7^e *Conférence Francophone de MOdélisation et SIMulation - MOSIM'08 - du* 31 *mars au* 2 *avril* 2008 *- Paris - France*
«Modélisation, Optimisation et Simulation des Systèmes : Communication, Coopération et Coordination»

INTEGRATED PRODUCTION AND PREVENTIVE MAINTENANCE IN PRODUCTION SYSTEMS SUBJECT TO RANDOM FAILURES

El-Houssaine AGHEZZAF, Carles SITOMPUL

Department of Industrial Management (IR18)
FirW, Ghent University, Technologiepark 903
Gent-Zwijnaarde B-9052, Belgium
`ElHoussaine.Aghezzaf@UGent.be`

Najib NAJID

Département GMP
IUT de Nantes 2 Av. du Professeur Jean Rouxel
PB. 595-44475 Carquefou-cedex, France
`Najib.Najid@iut-nantes.univ-nantes.fr`

RÉSUMÉ : *We consider the problem of integrating production and preventive maintenance plans in production systems subject to random failures. In particular, we are given a set of items to be produced on a set of capacitated production lines, which are subject to progressive deterioration. We assume that failure rate distributions of the production lines are known and that preventive maintenance is used to bring the system to its initial state. We also assume that any maintenance action, performed on a production line in a given period, reduces the available production capacity of the line during that period. We model this problem as a nonlinear mixed integer program when each production line implements a cyclical preventive maintenance policy, and suggest an approximate algorithm for its solution based on Lagrangian decomposition. We then reformulated the problem as a linear mixed integer program when a general preventive maintenance policy is implemented. The value of this last model provides a lower bound on the value of the first model. We finally conducted some computational tests to analyze the performance of the approximate algorithm and discussed some issues that still need investigation.*

MOTS-CLÉS : *Production Planning, Preventive Maintenance, Integration*

1. INTRODUCTION

Production and maintenance planning functions are often dealt with separately and carried out sequentially in production systems subject to failures. It is a prevailing practice that maintenance plans are developed based only on the system's reliability data without taking into account neither product type nor its volume. Production plans are then developed within the frame imposed by the system's maintenance plans. This results often in production and maintenance plans which are not optimal with respect to the objective of minimizing the combined production and maintenance cost. It is, therefore, essential to integrate production and preventive maintenance plans effectively in an attempt to achieve optimal tradeoffs between production and expected maintenance costs.

The issue of simultaneously planning production and preventive maintenance, in deteriorating production systems, can be tackled at both operational and tactical levels of the planning process. At the operational level, scheduling problems on unreliable machines have received a considerable attention in literature (Glazebrook 1987, Adiri et al. 1989, Birge et al. 1990, Li and Cao 1995, Graves and Lee 1999, Lee and Chen 2000). A large part of these scheduling problems assume that maintenance periods are known in advance, and thus reduce to scheduling problems with machine availability constraints (Adiri et al. 1989, Lee 1996, Qi et al. 1999). A recent survey on scheduling problems with limited machine availability can be found in (Schmidt, 2000). A general result is that scheduling problems, with availability constraints under different machine configurations and various objective functions, are NP-hard (Lee 1997, Kubiak et al. 1997), and thus various heuristics as well as exact methods for their solution were proposed.

At the tactical level, Wienstein and Chung (1999) presented a three-part model to resolve the conflicting objectives of system reliability and profit maximization. An aggregate production plan is first generated, then a master production schedule is developed to minimize the weighted deviations from the specified aggregate production goals, and finally work center loading requirements, determined through rough cut capacity planning, are used to simulate equipment failures during the aggregate planning horizon. Several experiments are used to test the significance of various factors for maintenance policy selection. These factors include the category of maintenance activity, maintenance activity frequency, failure significance, maintenance activity cost, and aggregate production policy. Aghezzaf et al. (2003) presented a production and maintenance planning model for single-machine systems subject to preventive maintenance with minimal repair at failure. They assumed that maintenance actions carried out on the machine reduce its yield. They proposed a mathematical programming model to establish optimal integrated production and maintenance plans for these systems.

In the following section, we propose a mathematical model to optimally integrate production and maintenance plans in production systems implementing cyclical preventive maintenance policies. In section 3, we suggest a Lagrangian decomposition method to solve this problem. Section 4 presents a mixed integer linear program which models the general integrated production and preventive maintenance planning problem. This latter model provides lowers bound for the first model. The last section presents some computational results related to the performance of the proposed Lagrangian decomposition algorithm. Finally, some possible extensions and remarks are discussed in the conclusion.

