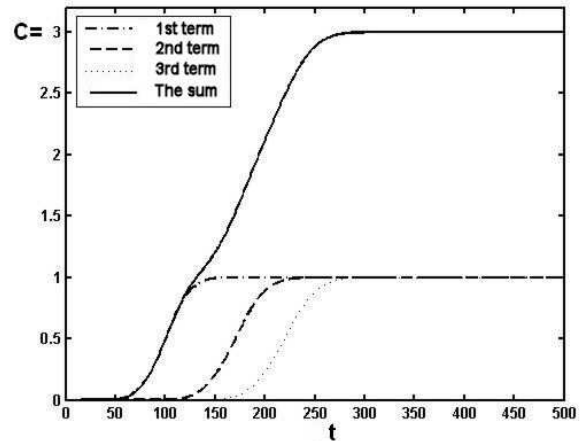


Figure 2. The terms of the quasi-renewal function (equation 3) and their sum for $c = 3$

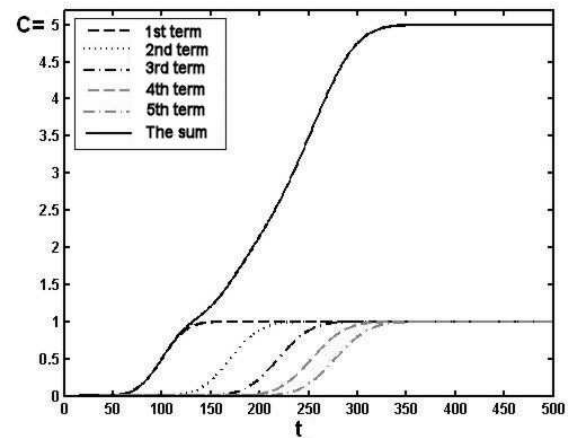


Figure 3. The terms of the quasi-renewal function (equation 3) and their sum for $c = 5$

Figure 4 shows the quasi-renewal function $Q(t)$ for the same example in cases $c = 1$; $c = 2$; $c = 3$ and $c = 4$

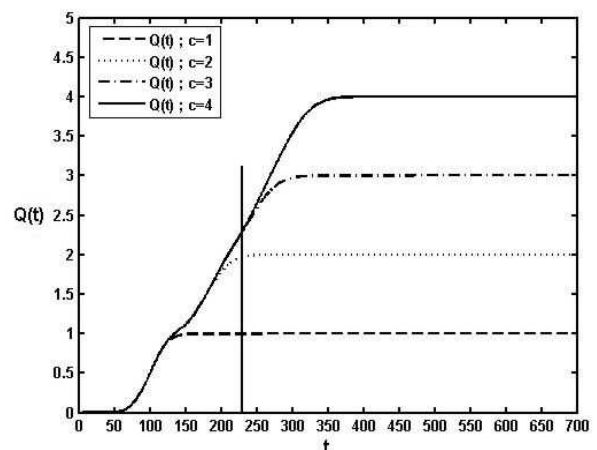


Figure 4. The quasi-renewal function for $c = 1$; $c = 2$; $c = 3$ and $c = 4$

It is clear that in case one is interested for example in the behaviour of the system (number of restarts) over a time horizon $T = 230$ time units, it would be necessary to

truncate the sum at $c \geq 3$. In fact, if the truncation is made at $c=1$ or $c=2$ one can not fully describe the behaviour of the quasi-renewal function on the considered time horizon $T = 230$ time units.

4. OPTIMIZATION OF THE SYSTEM STATIONARY AVAILABILITY FUNCTION SA(T)

Let us go back to the maintenance strategy defined in section 2. Consider a system whose time to first failure follows a probability distribution $F(t)$. This system is subjected to perfect preventive maintenance actions at instants kT ($k= 1,2,3,\dots$). If it fails between successive preventive maintenance actions, it undergoes an imperfect repair characterized by an efficiency indicator 'a' as previously defined in the context of a quasi-renewal process.

