

Discontinuity of SVD Embedding Mapping Used for Watermarks

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Abstract. This paper presents a verification test on a quasi-one-way characteristic of SD watermarking. Norm of distance between two matrices is one of measures for evaluating degree of computational burden against reverse analysis. The norm distance shows a kind of nonlinearity and even non-continuous phenomenon, in spite of the norm is smooth quadratic function.

Keywords: SVD, one-way function, mapping, norm, eigen-value.

1 Introduction

Singular Value Decomposition (SVD) watermarking is one of fields of research now found in academic publications. Among them, a semi-blind SVD watermarking method [1] shows robustness with difficulty of inversion attacks because of its quasi-one-way characteristic. The same SVD watermarking method is seen in [2]. The SVD is a specific operation, which maps image 'G' into SVD value matrix 'S' with positive values on the diagonal positions and all other zero values for off-diagonal positions. This specificity that all elements of the matrix S are non-negative prohibits using negative elements. This means that at least on a superficial level subtraction does not exist in the SVD world. From this observation, evaluation of the degree of the one-way characteristic started [3]. It is difficult to prove one-way characteristic in fully mathematical point of view. It is not necessary to accomplish one-way for practical applications. For watermarking applications, some degree of computational difficulty is sufficient to preventing reverse analysis, which is to find embedding way or embedding key. In this paper, numerical evaluations for discontinuous characteristics and error characteristics between the original data and the embedded data are carried out.

2 Verification of Quasi-One-Way Characteristic

Let us formalize the watermarking embedding procedure using SVD as a mapping. Embedding procedure is not a simple function that outputs a single value. So we call the procedure a mapping. Let m be a procedure mapping, $m : G \rightarrow G_w$.

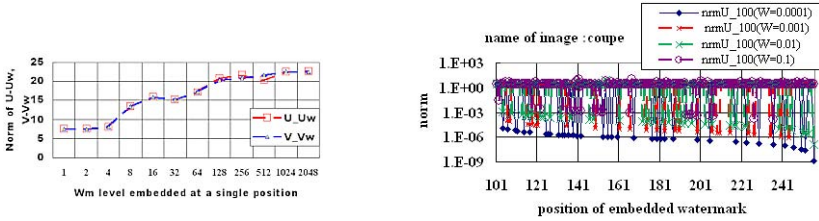


Fig. 1. (a) (Left) Norm vs. embedded level. (b) (Right) Norm vs. embedded position.

For an image G , SVD results $G = U \cdot S \cdot V^T$. For embedded image G_w , another result is $G_w = U_w \cdot S_w \cdot V_w^T$. The Euclidean norm between U and U_w , or V and V_w is evaluated. Fig.1 (a) shows the Euclidean Norm difference between one of the decomposing matrices U and U_w , which depend on the embedding magnitudes. Although the difference increases as the watermark magnitude increases, the tendency is not proportional, but a staircase pattern can be seen as the watermark magnitude increases. This result indicates that, because the Euclidean Norm is continuous since it is a root of the sum of the squared differences of the elements, the SVD process causes discontinuity. Embedded watermark level is on the horizontal axis, and Norm of difference is on the vertical axis.

Fig.1 (b) shows the Euclidean Norm of the difference between U and U_w versus the small difference of the watermark component. The positions of the embedded watermark are in the 100th row of the singular matrix S . Norm vs. embedded position. Discontinuity can be read by viewing vertically at every position. Discontinuous points occasionally exist for specific level transition from a lower position to a higher.

3 Discussions and Future Work

Without quantization, cause of the discontinuity can be from the change of elements of matrix U . Quantization in embedding may cause much discontinuous. Non-linear transform or decomposition will be more useful for quasi-one-way characteristic.

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