2. PRODUCTION AND CYCLICAL PREVENTIVE MAINTENANCE PLANNING

We are given a planning horizon H of length $T = N\tau$, covering N periods of fixed length τ, and a set of products $i \in P$ to be produced on a set of capacitated production lines $j \in L$ during this planning horizon. Each production lines $j \in L$ of the system produces each product $i \in P$ with a known production rate γ_{ij} expressed in product units per unit of time. During each period $t \in H$, a demand d_{it} of each product $i \in P$ must be satisfied. We assume that each production lines $j \in L$ has a known nominal capacity (given in time units) denoted by κ_j, and that each preventive or corrective maintenance action performed on the line consumes a certain percentage of this capacity. Consequently, each planned preventive maintenance action and each corrective maintenance action on the line $j \in L$ consume respectively $\theta_j^p = \alpha\kappa_j$ and $\theta_j^r = \beta\kappa_j$ capacity units (where $0 \leq \alpha \leq 1$ and $0 \leq \beta \leq 1$). We also assume that the failure distribution of each production line $j \in L$ is known, and we denote by $f_j(t)$ and $F_j(t)$ its corresponding probability and cumulative density functions respectively. Next, we let $r_j(t)$ denote the failure rate function of the production line $j \in L$ at time t. It is well known that $r_j(t)$ is given by:

$$r_j(t) = \frac{f_j(t)}{1 - F_j(t)}$$

Finally, we assume that preventive maintenance tasks are performed periodically on each production line $j \in L$, at periods $t = 1, (n_j + 1)\tau, (2n_j + 1)\tau, (3n_j + 1)\tau, \cdots, T$, and that corrective maintenance tasks are carried out on the line at failures.

The following mathematical program models the problem of determining optimal integrated production and cyclical preventive maintenance plans in a multi-line production system. The model assumes that each production line returns to its initial status after each preventive maintenance action, that any failure of equipment is immediately repaired, and that the expected failures increase with elapsed time since the last preventive maintenance.

To define the model parameters we let c_j^p be the cost of preventively maintaining production line $j \in L$ in the beginning of each maintenance cycle, and c_j^r be the cost of performing a corrective maintenance action on production line $j \in L$ when a failure occurs. We also let f_{it}^j be the production setup cost of item $i \in P$ in period $t \in H$ on production line $j \in L$, p_{it}^j be the production variable cost of item $i \in P$ in period $t \in H$ on production line $j \in L$, h_{it}^j be the inventory holding cost of item $i \in P$ in period $t \in H$, and ρ_{ij} be the processing time of item $i \in P$ on production line $j \in L$ (expressed in time units per item).

The model variables are given by: x_{it}^j the quantity of item $i \in P$ produced on line $j \in L$ during period $t \in H$, I_{it} the inventory of item $i \in P$ at the end of period $t \in H$, $T^j = n_j\tau$ the length of the preventive maintenance cycle for production line $j \in L$, and y_{it}^j a binary decision variable set to 1 if item $i \in P$ is produced on line $j \in L$ during period $t \in H$ and 0 otherwise.

The mathematical model:

$(PCPM)$: Minimize

$$Z_{PM} = \sum_{t \in H} \sum_{i \in P} \left(\sum_{j \in L} \left(f_{it}^j y_{it}^j + p_{it}^j x_{it}^j \right) + h_{it} I_{it} \right) +$$

$$\sum_{j \in L} \left\lceil \frac{T}{T^j} \right\rceil \left(c_j^p + c_j^r \int_0^{T^j} r_j(t) dt \right)$$

subject to:

$$\sum_{j\in L} x_{it}^{j} + I_{i,t-1} - I_{it} = d_{it}, \quad \forall i \in P, t \in H \qquad (1)$$

$$\sum_{i\in P} \rho_{ij} x_{it}^{j} \leq \kappa_j(t), \quad \forall j \in L, t \in H \qquad (2)$$

$$x_{it}^{j} - d_{tN}^{i} y_{it}^{j} \leq 0, \quad \forall i \in P, j \in L, t \in H \qquad (3)$$

$$\sum_{i\in P} y_{it}^{j} \leq 1, \qquad \forall j \in L, t \in H \qquad (4)$$

$$x_{it}^{j}, I_{it} \geq 0, \; y_{it}^{j} \in \{0,1\} \, \forall i \in P, j \in L, t \in H.$$

where $d_{tN}^{i} = \sum_{s\in H, s\geq t} d_{is}$, and the function $\kappa_j(t)$ defines the available capacity of the production line $j \in L$ in period $t \in H$ as is explained below.

