

A COMBINED APPROACH FOR THE TWO-DIMENSIONAL CUTTING-STOCK PROBLEM

Ahmed MELLOULI, Faouzi MASMOUDI

Mohamed HADDAR

U2MP

University of Sfax, Sfax Engineering School (E.N.I.S.)
B.P 3038-Sfax –TUNISIA
A. ahmed_tu2002@yahoo.fr,
B. faouzi.masmoudi@enis.rnu.tn

U2MP

University of Sfax, Sfax Engineering School (E.N.I.S.)
B.P 3038-Sfax –TUNISIA
C. mohamed.haddar@enis.rnu.tn

ABSTRACT: *This paper deals with the two-dimensional rectangular guillotine oriented cutting-stock problem with the objective of minimising the trim loss. For this class of problems, we propose a hybrid heuristic that combines two algorithms: the first with a good quality that can be applied to the problems of small and medium size, the second with a medium quality that can be applied to all size of the problems. An example has been used to illustrate and clarify each step of these algorithms. The effectiveness of the hybrid heuristic has been tested by a set of instances.*

KEYWORDS: *Cutting stock problems; Trim loss; Guillotine cutting; heuristics*

1. INTRODUCTION

The two-dimensional cutting stock problem consists of cutting a large supply of rectangular stock sheets to produce smaller pieces in quantities matching orders received. The objective of this process is to minimise the total cost such as the total number of stock sheets used. This problem arises in many industrial applications such as paper, wood, clothes, glass and sheet metal (for application see Farely, 1988, 1990b; Schutlz, 1995; Mornar, 1997; Menon, 2002; Dikili 2004; Yanasse, 2006).

In this paper, we study the two-dimensional rectangular guillotine oriented cutting-stock problem. This problem can be stated as follow. A set of rectangular order pieces with dimensions $w_1 \times l_1, \dots, w_N \times l_N$ are requested with quantity d_i ($i=1,2,\dots,N$). d_i is usually a very large number, more than one hundred, and it is to be cut from rolls of material with standard width W_j ($j=1,2,\dots,K$), each in sufficient length to satisfy the entire demand. The cutting patterns must satisfy the following technological constraints. First, the pieces are obtained from oriented guillotine cut. Second, the number of transversal cuts can not exceed the number of knives available in the machine, in this case the number is equal to six (Figure1). For this problem it is required to determine the production schedule that minimises the total material needed, while satisfying the given demand.

Before providing the details of the proposed heuristics, we shall briefly review the solution techniques available in the literature.

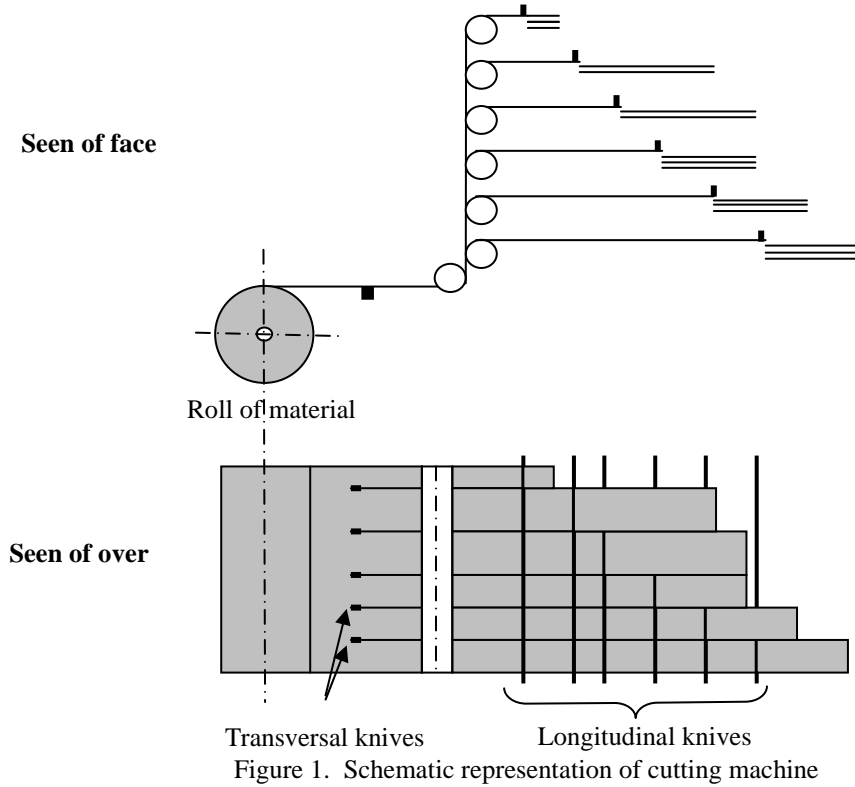
In the 1960's a number of pioneering papers on cutting stocks problems were written by Gilmore and Gomory (1961, 1963, 1965, 1966). A set of pieces is to be cut, using the least numbers of identical sheets. The solution is an integer programming model with a column generation procedure.

Since integer linear programming models are not practical to solve large size problems, some heuristics have been proposed for the generation of good cutting patterns to produce the columns of an integer programming problem. For example (Farely, 1990b; Wang, 1983).

Other researchers proposed a two-stage approach for the integer programming cutting stock problems, based on a rounding up procedure after a linear program (LP) relaxation (Johnson, 1986, 1997). To construct the matrix of constraint, Suliman (2001) proposed a branch and bound column generation method for the one-dimensional CSP that can be projected to the 2DCSP.

Suliman (2006) developed a three-stage sequential heuristic. In the first stage, a width-cutting pattern is determined. Determining the table length and the associated layout of the pieces length wise to produce a good cutting pattern is the second stage. In the final stage, the number of times in which the generated cutting pattern will be used is determined.