4.1. General behavior of the system stationary availability SA(T)

As formulated in section 2, the system stationary availability, $SA(T)$, under the considered maintenance policy is :

$$\begin{cases} SA(T) = 1 - \frac{T_p + T_c \cdot Q(T)}{T} & \text{for } T \geq T_p \\ SA(T) = 0 & \text{for } T \leq T_p \end{cases} \quad (1)$$

T_p and T_c being respectively the preventive and corrective maintenance actions average durations.

$Q(T)$ is a sum of continuous distribution functions, it is thus a continuous function. Hence, $SA(T)$ is a continuous function.

By definition, $SA(T_p) = 0$

Theorem:

In the case of a periodic preventive maintenance strategy with imperfect repair of non negligible duration at failure (Quasi-Renewal), the system stationary availability tends to 0 when the preventive maintenance period tends to infinity:

$$\lim_{T \rightarrow \infty} SA(T) = 0$$

Proof :

Figure 5 represents the quasi-renewal process with a constant non negligible corrective maintenance duration T_c .

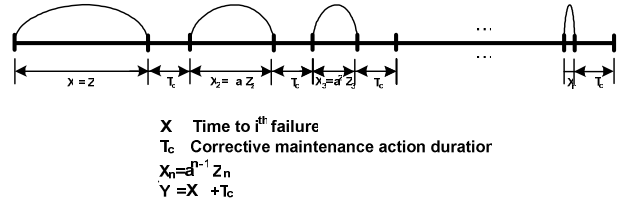


Figure 5. The quasi-renewal process model with constant non negligible corrective maintenance duration.

$$\lim_{T \rightarrow \infty} SA(T) = \lim_{T \rightarrow \infty} 1 - T_c \frac{Q(T)}{T}$$

The expression $\frac{Q(T)}{T}$ represents the average number of restarts per time unit in the time interval $[0,T]$. We can also consider this term as the inverse of the average period between successive restarts over the same interval $[0,T]$. Thus, according to the quasi-renewal process defined in section 3, and considering the following:

X_i : the system time to the i^{th} failure following a given probability distribution

Y_i : a random variable defined as the sum of X_i and T_c .

We have:

$$\lim_{T \rightarrow \infty} \frac{Q(T)}{T} = \lim_{n \rightarrow \infty} \frac{1}{\sum_{i=1}^n Y_i / n} = \lim_{n \rightarrow \infty} \frac{n}{\sum_{i=1}^n (X_i + T_c)}$$

$$\lim_{T \rightarrow \infty} \frac{Q(T)}{T} = \lim_{n \rightarrow \infty} \frac{n}{\sum_{i=1}^n X_i + n T_c}$$

$$\lim_{T \rightarrow \infty} \frac{Q(T)}{T} = \lim_{n \rightarrow \infty} \frac{n}{\sum_{i=1}^n a^{i-1} Z_i + n T_c}$$

With $\lim_{n \rightarrow \infty} a^{n-1} Z_n = 0$ for $0 < a < 1$ then $\sum_{i=1}^n a^{i-1} Z_i$ is finite.

$$\text{Hence, } \lim_{T \rightarrow \infty} \frac{Q(T)}{T} = \frac{1}{T_c}$$

$$\lim_{T \rightarrow \infty} SA(T) = \lim_{T \rightarrow \infty} 1 - T_c \frac{Q(T)}{T} = 0, \text{ for } 0 < a < 1$$

These calculated limits show that the system availability function $SA(T)$ admits necessarily at least a maximum (Curve 1 figure 6). This means that the considered maintenance policy admits in every situation (for any given set of input parameters: $F(\cdot)$, a , T_p and T_c) a finite optimal period T^* of

preventive maintenance which maximizes the system stationary availability. Curves 2 and 3, in figure 6, show both possible behaviors of the system stationary availability in the case of perfect corrective maintenance actions (the block replacement policy (Barlow and al. 1965)). It is interesting to notice that if an infinite optimal solution (no corrective maintenance) is possible in the case of perfect repairs, it can not be envisaged in an imperfect repair situation.