Constraints (1) are the flow conservation constraints defined for any item $i \in P$ in any period $t \in H$. They guarantee that the available inventory of an item augmented with the quantity produced, on each production line, of that item is sufficient to satisfy its demand in period t. The remainder is stocked for the subsequent periods. Constraints (2) are the capacity restrictions defined for each production line $j \in L$ in each period $t \in H$. They guarantee that the quantity produced on a production line does not exceed the available capacity of that line in period t. Constraints (3) force payments of the fixed costs, in any period $t \in H$, for any item $i \in P$ that is produced during that period. Constraints (4) ensure that at any moment no more than one product is scheduled on a production line.

We now define exactly how the available capacity function of each production line in each period is determined. Consider a production line $j \in L$, and assume that its preventive maintenance cycle is given by $T^j = n_j\tau$. Assume also that a maintenance action on this line took place in the beginning of a period $s \in H$, and that it covers the periods $\{s, s+1, \cdots, (s+n_j-1)\}$. The available capacity of the production line $j \in L$ in period s is then given by equation:

$$\kappa_j(s) = \kappa_j - \theta_j^p - \theta_j^r \int_0^\tau r_j(u)\, du \qquad (5)$$

The capacity $\kappa_j(t)$ for the periods $t \in \{s+1, \cdots, (s+n_j-1)\}$, during which only the necessary corrective maintenance actions take place, is given by equation:

$$\kappa_j(t) = \kappa_j - \theta_j^r \int_0^\tau r_j(u + (t-s)\tau)\, du \qquad (6)$$

Notice that when the maintenance periods are given, the available capacity of each production line during each period can be determined, and that the expected maintenance cost becomes a constant. In this case the problem $(PCPM)$ reduces to the usual multi-item multi-line capacitated lot-sizing problem. Thus, problem $(PCPM)$ belongs to the class of NP-hard problems since it contains, as a special case, the single-line capacitated lot-sizing problem known to be NP-hard (Bitran and Yanasse 1982).

An exact algorithm for problem $(PCPM)$:

Clearly, the variables $(T^j)_{j\in L}$ defining the preventive maintenance cycles, for the production lines of the system, cause the objective function of the problem $(PCPM)$ to be nonlinear. Now, recall that if these preventive maintenance cycles are known in advanced the problem reduces to a capacitated multi-item multi-line lot-sizing problem. This latter problem can be solved with any usual linear mixed integer solver. Also, many branch and bound, branch and cut, and branch and price exact methods as well as many heuristic methods were proposed to solve the single-line version of this problem (Florian and Klein 1971, Lambert and Luss 1982, Eppen and Martin 1987, Aggarwal and Park 1993, Van Hoesel and Wagelmans 1996, Belvaux and Wolsey 2000). A recent survey on the multi-item capacitated lot-sizing problem can be found in (Wolsey 2002). The following steps describes an algorithm to solve the problem $(PCPM)$:

- For each combination of the preventive maintenance cycles $(n_1, \ldots, n_j, \ldots, n_N)$ of the production lines $j \in L$, with $n_j \in \{1, 2, \ldots, N\}$, perform the following two steps:
    1. Compute the expected maintenance cost corresponding to the selected combination, and determine for each production line $j \in L$ the available capacity $\kappa_j(t)$ in each period $t \in H$.
    2. Solve the resulting pure production planning problem, with the above computed available capacities $(\kappa_j(t)_{t\in H})_{j\in L}$, and add together the resulting expected maintenance and production costs.
- Rank these total costs and select the plan resulting in the least cost, this is the optimal integrated production and cyclical preventive maintenance plan.

The number of possible combinations of the preventive maintenance cycle sizes is exponential of order $N^{|L|}$, where N is the number of periods in the planning horizon. Now, observe that when the number of production line is very small (2 or 3), then the complexity of the above procedure depends actually on the complexity of the algorithm used to solve the pure capacitated lot-sizing problems.

In the following section we discuss a Lagrangian decomposition based approximation algorithm for the problem ($PCPM$). Since the single-line production and cyclical preventive maintenance planning problem play a central role in this procedure, we first start by detailing how this latter problem can be efficiently solved.