These works consider only trim loss as the main objective, several works include other factors and a set of methods for multi-criteria cutting stock problems are presented by Gasimov (2007); Yanasse (2006) and Franca (2007).



2. PROBLEM FORMULATION

The following notations are used in this paper to facilitate communication:

$\lfloor x \rfloor$: Biggest integer lower than x .

$\lceil x \rceil$: Lowest integer bigger than x .

V_{kji} : Number of units of width w_i being cut according to the j th pattern from the k th roll.

L_{kj} : Length produced with the pattern V_{kj}

X_{kji} : If $V_{kji} > 0$, X_{kji} denote the integer value $\left\lfloor \frac{L_{kj}}{l_i} \right\rfloor$, so

that $X_{kji} V_{kji}$ represents the number of pieces of width w_i contained in the pattern V_{kj} through the length L_{kj} .

J_k : Number of non dominated patterns of W_k roll width.

This problem can be formulated as follows:

$$\text{Minimise } \sum_{k=1}^K W_k \sum_{j=1}^{J_k} L_{kj}$$

Subject to

$$\sum_{k=1}^K \sum_{j=1}^{J_k} X_{kji} V_{kji} \geq d_i \quad \text{For all } i \quad (1)$$

$$l_i X_{kji} \leq L_{kj} \quad \text{For all } k, j \text{ and } i \quad (2)$$

$$\sum_{i=1}^N V_{kji} \leq 6 \quad \text{For all } k \text{ and } j \quad (3)$$

$$W_{k-1} < \sum_{i=1}^N V_{kji} W_i \leq W_k \quad \text{For all } k \text{ and } j \quad (4)$$

$$L_{kj} \geq 0 \quad X_{kji} \in \mathbf{Z}_+ \quad (5)$$

The objective function represents the total area utilised from the different rolls which represent the sum of the area demand and the waste. Since the total area demand is constant, minimising the total area is equivalent to minimise the waste.

The constraints have the following meaning. Constraints (1) guarantee that the demand of each type of items is satisfied. Constraints (2) assure that the variables L_{kj} assume the appropriate values with respect the corresponding integer variables X_{kji} . Constraints (3) specify that the number of transversal cuts in each pattern can not exceed the number of knives available in the machine (in this case, the number is equal to six). Constraints (4) reduce the number of patterns by selecting for each pattern width the closest feasible roll. Finally, constraints (5) introduce non-negative and integrality conditions.

In order to solve this problem in a reasonable time, heuristics are used.

3. THE FIRST HEURISTIC APPROACH (H1)

The first heuristic is based on three stages, involving an enumeration of all feasible non-dominated patterns of the different widths to construct the matrix of constraints at the first stage followed by a LP relaxation at the second stage, and finally a generation of a solution to the basic

problem through the solution of the associated problem with relaxed constraints.

The linear relaxation of the basic problem can be formulated as follow:

$$\text{Minimise } \sum_{k=1}^K W_k \sum_{j=1}^{J_k} L_{kj}$$

Subject to

$$\sum_{k=1}^K \sum_{j=1}^{J_k} L_{kj} V_{kji} \geq d_i l_i \quad \text{For all } i \quad (6)$$

$$\sum_{i=1}^N V_{kji} \leq 6 \quad \text{For all } k \text{ and } j \quad (3)$$

$$W_{k-1} < \sum_{i=1}^N V_{kji} W_i \leq W_k \quad \text{For all } k \text{ and } j \quad (4)$$

$$L_{kj} \geq 0 \quad (7)$$

From an optimal solution L^* of the relaxed problem, it is easy to construct a feasible solution for the basic problem as follows. Since the number of items produced according to L^* is

$$\sum_{k=1}^K \sum_{j=1}^{J_k} \left\lfloor \frac{L_{kj}}{l_i} \right\rfloor V_{kji} \leq \sum_{k=1}^K \sum_{j=1}^{J_k} \frac{L_{kj}}{l_i} V_{kji}, L^* \text{ may violate}$$

the demand constraints for some item types. However, it is easy to transform L^* in a feasible solution to the basic problem by suitably augmenting some of its components. The following algorithm named rounding up is designed for this aim.

Procedure rounding up:

Step 1. For each $i=1, \dots, N$ compute the total number

$$\bar{d}_i = \sum_{k=1}^K \sum_{j=1}^{J_k} \left\lfloor \frac{L_{kj}}{l_i} \right\rfloor V_{kji}, \text{ and set } Q = \{i: r_i = d_i - \bar{d}_i > 0\}.$$

Step 2. For each pattern V_{kj} with $L_{kj} > 0$, calculate

$$H_{kj} = \sum_{i \in Q} h_i \text{ where } h_i = \inf(V_{kji}, n_i).$$

Step 3. Select the pattern with the greatest H_{kj} , and increase its length L_{kj} as much as necessary to make $r_i \leq 0$ for all $i \in Q$ with $V_{kji} > 0$.

Update, r_i, Q and L_{kj} .

If $Q = \Phi$ go to step 4. Else go to step 2.

Step 4. For each pattern V_{kj} with $L_{kj} > 0$, reduce L_{kj} as much as possible while maintaining the feasibility of the solution with respect to the demand constraints.

The heuristic approach flowchart is shown in Figure 2.

3.1 Example

To illustrate and clarify each step of this heuristic the following example is used. Four rectangular pieces with the specifications shown in Table 1 are to be cut from rolls with three widths: $W_1 = 2.5\text{m}$; $W_2 = 2.25\text{m}$; $W_3 = 2\text{m}$.