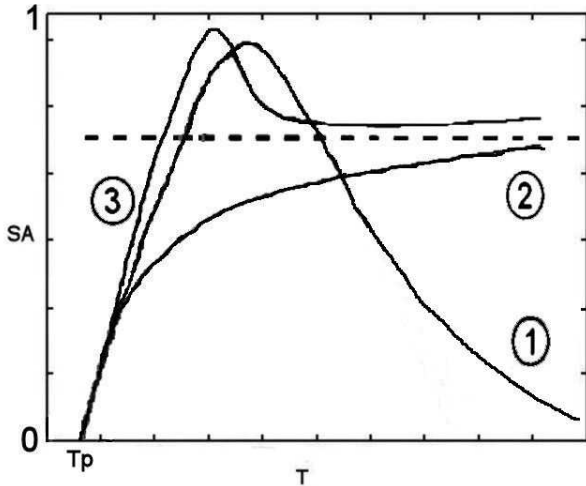


Figure 6. General behaviours of system stationary availability function $SA(T)$ in the situation of perfect repairs (curves 2 and 3) and imperfect repairs (curve 1)

4.2. Numerical Example

For illustration purpose, let's consider a randomly failing system whose time to first failure follows a Gamma distribution with shape parameter $\alpha=3$ and scale parameter $\beta=3$.

The following average durations are considered:

Average duration of a corrective maintenance action:
 $T_c=0.35$ time unit ;

Average duration of a preventive maintenance action:
 $T_p=0.01$ time unit ;

The corrective maintenance actions are characterized by a constant average efficiency factor $a=0.6$.

The following figure shows the evolution of the system stationary availability $SA(T)$ considering a truncation in the quasi-renewal function at $c=10$ (equation 8).

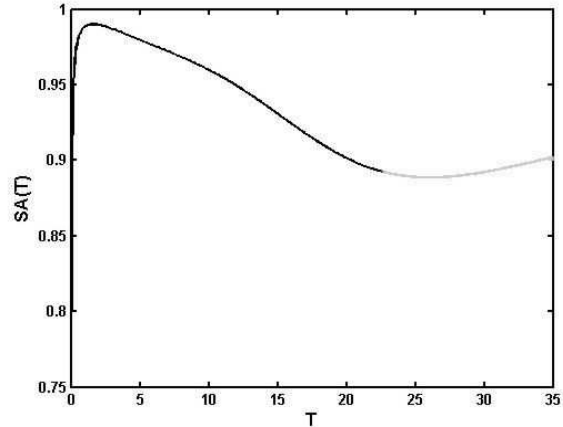


Figure 7. Evolution of the system stationary availability $SA(T)$ with a truncation at $c=10$ in equation 8.

Figure 7 shows that there is an optimal preventive maintenance period T^* (1.6 time unit) which maximizes the system availability $SA(T)$. The minimum of the curve of $SA(T)$ corresponds to the moment t_i when the curve of the quasi-renewal function $Q(t)$ reaches the limit $Q(t_i)=c$ (such as shown in the previous section). Note that from $T=t_i$ (the part of the curve in gray), the $SA(T)$ curve is not exploitable.

Figure 8 below shows that, as expected, for the same input parameters $F(\cdot)$, T_p and T_c , as the corrective maintenance actions become more and more efficient (increasing 'a'), the optimal preventive maintenance period T^* becomes larger and the corresponding availability $SA(T^*)$ increases. It is also interesting to notice that beyond a certain level of the repair efficiency factor, the variations, especially at the level of the availability, tend to stabilize. This allows concluding that it is possible to tolerate a certain degree of ineffectiveness of the corrective actions without having a notable availability loss.

Besides, we also notice that for the same system and for the same repair efficiency degree a , as the ratio T_c/T_p increases, the optimal period of preventive maintenance T^* and the corresponding optimal stationary availability decrease.