3. A LAGRANGIAN RELAXATION BASED ALGORITHM FOR PROBLEM ($PCPM$)

The mathematical program ($PCPM$) models the integrated production and preventive maintenance planning problem on parallel production lines implementing cyclical preventive maintenance policies. To take advantage of the properties of the single line version of the problem, we reformulate the problem ($PCPM$) to make these single line versions appear as sub-problems. We therefore introduce new variables δ_{it}^j and η_{it}^j both defining the fraction of demand of item $i \in P$ produced on production line $j \in L$ in period $t \in H$, (i.e. $\delta_{it}^j = \eta_{it}^j$). We also introduce the variables I_{it}^j defining the proportion of inventory of item $i \in P$ at the end of period $t \in H$ resulting from the production line $j \in L$. The resulting model becomes then:

A reformulated mathematical model ($PCPM_R$):

$PCPM_R$: Minimize

$$Z_{PM} = \sum_{j\in L}\sum_{t\in H}\sum_{i\in P}\left(f_{it}^j y_{it}^j + p_{it}^j x_{it}^j + h_{it} I_{it}^j\right) + \sum_{j\in L}\left\lceil \frac{N}{n_j}\right\rceil\left(c_j^p + c_j^r\int_0^{n_j\tau} r_j(t)dt\right)$$

subject to:

$$x_{it}^j + I_{i,t-1}^j - I_{it}^j = \delta_{it}^j, \qquad \forall i \in P, j \in L, t \in H \quad (7)$$

$$\sum_{i\in P}\rho_{ij}x_{it}^j \le \kappa_j(t), \qquad \forall j \in L, t \in H \quad (8)$$

$$x_{it}^j - d_{tN}^i y_{it}^j \le 0, \qquad \forall i \in P, j \in L, t \in H \quad (9)$$

$$\sum_{i\in P} y_{it}^j \le 1, \qquad \forall j \in L, t \in H \quad (10)$$

$$\delta_{it}^j - \eta_{it}^j = 0, \qquad \forall i \in P, j \in L, t \in H \quad (11)$$

$$\sum_{j\in L}\eta_{it}^j = d_{it}, \qquad \forall i \in P, t \in H \quad (12)$$

$$x_{it}^j, I_{it}^j, \delta_{it}^j, \eta_{it}^j \ge 0,\ y_{it}^j \in \{0,1\}\, \forall i \in P, j \in L, t \in H.$$

Constraints (7) are the new flow conservation constraints defined for each production line $j \in L$. The constraints (11) and (12) guarantee, for each item $i \in P$ and each period $t \in H$, that the sum of the quantities shipped from inventories accumulated by the production lines $j \in L$ are sufficient to cover the demand of item $i \in P$ in period $t \in H$.

Examining carefully the problem ($PCPM_R$), we observe that if the constraints (11) are relaxed, the problem splits up into $(|L| + 1)$ sub-problems. The first $|L|$ sub-problems consist of planning production and cyclical preventive maintenance on each single production line $j \in L$ for a given demand vector δ^j. The last sub-problem is a simple linear program, which assigns the demands to be produced to the production lines with the objective of minimizing the marginal production costs of each production line.

Now, let λ_{it}^j be the Lagrangian multipliers associated with constraints (11). The resulting sub-problem for each production line $j \in L$ is given by:

$PCPM_{LR}^j(\lambda)$: Minimize

$$V_{PM}^j(\lambda) = Z_{PM}^j - \sum_{t\in H}\sum_{i\in P}\lambda_{it}^j\delta_{it}^j$$

subject to:

$$x_{it}^j + I_{i,t-1}^j - I_{it}^j = \delta_{it}^j, \qquad \forall i \in P, j \in L, t \in H \quad (13)$$

$$\sum_{i\in P}\rho_{ij}x_{it}^j \le \kappa_j(t), \qquad \forall j \in L, t \in H \quad (14)$$

$$x_{it}^j - d_{tN}^i y_{it}^j \le 0, \qquad \forall i \in P, j \in L, t \in H \quad (15)$$

$$\sum_{i\in P} y_{it}^j \le 1, \qquad \forall j \in L, t \in H \quad (16)$$

$$x_{it}^j, I_{it}^j, \delta_{it}^j \ge 0,\ y_{it}^j \in \{0,1\}\, \forall i \in P, j \in L, t \in H.$$

where $\kappa_j(t)$ is the available capacity of the production line $j \in L$ in period $t \in H$, defined by equations (5) and (6), and Z_{PM}^j given by:

$$Z_{PM}^j = \sum_{t\in H}\sum_{i\in P}\left(f_{it}^j y_{it}^j + p_{it}^j x_{it}^j + h_{it} I_{it}^j\right) + \left\lceil \frac{N}{n_j}\right\rceil\left(c_j^p + c_j^r\int_0^{n_j\tau} r_j(t)dt\right)$$