Item number	Width w_i	Length l_i	Demand d_i
1	1.35	0.5	200
2	1.05	0.4	150
3	0.78	0.8	400
4	0.37	0.9	400

Table 1. Example data

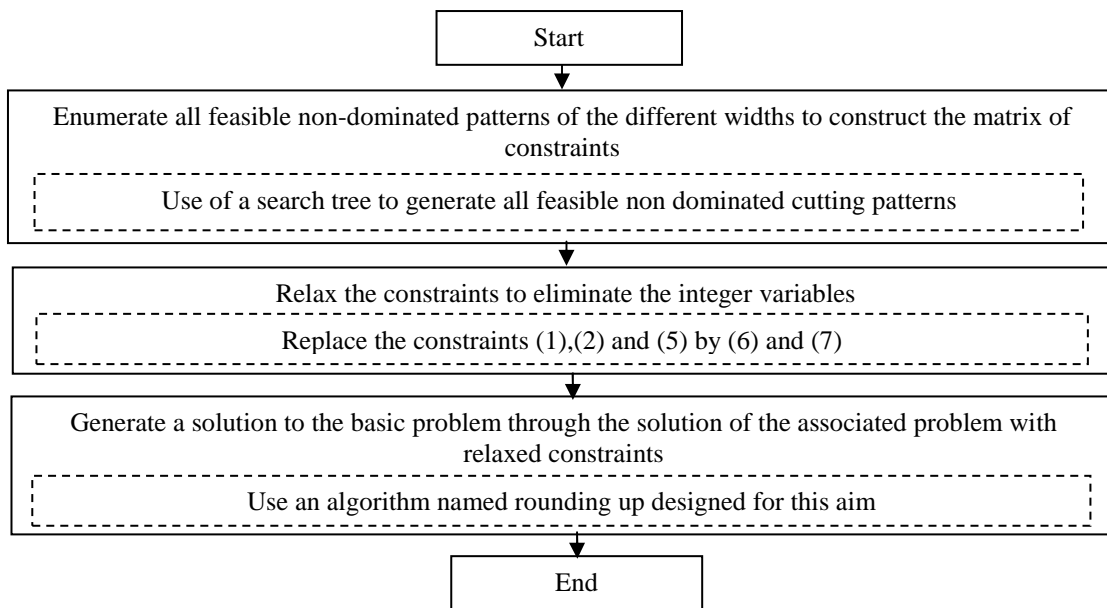


Figure 2. Heuristic approach flowchart

Before providing the details of this example, it is necessary to show the definition of the dominated patterns.

Definition:

a pattern V_1 is dominated by a pattern V_2 if the first is included in the second and each of the two patterns is feasible with the same roll width.

3.1.1 First stage

The enumeration of all feasible non-dominated patterns of the different widths is achieved through a search tree with a number of levels equal to the number of orders.

The non-dominated feasible patterns of different widths are presented in the following matrix:

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 3 & 2 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 3 & 2 & 1 & 1 & 0 & 1 & 0 & 3 & 2 & 0 & 2 & 1 & 4 & 3 & 6 & 5 \end{pmatrix}$$

The associated roll widths of these patterns are presented in the following line Matrix:

$$C = [2.5 \ 2.5 \ 2.25 \ 2.5 \ 2.25 \ 2.0 \ 2.5 \ 2.25 \ 2.25 \ 2.0 \ 2.25 \ 2.0 \ 2.5 \ 2.5 \ 2.0 \ 2.5 \ 2.0 \ 2.25 \ 2.0]$$

3.1.2 Second stage

Relax the constraints of our problem to obtain a linear formulation by replacing the constraints (1),(2) and (5) by (6) and (7).

The relaxed problem is:

$$\begin{cases} \text{Min } (C L) \\ \text{Subject to} \\ A L \geq B \\ L \geq 0 \end{cases}$$

- A: Matrix of constraints
- C: Associated rolls widths of patterns presented in the matrix A
- L: Lengths to produce with the different patterns to satisfy the accumulated lengths of orders.
- B: Accumulated length of each rectangular type of pieces.

For our example: B is presented in the following column matrix:

$$B = \begin{pmatrix} d_1 x l_1 = 100 \\ d_2 x l_2 = 60 \\ d_3 x l_3 = 320 \\ d_4 x l_4 = 360 \end{pmatrix}$$

The solution of the linear problem is:

$$L = [0 ; 100 ; 0 ; 0 ; 0 ; 0 ; 21.0092 ; 0 ; 17.99 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 101.1 ; 0 ; 0 ; 20 ; 0]$$

3.1.3 Third stage

Generate a solution to our problem from the solution of the associated problem with relaxed constraints through the procedure rounding up (Figure 3).

