

# SUR LES SYSTEMES MIN-MAX-PLUS

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The theory of Discrete Event Systems (DESS) is a research area of current interest. The development of this theory is largely stimulated by discovering general principles which are useful to a wide range of application domains. In particular, technological and/or 'man-made' manufacturing systems, communication networks, transportation systems, and logistic systems, all fall within the class of DESS. One of the key features that characterize Discrete Event Systems (DESS) is that their dynamics are event-driven as opposed to time-driven, i.e., the behavior of a DES is governed only by occurrences of different types of events over time rather than by ticks of a clock.

We will concentrate on modelling and analysis issues of DESS within the min-max-plus algebra setting. Min-max-plus systems are defined by the operations minimization, maximization and addition over the real numbers. As such these models are an extension of the more well-known max-plus systems defined by the operations maximization and addition only.

A very brief introduction to max-plus systems will be given, with some applications, and we will then extend this to min-max-plus systems. The extra minimization operator gives more flexibility with respect to modelling issues. The analysis, however, becomes more complex.

Min-max-plus systems will be defined properly and eigenvalues for such systems will be defined and their existence discussed. Such eigenvalues are an indication as to how "fast" the system can "work". Specific results will be given for some subclasses, such as bipartite systems. For the more mathematical oriented reader: min-max-plus systems lie dense in the class of systems described by non-expansive mappings. The behavior of these systems becomes periodic after a transient part.

A popular application of max-plus algebra is the design of timetables for trains on an existing net and answering related questions with respect to capacity issues, propagation of delays. We will discuss whether (and how) more practical requirements can be incorporated in the modelling if the minimization operator is added.