As already mentioned, observe that if the vector δ^j is given, then the quantity $\sum_{t\in H}\sum_{i\in P}\lambda_{it}^j\delta_{it}^j$ in the objective function of $\left(PCPM_R^j(\lambda)\right)$ becomes a constant. The resulting problem reduces then to the problem of planning production and cyclical preventive maintenance on a single production line, in which δ_{it}^j are the demands for each item $i \in P$ in each period $t \in H$.

The last sub-problem $(DASP(\lambda))$ determines, for each item $i \in P$ in each period $t \in H$, the best assignment of the demand d_{it} to the production lines $j \in L$, and is given by:

$DASP(\lambda)$: Minimize

$$V^0_{PM}(\lambda) = \sum_{j \in L} \sum_{t \in H} \sum_{i \in P} \lambda^j_{it} \eta^j_{it}$$

subject to:

$$\sum_{j \in L} \eta^j_{it} = d_{it}, \qquad \text{for all } i \in P, t \in H \tag{17}$$

$$\eta^j_{it} \geq 0 \quad \text{for all } i \in P, j \in L, t \in H.$$

If we let $\hat{V}^j_{PM}(\lambda)$ and $\hat{V}^0_{PM}(\lambda)$ be, respectively, the optimal values of the problems $\left(PCPM^j_{LR}(\lambda)\right)$ and $(DASP(\lambda))$ for a given $\lambda \in \mathbf{R}$, then the Lagrangian dual associated with the above relaxed problem is given by:

$$W_{LD} = \text{Maximize}_{\lambda \in \mathbf{R}} \left(\sum_{j \in L} \hat{V}^j_{PM}(\lambda) + \hat{V}^0_{PM}(\lambda) \right)$$

In the following we discuss an approximation solution procedure to solve the problem $(PCPM)$ using the subproblems $\left(PCPM^j_{LR}(\lambda)\right)$ and $(DASP(\lambda))$.

A Lagrangian based solution procedure for the problem ($PCPM$): To launch the solution process explained below, we need to start with a first assignment of demands to the production lines. We start with an assignment of demands to the production lines in proportions of their processing rates (fair share):

$$\delta^j_{it} = \frac{\rho_{ij}}{\sum_{j' \in L} \rho_{ij'}} d_{it} \quad \text{for all } i \in I, t \in H, j \in L.$$

Step 1: Solve the linear programming relaxations of the problems $\left(PCPM^j_{LR}\right)$ obtained from the problems $\left(PCPM^j_{LR}(\lambda)\right)$ by letting $\lambda = 0$ and δ^j as defined above. We use the single production line algorithm as described above but where the pure production planning problems $\left(PCPM^j(n_j)\right)$, for fixed values of n_j, are solved as linear programs (LP-relaxation).

Step 2: Get the shadow prices λ^j_{it} associated with the constraints (17) for the optimal problem $\left(PCPM^j\right)$, i.e. the optimal $\left(PCPM^j_{SL}(\hat{n}_j)\right)$.

Step 3: Using these shadow prices λ^j_{it} solve the problem $(DASP(\lambda))$ and get the new values for δ^j. These values are used in step 1 to get the new shadow prices as explained in step 2, and the process is continued until the values of the problems $\left(PCPM^j_{LR}\right)$ are not changing.

Step 4: Once the final values of δ^j are obtained, then for each line we use the single production line algorithm described above, but now the problems $\left(PCPM^j\right)$ are solved as integer linear programs. The collection of optimal plans for each production line $j \in L$ is taken as the best approximate to the optimal solution for the problem $PCPM$.

The values obtained with this solution procedure is compared to a very good lower bound on the optimal solution of the problem to assess the optimality gap.