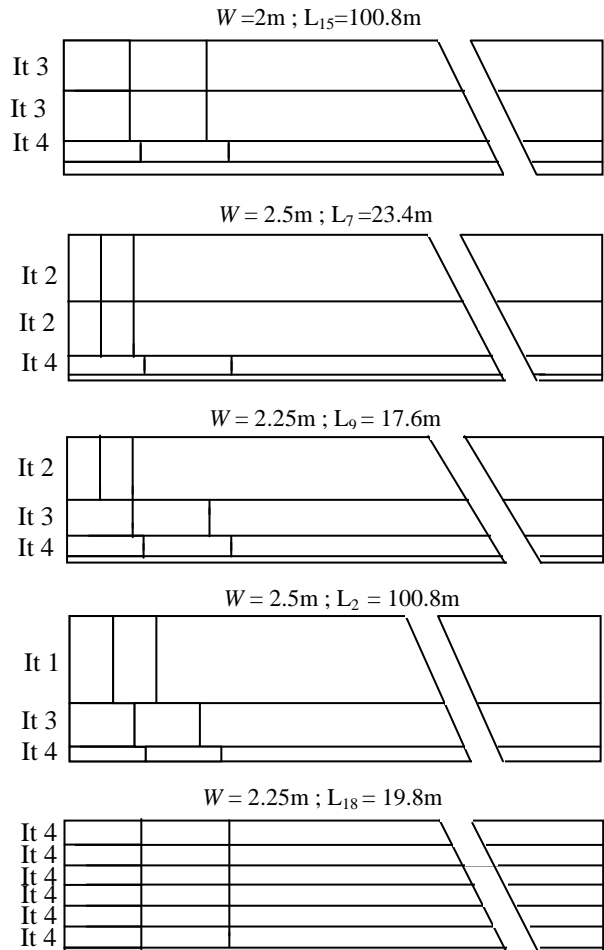


Figure 3. Production schedule

4. THE SECOND HEURISTIC APPROCH (H2)

The second heuristic is based on a sequential approach with four-stage. In the first stage a set of K width-cutting pattern layouts with the minimum trim loss is identified. In the second stage the pattern with the least width trim loss is selected from the set. In the third stage the length of this pattern layout is calculated from the way that at least the demand of one order is fulfilled. As a result of these stages a good cutting pattern is produced that can be used for the solution of the two-dimensional cutting-stock problem. Finally, the pieces produced with the cutting pattern are decreased from the demand orders. A new iteration is started to produce the following cutting pattern for the reduced problem.

The heuristic approach flowchart is shown in Figure 4.

For each specific standard roll width W_j ($j=1\dots K$), a width cutting pattern is generated as follows. Let w_i ($i=1\dots N$) be the width list of the required rectangles arranged in a decreasing order $w_1 > w_2 > \dots > w_N$. The maximum number of pieces of width w_i that can be obtained across the width W_i of the stock roll is:

$$V_k(i) = \min\left(C6, \left\lfloor \frac{W_k - W'_k}{w_i} \right\rfloor\right) \quad (8)$$

Where W'_k represents the width of the cutting pattern, and it is initially set equal to 0. C6 represents the number of available knives, and it is initially set equal to 6. The width of the cutting pattern W'_k and the number of available knives after the cutting pieces of w_i width become:

$$W'_k = W'_k + V_k(i) \times w_i \quad (9)$$

$$C6 = C6 - V_k(i) \quad (10)$$

The Width-cutting pattern flowchart is shown in Figure 5.

4.1 Example

To illustrate and clarify each step of this heuristic the example presented in 3.1 is used.

The procedure goes along the steps shown in the flowcharts of figure 7.

First iteration:

In the first stage a set of K width-cutting pattern layouts is identified by the procedure presented in the figure 8. For each specific standard roll width W_j , a pattern layout is generated.

$$V_1 = \begin{vmatrix} 1 \\ 1 \\ 0 \\ 0 \end{vmatrix} \begin{matrix} W_1=2.5m \\ W'_1=2.4m \end{matrix} \quad V_2 = \begin{vmatrix} 1 \\ 0 \\ 1 \\ 0 \end{vmatrix} \begin{matrix} W_2=2.25m \\ W'_2=2.13m \end{matrix} \quad V_3 = \begin{vmatrix} 1 \\ 0 \\ 0 \\ 1 \end{vmatrix} \begin{matrix} W_3=2m \\ W'_3=1.72m \end{matrix}$$

In the second stage the pattern with the least width trim loss is selected from the set. For this example the first pattern is selected because:

$$(W_j - W'_j) = \min\{(W_i - W'_i) ; i=1,2,3\} = \min\{(2.5-2.4), (2.25-2.13), (2-1.72)\}.$$

In the third stage the length of this pattern layout is calculated from the way that at least the demand of one order is fulfilled