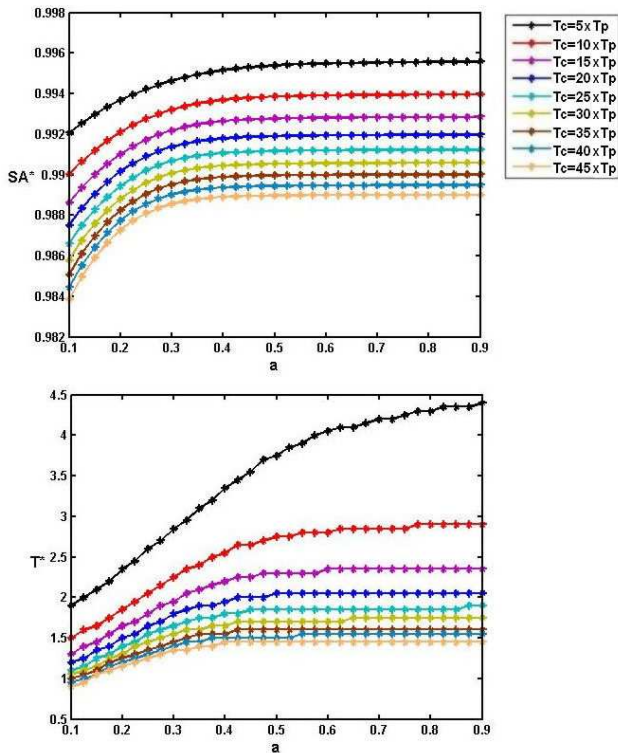


Figure 8. T^* and $SA(T^*)$ evolution according to the repair efficiency factor, for various ratios T_c/T_p

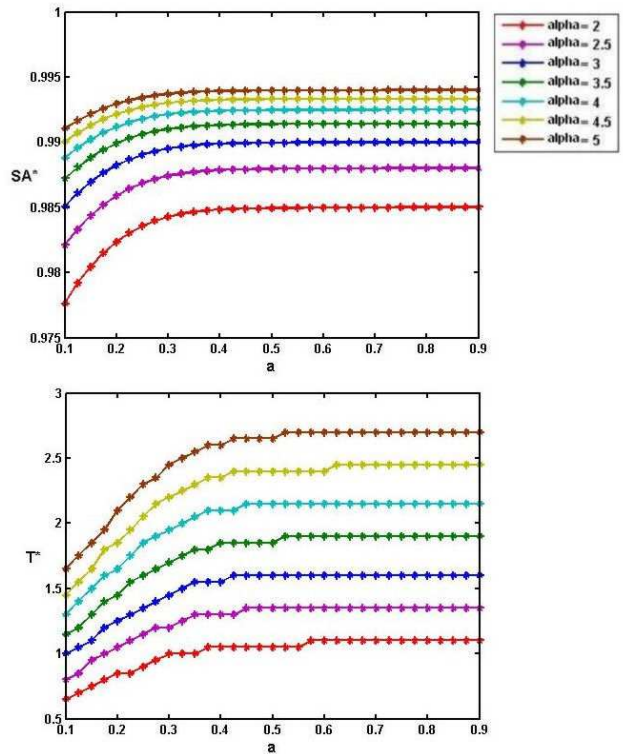


Figure 9. T^* and $SA(T^*)$ evolution according to the repair efficiency factor 'a' for various Gamma distributions (with different shape parameters)

Figure 9 shows the evolution of the optimal preventive maintenance period T^* according to the shape parameter α of the Gamma distribution associated with the first time to failure of the system. We notice essentially that for all the considered systems, the optimal periods of preventive maintenance and their corresponding system availability level increase as the repairs are considered more and more efficient. This is accomplished until having a stability of its two parameters (T^* et $SA(T^*)$) from a certain repair efficiency factor approximately equal to 0.35. Here again, the following important conclusion can be drawn: for several systems with different failure rates, the degree of the repair efficiency stops affecting the optimal strategy from a certain threshold (here: 0.35).