4. PRODUCTION AND GENERAL PREVENTIVE MAINTENANCE PLANNING

In many situations additional constraints, such as limited size and number of maintenance crews combined with the productivity requirement, can make it difficult to implement cyclical preventive maintenance strategies for all production lines of the systems. In the following paragraph, we present a model that is more general in the sense that it relaxes the cyclical restriction, and can handle these additional constraints. Before developing the model, let us first define some necessary additional parameters and variables:

Let $\kappa^j_s(t)$ be the expected loss in production capacity of line j, in period t, when the last preventive maintenance action before time t on the line j has taken place in period s, $s \leq t$. Thus, this expected loss is then given by:

$$\kappa^j_s(t) = \begin{cases} \theta_p + \theta_r \int_0^\tau r_j\,(u)\,du & \text{if } t = s, \\ \theta_r \int_0^\tau r_j\,(u + (t-s)\tau)\,du & \text{if } t > s. \end{cases} \tag{18}$$

Let $c^j_s(t)$ be the expected maintenance cost of line j, in period t, when the last preventive maintenance action before time t on the line j has taken place in period s, $s \leq t$. Thus, this expected cost is then given by:

$$c^j_s(t) = \begin{cases} c^p + c_r \int_0^\tau r_j\,(u)\,du & \text{if } t = s, \\ c_r \int_0^\tau r_j\,(u + (t-s)\tau)\,du & \text{if } t > s. \end{cases} \tag{19}$$

Finally, let y^j_{st} be a binary decision variable set to 1 if the preventive maintenance cycle on line j covering period t has started at period s, and 0 otherwise. The general integrated production and maintenance planning model ($PGMP$) is given by:

$PGMP$: Minimize

$$Z_{PM}^G = \sum_{t\in H}\sum_{i\in I}\left(\sum_{j\in L}\left(f_{it}^j y_{it}^j + p_{it}^j x_{it}^j\right) + h_{it} I_{it}\right) + \sum_{t\in H}\sum_{s\in H, s\leq t} c_s^j(t) z_{st}^j$$

subject to:

$$\sum_{j\in L} x_{it}^j + I_{i,t-1} - I_{it} = d_{it}, \quad \forall i \in P, t\in T \quad (20)$$

$$\sum_{i\in I}\rho_{ij} x_{it}^j + \sum_{s\in H, s\leq t}\kappa_s^j(t) z_{st}^j \leq \kappa_j, \quad \forall j\in L, t\in T \quad (21)$$

$$x_{it}^j - d_{tN}^i y_{it}^j \leq 0, \quad \forall i\in P, j\in L, t\in T \quad (22)$$

$$\sum_{i\in I} y_{it}^j \leq 1, \quad \forall j\in L, t\in T \quad (23)$$

$$\sum_{s\in H, s\leq t} z_{st}^j = 1, \quad \forall j\in L, t\in T \quad (24)$$

$$z_{st}^j - z_{st-1}^j \leq 0, \quad \forall j\in L, s\in T, t\in T, s<t \quad (25)$$

$$x_{ij}^t, I_{it} \geq 0,\ y_{ij}^t, z_{st}^j \in \{0,1\}\, \forall i\in P, j\in L, s,t\in H.$$

Constraints (21) are the new capacity constraints. These constraints define for each production line $j \in L$ the available capacity during each period. Constraints (24) determine the preventive maintenance periods for each production line. Constraints (25) guarantee, that if a period s is the last preventive maintenance period before $t > s$, then s is also the last preventive maintenance period before $t-1$.

The advantage of this problem is the fact that the objective function is not any more nonlinear, we can then solve it using any solver for linear mixed integer programs. In the following section we will use this model to generate lower bounds for the test problem ($PCPM$) to assess the optimality gap for our approximation procedure.

5. DESIGN OF EXPERIMENT AND COMPUTATIONAL RESULTS ANALYSIS

To assess the quality of the solutions of the problem ($PCPM$) obtained with the Lagrangian approximation procedure, we developed a design of experiment based on some critical planning parameters which are thought likely to have a significant impact on these approximate solutions. Although the number of products, the number of production lines, and the number of periods in the planning horizon are important factors to investigate, we limited our experimental design to study the impact of the cost structure, the demand structure, the capacity tightness, and the failure distribution of each production line. We assume that the failure distribution of each production line $j \in L$ can be either a Gamma distribution with a shape parameter of 2 and a rate parameter which can randomly be selected from the set $\{1,2\}$, or a Weibull distribution with a shape parameter of 2 and the scale parameter which can randomly be selected from the set $\{1,2\}$.