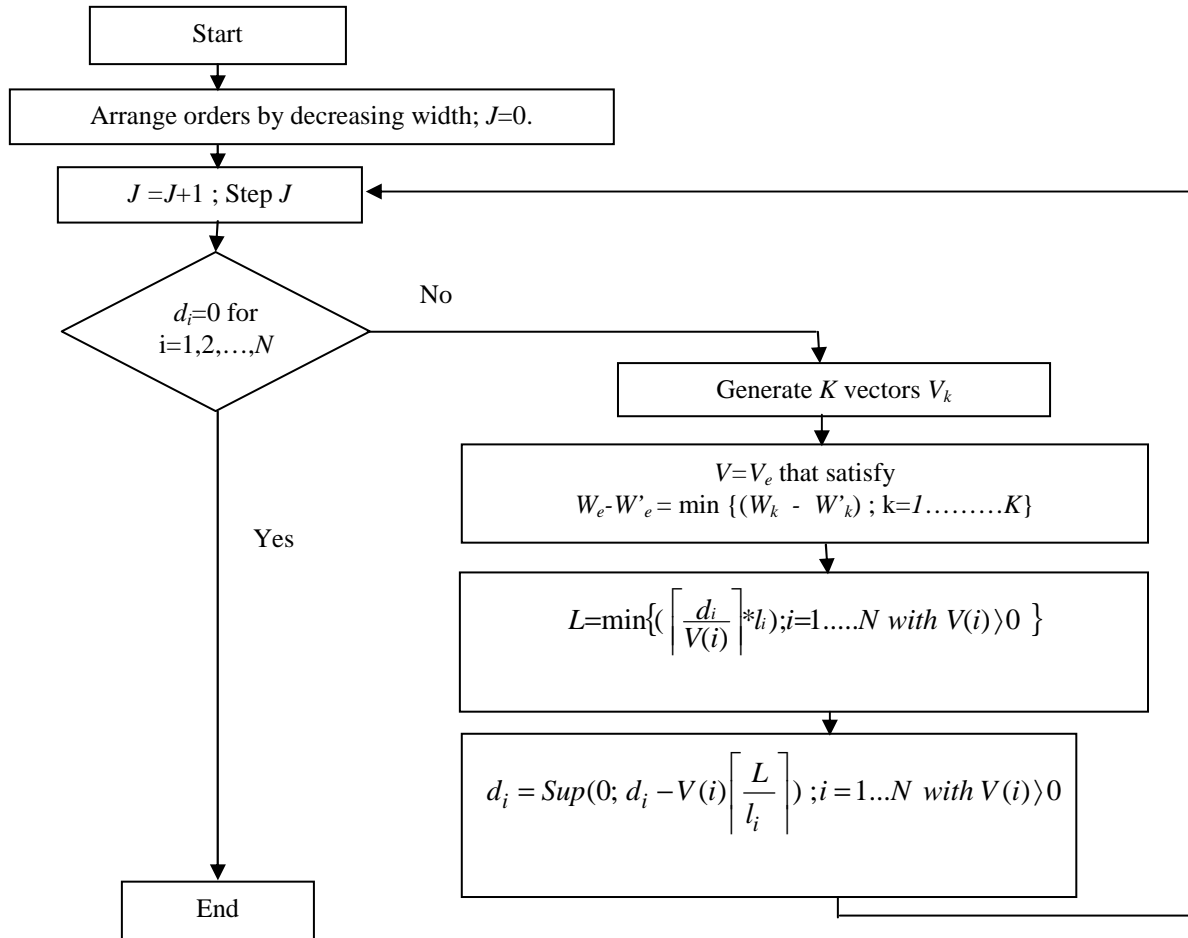


Figure 4. Heuristic approach flowchart

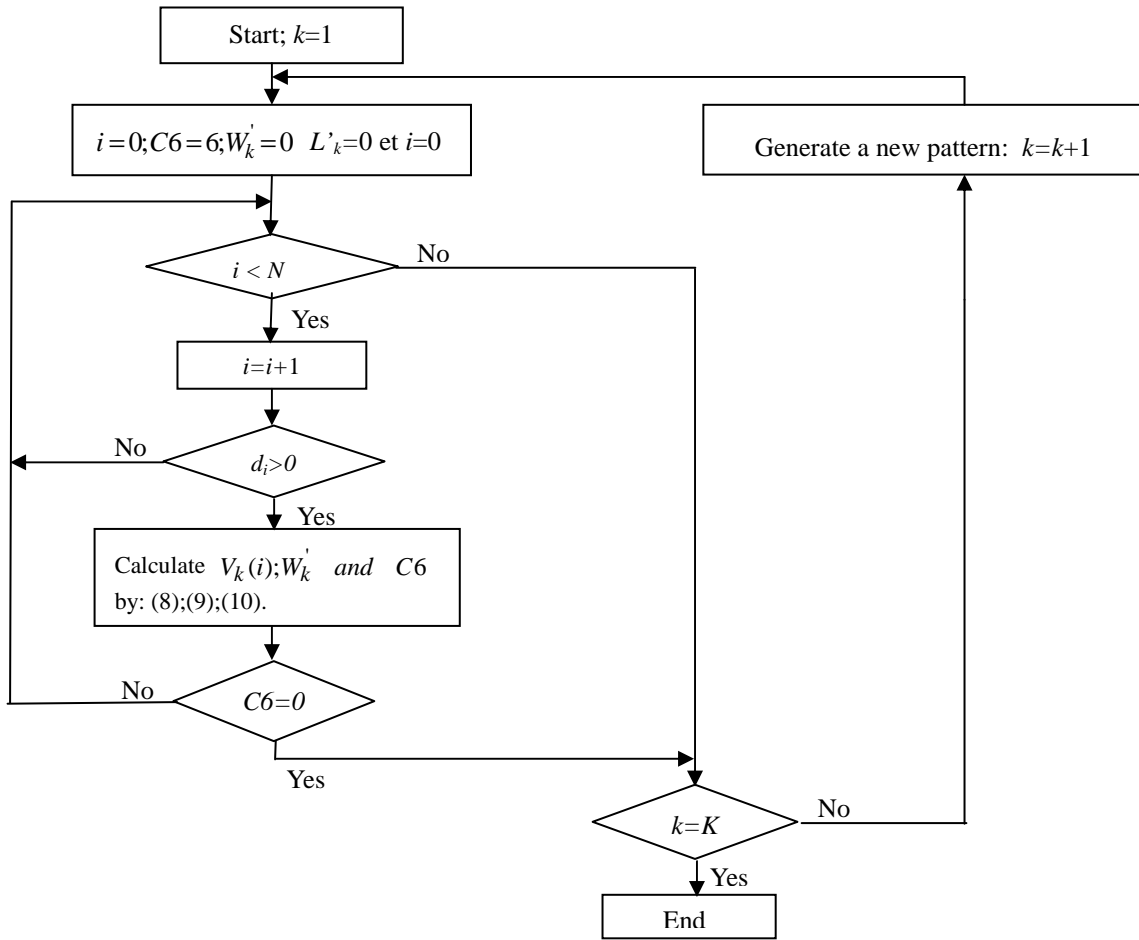


Figure 5. Width-cutting pattern flowchart

$$L = 60m = \min \left\{ \left\lceil \frac{d_i}{V(i)} \right\rceil * l_i \right\}; i = 1, 2 \}$$

Finally, the pieces produced with the cutting pattern are decreasing from the demand orders. The update demands are presented in the following table:

Item number	Width w_i	Length l_i	Update demand d_i
1	1.35	0.5	Sup(0,(200-[60/0.5]))=80
2	1.05	0.4	Sup(0,(150-[60/0.4]))=0
3	0.78	0.8	400
4	0.37	0.9	400

Table 2. Reduced problem data

A new iteration is started to produce the following cutting pattern for the reduced problem. The demand is satisfied by four iterations.