5. CONCLUSION

In this work, we studied a preventive maintenance policy for randomly failing systems evolving in a context where the preventive maintenance actions performed every T time units restore the system to its as good as new state, while in case of failure between successive preventive maintenance actions, imperfect repair is performed restoring the system to a state between as good as new and as bad as old. The imperfection of repair actions has been modeled by using the quasi-renewal process based on a deterministic repair efficiency factor 'a' taken between 0 and 1. We studied the evolution of the system stationary availability under the proposed maintenance policy. We considered the general case and the particular situations of systems whose times to first failure follow a Gamma distribution. We derived for this set of systems a new expression to approximate and easily compute the quasi-renewal function which is the average number of restarts following failures during a time interval $[0, t]$.

This study showed that for any given system and any given repair effectiveness factor and for any given downtime durations for preventive and corrective maintenance, there is necessarily a finite optimal period T^* of preventive maintenance which maximizes the system stationary availability.

Obtained numerical results corresponding to a set of particular cases with systems whose times to first failure follow Gamma distributions, allowed concluding that it is possible to tolerate a certain degree of ineffectiveness of the corrective actions without having a notable

availability loss. The knowledge of the interval of tolerance of the repair inefficiency degree not causing significant availability loss represents an important source of flexibility for the maintenance management.

Finally, the obtained results also showed that for several systems with different failure rates (different Gamma shape parameters), the degree of the repair efficiency stops affecting the optimal strategy from a certain threshold.

Several extensions of this work are currently under consideration.

REFERENCES

- Barlow, R., Proschan, F., *Mathematical theory of reliability*, New York, John Wiley & Sons, 1965.
- Chan, J. K. and Shaw L.. *Modeling repairable systems with failure rates that depend on age and maintenance*. IEEE Transactions on Reliability, 42(4), , 1993 p. 566-571.
- Ian Jon Rehmert, *Availability Analysis for the Quasi-Renewal Process*, Ph.D. thesis in Industrial and Systems Engineering, Virginia Polytechnic Institute and State University, October 2000.
- Kijima M., Morimura H. and Suzuki Y.. *Periodical replacement problem without assuming minimal repair*. European Journal of Operational Research, vol.37, p194-203, 1988.
- Kijima M.. *Some results for repairable systems with general repair*. Journal of Applied Probability, vol.26, p.89-102, 1989.
- Pierrat L, Aubigny G, *Somme de lois Gamma de paramètres d'échelle distincts et détermination des paramètres d'une distribution Gamma approchée*, 38^{ème} Journées de Statistique, Clamart, 2006.
- Malik M.A.K, *Reliable preventive maintenance scheduling*, AIIE Transactions, vol. 11(3), p 221-228, 1979.
- Moschopoulos P. G., *The distribution of the sum of independent Gamma random variables*, Ann. Inst. Statist. Math, 37 (1985), Part A, 541-544.
- Pham H, et Wang H., *Imperfect maintenance*, *European Journal of Operational Research*, 1996, Vol.94: p.425–38.
- Shey-Huei S., Yuh-Bin L. and Gwo-Liang L., *Optimum policies for a system with general imperfect maintenance*, Reliability Engineering and System Safety, ELSEVIER, 2005.
- Shin I., Lim T.J. and Lie C.H., *Estimating parameters of intensity function and maintenance effect for repairable unit*, Reliability Engineering and System Safety, vol. 54, p. 1-10, 1996
- single-unit systems, Naval research logistics, 36, 419-446, 1989.
- Wang, H. Pham, *A quasi renewal process and its applications in imperfect maintenance*, International Journal of systems Science, volume 27, number 10, 1996, pages 1055-1062.
- Wang, H. Pham, *Correspondence Changes to : "A quasi renewal process and its applications in imperfect maintenance"*, International Journal of systems Science, 1997 volume 28, number 12, pages 1329.
- Wang Hongzhou. *A survey of maintenance policies of deteriorating systems*, European Journal of Operational Research, 139 (2002) 469-489, 2002.