The cost structure is assumed to have the following properties: the setup cost f_{it}^j for each product is selected randomly from $[25,100]$, the production cost p_{it}^j is selected randomly from $[5,10]$, and the holding cost is then defined as $h_{it} = \alpha\% max_j(p_{it}^j)$, where α is uniformly distributed between 5 and 20. The demand for each product $i \in P$ is assumed to have a stationary mean $\bar{d}_i$. The mean demands are randomly sampled from the interval $[75,100]$, and the period-by-period demands for each product $i \in P$ are generated from a truncated normal distribution having mean $\bar{d}_i$ and a standard deviation randomly sampled from the interval $[0.25\bar{d}_i, 0.5\bar{d}_i]$.

The capacity is the last parameter set for each production line in each test problem. We first introduce the target average utilization of capacity factor β to define the capacity tightness. The factor β is set to 0.75, 0.85 and 0.95 corresponding respectively to situations with loose, moderately loose and tight capacity constraints. We then compute the capacity required, in each period, as if a lot-for-lot solutions were implemented. These values are averaged over all periods and the result is divided by the number of production lines. The regular capacity of each production line in each period is then obtained by dividing the later result by the target average utilization of capacity factor β. The remaining parameters are given as follows: the number of production lines is 3, number of different products is set to 5, and the length of the planning horizon is set to 12 periods.

For each capacity tightness level, we first generate a group of 10 test problems in which the failure distributions of all production lines are Gamma distributions, then a group of 10 other test problems in which the failure distributions of all production lines are Weibull distributions, and finally a group of 10 test problems in which the failure distribution of each line is randomly selected randomly to be a Gamma or a Weibull distribution (Mixed). In this last group, test problems in which the failure distribution are all the same are discarded. Each test problem is solved with the Lagrangian relaxation based approximation solution to get a very good upper bound $\tilde{Z}_{PM}$ on the optimal value the problem $(PCPM)$. We then solve the model $(PGMP)$ for the same instance to get a lower bound on the optimal value the problem $(PCPM)$. We finally compute the gap as follows:

$$Gap = 100\frac{\tilde{Z}_{PM} - Z^{G}_{PM}}{\tilde{Z}_{PM}}\%$$

The following table shows the summary of the results of the averages of above ratios computer for each test group:

	Capacity tightness		
Failure Distribution	Loose	Mod. loose	Tight
Gamma	1.24	1.97	2.26
Weibull	1.07	2.15	2.37
Mixed	1.19	1.92	2.71

Table 5 : Summary of the Solution gaps (mean).

The average solution gap ratios shown on table 5 indicate that the tightness of the capacity constraint has a significant effect on the solution gap. Test problems with tight capacity constraints have much larger solution gaps than those with loose capacity. Actually, this result was expected since, for test problems with loose capacity, usually the two independently obtained optimal production and maintenance plans produce a feasible optimal integrated plan for the problem $(PCPM)$. The table shows also that when Weibull distributions are used, the solution gaps become larger than when Gamma distributions are used. But this is also expected at some extent since the expected capacity loss, as a function of the age of the system, is larger in case of a Weibull distribution. Overall, the results obtained are fairly good in terms of optimality gaps but also in terms of computational time in comparison with the time required to exactly solve the problem.

In our Lagrangian based approximation procedure, the assignment of demands to the production lines is carried out on the basis of marginal costs obtained through the solutions of LP-relaxations of the single-line linear mixed integer sub-problems. When no improvement is realized we keep the last assignment of demands and solve the latter sub-problem as mixed integer problems. The test problems, at least those we solved exactely, indicate that a large part of the gaps resulted from non-optimal assignments of the demands to the lines. To improve this gap we are currently investigating the Lagrangian dual of the problem $(PCPM_R)$ in which the variables η are eliminated and the constraints (16) are relaxed.