The solution of our problem is:

$$V_1 = \begin{array}{l|l} \begin{array}{c} 1 \\ 1 \\ 0 \\ 0 \end{array} & \begin{array}{l} L_1=60 \text{ m} \\ W=2.5\text{m} \end{array} \end{array} \quad V_2 = \begin{array}{l|l} \begin{array}{c} 1 \\ 0 \\ 1 \\ 1 \end{array} & \begin{array}{l} L_2=40 \text{ m} \\ W=2.5\text{m} \end{array} \end{array}$$

$$V_3 = \begin{array}{l|l} \begin{array}{c} 0 \\ 0 \\ 2 \\ 1 \end{array} & \begin{array}{l} L_3=140 \text{ m} \\ W=2\text{m} \end{array} \end{array} \quad V_4 = \begin{array}{l|l} \begin{array}{c} 0 \\ 0 \\ 0 \\ 6 \end{array} & \begin{array}{l} L_4=30.6 \text{ m} \\ W=2.25\text{m} \end{array} \end{array}$$

4. EXPERIMENTAL RESULTS

The effectiveness of these proposed heuristics has been tested to check both the computing time and the quality of the solution.

4.1 Computing time

For each heuristic the computing time has been evaluated by a set of problems which are randomly generated with different number of orders.

The experimental results are presented in the following table and graph:

Number of orders	5	10	15	20	25	30
Computing time in sec (H1)	0,5	1,5	2	5	30	346
Computing time in sec (H2)	0,3	0,6	0,9	1,2	1,4	1,6

Table 3. Computing time for H1 an H2

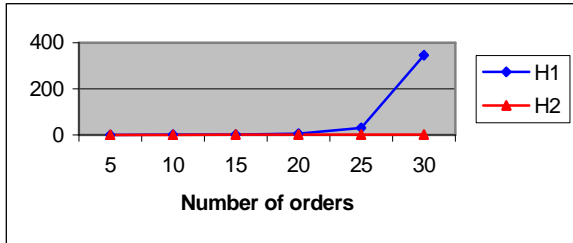


Figure 6. Computing time

The experimental results presented in Table 5 show that the first heuristic is suitable for only problems with low size because the computing time increases exponentially with the size of the problem, however the second heuristic is suitable for all sizes problems.

4.2 Quality of the solutions

For each heuristic the quality of the solutions has been evaluated through the value of the relative error witch represents a comparison between the solution presented by a heuristic and the solution of the relaxed problem that represent a lower bound for the optimal solution.

$$VREH_i = \frac{MNH_i - LB}{MNH_i}$$

Where $VREH_i$, MNH_i and LB denote respectively, the value of the relative error obtained with H_i , the material needed with the heuristic H_i and a lower bound of the optimal solution.

The experimental results of the application of these two heuristics on some problems witch are randomly generated are presented in Table 5.

Problem number	Lower bound	Square of solution (H1)	$VREH_1$ (%)	Square of solution (H2)	$VREH_2$
1	25031	25078	0.19	25241	0.83
2	92879	92973	0.10	92956	0.08
3	19772	19804	0.16	20069	1.48
4	51922	52414	0.94	54870	5.37
5	23074	23262	0.81	23930	3.58
6	15016	15039	0.15	15422	2.63
7	16548	16570	0.13	16761	1.27
8	13634	13682	0.35	13806	1.25
9	16106	16110	0.02	16544	2.65
10	22504	22584	0.35	22680	0.78

Table 4. Quality of the solutions generated by H1 and H2.

The comparison of the results given by these heuristics with the lower bounds of the optimal solutions shows a good quality of the first heuristic and a medium quality of the second heuristic, because the VREH1 does not exceed a rate of 1% and VREH2 does not exceed a rate of 6%.

5. THE COMBINED APPROACH

The basic idea of the new method is to find a temporary solution with the second heuristic (H2) and then improve it by solving a small, critical part of the problem (sub-problem) again with the first heuristic (H1).

The sub-problem contains less than S different stock and order sizes. So, we expect substantially better results at the expense of low increased time complexity. Therefore, the proposed method would still be suitable for all sizes of the problem.

As the size of the sub-problem remains unchanged regardless of the size of the whole problem, total time complexity depends on time complexity of H2 witch is suitable for problems with large size. Therefore, the combined approach can be used for problems with large size.

The combined approach flowchart is shown in Figure 7.

6. EXPERIMENTAL RESULTS

A set of experimental tests have been executed to compare the effectiveness of the proposed combined approach with the previous heuristics H1 and H2 on both the computing time and the quality of the solution.

The experimental results of the application of these heuristics on some problems witch are randomly generated are presented in Table 5.

The comparison of the results given by the combined approach and the previous heuristics show a great success of the combined approach to determine a good solution close to the optimal solution in a reasonable time, because the VREH of the set of experimental tests don't exceed a rate of 2% with a low computing time.

For this combined approach the user can choose the value of S to get a medium solution more quickly or to perform the solution by a grew in the computing time.

To proof these performances, the heuristic H has been tested on the following three problems collected from the literature (Benati, 1997).

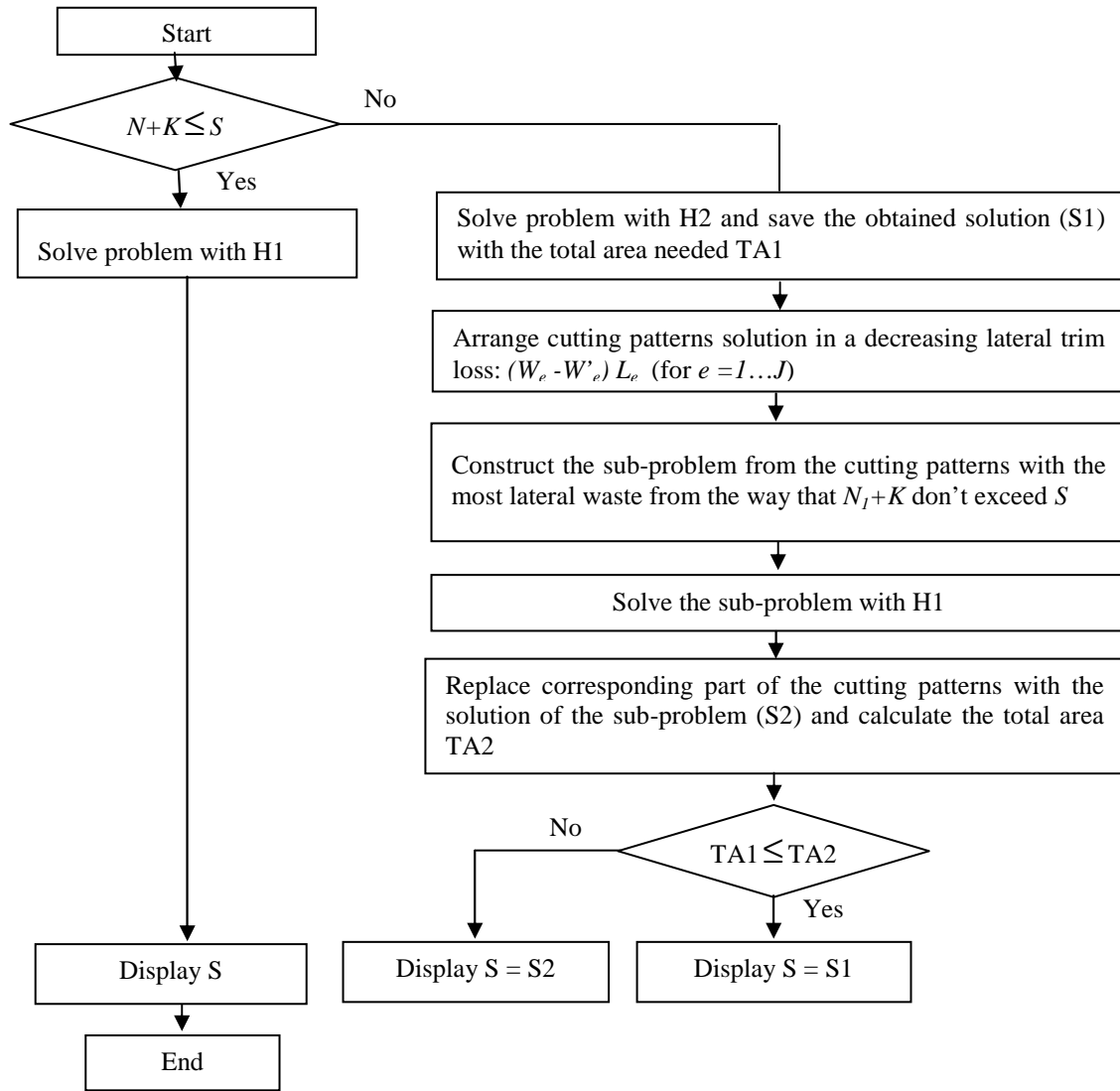


Figure 7. Combined approach flowchart

For the following examples S is equal to 15.

N°	N	K	Lower bound (LB)	Material needed (m ²)			value of the relative error (%)			Computing time (sec)		
				H1	H2	H	VREH ₁	VREH ₂	VREH	H1	H2	H
1	10	1	16379	16430	16401	16430	0,31	0,13	0,31	1,3	0,8	1,3
2	10	2	16048	16093	16213	16093	0,28	1,02	0,28	1,3	1	1,3
3	10	3	14581	14616	14691	14616	0,24	0,75	0,24	1,8	1,2	1,8
4	10	4	14754	14766	14864	14766	0,08	0,74	0,08	2	1,2	2
5	20	1	22805	22993	23450	23033	0,82	2,75	0,99	22	1,6	7
6	20	2	22774	22868	23064	23064	0,41	1,26	1,26	30	1,6	2,8
7	20	3	22267	22344	22703	22361	0,35	1,92	0,42	40	1,6	2,2
8	20	4	22562	22617	22854	22794	0,24	1,28	1,02	50	1,8	2,5
9	30	1	50228	50690	51517	50826	0,911	2,502	1,177	249	1,8	2,8
10	30	2	49957	50144	50941	50211	0,373	1,932	0,506	346	1,6	2
11	30	3	49204	49278	50160	49529	0,150	1,906	0,656	440	1,7	2,2
12	30	4	49475	49780	50267	49876	0,613	1,576	0,804	580	1,9	2,3

Table 5. Experimental results

Problem 1

Item number	Width w_i	Length l_i	Demand d_i
1	0.8	1.03	300
2	0.4	0.51	200
3	0.66	0.55	100
4	1.035	1.27	300
5	0.324	0.46	200
6	0.233	1.07	600
7	0.615	0.724	100
8	0.8	1.03	300
9	0.7	0.9	200
10	0.77	1.03	1000
11	0.37	0.45	2000
12	0.3	0.4	1000
13	0.55	0.65	2000
14	0.8	1.05	1000
15	0.388	0.51	100
16	0.605	0.73	100
17	0.795	1.03	100

Table 6. Problem 1 data

The testing has been carried out with the following sets of roll widths: A1=(1.33), A2=(1.33,1.25), A3=(1.33,1.25,0.77), B1=(1.15), B2=(1.15,1.05), B3=(1.15,1.05,1.01).

Problem 2

Item number	Width w_i	Length l_i	Demand d_i
1	0.37	0.45	2000
2	0.24	0.3	500
3	0.3	0.4	3000
4	0.35	0.45	4000
5	0.4	0.5	4000
6	0.4	0.512	4000
7	0.5	0.6	500
8	0.55	0.65	6000
9	0.4	0.51	2000
10	0.415	0.521	1000
11	0.52	0.64	600
12	1.05	0.5	500

Table 7. Problem 2 data

The problem has been tested with the same sets of rolls as in problem 1.