6. CONCLUSION

In this paper we studied the problem of integrating production and preventive maintenance plans in a multi-line production system, in which the production lines are subject to progressive deterioration. We proposed two mathematical programming models for the problem. The first model assumes that each production line of the system implements a periodic preventive maintenance policy with a fixed cycle size. The second model relaxes this last restriction and establishes for each production line specific maintenance periods, which do not necessarily fall at equally distant epochs. The first model is an NP-hard nonlinear mixed integer program, for which we developed an approximation method using Lagrangian relaxation. The sub-problems, used to iteratively construct an approximate optimal solution, are single-line versions of the problem, one for each production line, and a last sub-problem is a linear program used to assign demands to the production lines. A series of test problems, with moderate size, were generated and solved with the Lagrangian based approximation procedure. The optimality gaps were assessed using lower bounds obtained with the second model, which is a linear mixed integer program. The results show that the procedure performs fairly well in terms of both computational time and solution quality. Observe that the integration of production and preventive maintenance plans is tackled here at the tactical level, and through a very precise definition of the available capacities of the production process at any period of time horizon. There is no doubt that this approach performs better than tackling the problem only at the operational level without this precious information on the capacities. Of course this does not mean that the problem at the operational level will become too easy, but it will certainly be simplified. The question of integrating the tactical and operational levels needs to be investigated. Many other technical questions regarding the structure and the complexity of the models obtained with different maintenance policies need to be addressed. Questions regarding the convergence of the solution improvement process used in our approximation procedure and many others are on our agenda for future research.

References

Adiri, I., J. Bruno, E. Frostig, A.H.G. Rinnooy Kan. 1989. Single machine flow-time scheduling with a single breakdown. *Acta Informatica* **26** 679-696.

Aggarwal, A., J.K. Park. 1993. Improved Algorithms For Economic Lot Size Problems. *Oper. Res.* **41** 549-571.

Aghezzaf, E.H., A. Jamali, D. Ait-Kadi. 2006. An integrated production and preventive maintenance planning model. To appear in *European journal of operational research.*

Belvaux, G., L.A. Wolsey. 2000. BC-PROD: A Specialized Branch-and-Cut System for Lot-Sizing Problems. *Management Sci.* **46** 724-738.

Birge, J., J.B.G. Frenk, J. Mittenthal, A.H.G. Rinnooy Kan. 1990. Single-Machine Scheduling Subject to Stochastic Breakdowns. *Naval Res. Logist.* **37** 661-677.

Bitran, G.R., H.H. Yanasse. 1982. Computational Complexity Of The Capacitated Lot Size Problem. *Management Sci.* **28** 1174-1186.

Eppen, G.D., R.K. Martin. 1987. Solving Multi-Item Lot-Sizing Problems Using Variable Redefinition. *Oper. Res.* **35** 832-848.

Florian, M., M. Klein. 1971. Deterministic Production Planning With Concave Costs and Capacity Constraints. *Management Sci.* **26** 669-679.

Glazebrook, K.D. 1987. Evaluating the effects of Machine Breakdowns in Stochastic Scheduling Problems. *Naval Res. Logist.* **34** 319-335.

Graves, G.H., C.Y. lee. 1999. Scheduling maintenance and semiresumable jobs on single machine. *Naval Res. Logist.* **46** 845-862.

Kubiak, W., J. Blazewicz, P. Formanowicz, J. Breit, G. Schmidt. 2002. Two-machine flow shops with limited machine availability. *Eur. J. Oper. Res* **136** 528-540.

Lambert, A., H. Luss. 1982. Production Planning With Time-Dependent Capacity Bounds. *Eur. J. Oper. Res* **9** 275-280.

Lee, C.Y. 1996. Machine scheduling with an availability constraint. *J. of Global Opt.* **9** 395-416.

Lee, C.Y. 1997. Minimizing the makespan in the two machines flowshop scheduling problem with an availability constraint. *Oper. Res. Lett.* **20** 129-139.

Lee, C.Y., Z.L. Chen. 2000. Scheduling jobs and maintenance activities on parallel machines. *Naval Res. Logist.* **47** 145-165.

Li W., J. Cao. 1995. Stochastic Scheduling on a single machine subject to multiple breakdowns according to different probabilities. *Oper. Res. Lett.* **18** 81-91.

Qi, X., T. Chen, F. Tu. 1999. Scheduling the maintenance on single machine. *J. Oper. Res. Soc.* **50**(10) 1071-1078.

Schmidt, G. 2000. Scheduling with limited machine availability. *Eur. J. Oper. Res* **121** 1-15.

Van Hoesel, C.P.M., A.P.M. Wagelmans. 1996. An O(T3) Algorithm For The Economic Lot-Sizing Problem With Constant Capacities. *Management Sci.* **42** 142-150.

Wienstein, L., C.H. Chung. 1999. Integrated maintenance and production decisions in a hierarchical production palnning environnement, *Comp. Oper. Res.* **26** 1059-1074.

Wolsey, L.A. 2002. Solving Multi-Item Lot-Sizing Problems with an MIP Solver Using Classification and Reformulation. *Management Sci.* **48**, 1587-1602.