Problem 3

Item number	Width w_i	Length l_i	Demand d_i
1	1.05	1.315	400
2	0.78	1.08	200

3	0.82	1.03	500
4	0.4	0.51	2000
5	0.37	0.45	2000
6	0.74	0.935	3000
7	0.8	1.05	3000
8	0.55	0.65	5000
9	0.6	0.8	500
10	0.615	0.73	3000
11	0.72	1	3000
12	0.77	1.03	2000
13	0.55	0.65	2000
14	0.4	0.5	2000
15	0.605	0.73	500
16	0.388	0.51	500
17	0.55	0.65	500
18	0.37	0.45	1000
19	0.43	0.57	500
20	0.72	0.72	1500
21	1	0.7	500
22	0.8	1	300

Table 8. Problem 3 data

Roll widths on which it has been tested are the following: C1=(1.33), C2=(1.33,1.12), C3=(1.33,1.12,1.05) and then B1, B2, B3.

The experimental results are presented in the Table below.

Code	Lower bound (LB)	Material needed (m ²)	Value of the relative error (%)
P1-A1	4664,6	4670	0,12
P1-A2	4499,7	4514,7	0,33
P1-A3	4499,7	4601,4	2,21
P1-B1	4844,3	4846,6	0,05
P1-B2	4604,5	4624,4	0,43
P1-B3	4545,8	4574,8	0,63
P2-A1	6571,5	6597,8	0,40
P2-A2	6459	6484,1	0,39
P2-A3	6459	6486,2	0,42
P2-B1	6541,1	6550,3	0,14
P2-B2	6473,4	6483,8	0,16
P2-B3	6473,4	6486,8	0,21
P3-B1	22074	22077	0,01
P3-B2	20553	20711	0,76
P3-B3	19990	20210	1,09
P3-C1	20079	20085	0,03
P3-C2	18405	18535	0,70
P3-C3	18093	18226	0,73

Table 9. Experimental results

In table 9 the value of the relative error doesn't exceed a rate of 2.5 % with an average of 0.52 %. These results confirm the effectiveness of the proposed heuristic specially that the computing time for each example doesn't exceed 2 seconds.

7. CONCLUSION

In this work an algorithm to solve the two-dimensional guillotine oriented cutting-stock problem was proposed. This algorithm combines two heuristics H1 and H2 which have the following performances. The first is suitable for problems with low size and get a solution with a good quality, the second is suitable for all size problems and get a solution with a medium quality. The basic idea of the new method is to find a temporary solution with the second heuristic (H2) and then improve it by solving a small, critical part of the problem again with the first heuristic (H1). This algorithm has been tested on a set of problems. The experimental work shows the following performances of the combined approach: a good quality of the solution (the VREH is less than 2%) and a low computing time to be suitable for all size problems.

REFERENCES

- Benati,S, 1997. An algorithm for a cutting stock problem on a strip. *Journal of Operational Research Society*, 48, p. 288-294.
- Dikili.A.C, 2004. A new approach for the solution of the two-dimensional guillotine-cutting problem in ship production. *Ocean Engineering*, 31, p. 1193-1203.
- Farley,A, 1988. Mathematical programming models for cutting stock problems in the clothing industry. *Journal of Operational Research Society*, 39, p. 41-53.
- Farley,A, 1990b. The cutting stock problem in the canvas industry. *European Journal of Operational Research*, 44, p. 247-255.
- Franca.R and Annamaria.F, 2007. A two-dimensional strip cutting problem with sequencing constraint. *European Journal of Operational Research*, Elsevier, in press.
- Gasimov.R.N., Sipahioğlu.A & Saraç.T, 2007. A multi-objective programming approach to 1.5-dimensional assortment problem. *European Journal of Operational Research*, 179(1), p. 64-79.
- Gilmore.P.C. and Gomory.R.E, 1963. A linear programming approach to the cutting stock problem. part II. *Operation Research*, 11, p. 863-888.
- Gilmore.P.C. and Gomory.R.E, 1965. Multistage cutting-stock problems of two and more dimensions. *Operation Research*, 13, p. 94-120.
- Gilmore.P.C. and Gomory.R.E, 1966. Theory and computation of knapsack functions. *Operation Research*, 14, p. 1045-1074.
- Gilmore.P.C. and Gomory.R.E, 1961. A linear programming approach to the cutting stock problem. *Operation Research*, 9, p. 849-859.
- Johnson.R.E, 1986. Rounding algorithms for cutting stock problems. *Asia-Pacific Journal of Operational Research*, 3, p. 166-171.
- Johnson.M.P., Rennick.C. & Zak.E, 1997. Skiving addition to the cutting stock problem in the paper industry. *Journal of Siam Review*, 39(3), p. 472-483.
- Menon.S. & Schrage.L, 2002. Order allocation for stock cutting in the paper industry. *Operation Research*, 50, p. 324-332.
- Mornar.V and Khoshnevis.B, 1997. A cutting stock procedure for printed circuit board production. *Computers & Industrial Engineering*, Volume 32, p. 57-66.
- Schultz.T.A, 1995. Application of linear programming in a gauge splitting operation. *Operations Research*, 43 (5), p. 752-757.
- Suliman. Saad.M.A, 2001. Pattern generating procedure for the cutting stock problem. *International Journal of Production Economics*, 74, p. 293-301.
- Suliman. Saad.M.A, 2006. A sequential heuristic procedure for the two-dimensional cutting-stock problem. *International Journal of Production Economics*, 99, p. 177-185.
- Wang,P.Y, 1983. Two algorithms for constrained two dimensional cutting stock problems. *Operation Research*, 31, p. 573-586.
- Yanasse.H.H. and Limeira.M.S, 2006. A hybrid heuristic to reduce the number of different patterns in cutting stock problems. *Computers and operations research*, 33, p. 2744-2756.