Modélisation boîte noire de la relation pluie-débit : un état de l'art

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Résumé— **L'obtention de modèles mathématiques fiables de la relation pluie-débit est une problématique importante présente dans de nombreuses applications liés à la gestion des ressources en eau. Parallèlement aux approches classiques de modélisation fondées sur les lois de la physique se sont développées ces dernières années des méthodes de type boîte noire faisant appel, par exemple, à des structures de modèles non linéaires ou linéaires à paramètres variants. Dans cette communication, un état de l'art des techniques d'identification de modèle pluie/débit est proposé, puis une analyse des différents problèmes restant à traiter est exposée. Enfin, de nouvelles pistes de travail fondées sur des structures de modèles de type boîte grise sont discutées.**

Mots-clés— **Environmental systems, water resources, system identification, LPV models.**

I. Introduction

In the hydraulic practice, modelling of the Rainfall-Runoff Relationship (RRR) is a challenging issue which has great importance in the management and failure detection of urban drainage networks. In fact, urban catchments are usually designed to convey medium intensity rainfall, but increasing urbanization made possible large flow inside drainage networks. So, precise models are necessary to perform network fault detection and to design control systems for real-time storm effects management (see, e.g., [32], [30], [24]).

In the hydraulic practice, a number of approaches have been proposed to model the rainfall-runoff relationship. A first possibility would be to describe the RRR through physical based models completely characterised by a set of design parameters. Good estimates of parameters for these models should be obtained through long term surveys on existing networks, which would be able to provide reliable parameter values in order to design networks on the basis of a previously assigned "design risk" [7]. Unfortunately this approach is not always easily feasible, for three main reasons :

– it would be very expensive because of the required instrumentation and the need for good and frequent maintenance of the data acquisition set-up ;
– if a design of a new sewer network is required, nothing but rainfall can be measured and a suitable model from measures on similar networks must be at hand ;
– when the modelling aim is the rehabilitation of an existing sewer network, survey results may be not homogeneous because of changes occurred in the network due to urbanization.

Therefore, in network design very simple models are used in order to obtain rainfall-runoff relationship models on the basis of a restricted data set. Usually one of the following ways is chosen [6], [12] :

1. to represent the overall process behaviour through simple mathematical relationships (the so-called "systemic approach") ;
2. to model the process in a simplified way, in strict analogy with simple hydraulic systems (the so-called "conceptual modelling approach").
3. black-box modelling.

The main problem concerning systemic and conceptual modelling is that they rely on simple mechanistic models which have poor predicting capability ([22], [29]). Also, for use in network design and maintenance, parameter tuning must be performed very carefully.

Black-box models of the RRR, on the other hand, have been successfully used (see [6], [29]), showing increased prediction accuracy with respect to conceptual model based predictions. More specifically, black-box models have been used with a twofold aim :

– to design new and more precise calibration procedures for conceptual models, as in [29].
– To fulfil the need for accurate models of flow in urban networks, providing information on the network capability to cope with unexpectedly large rainfall events and to develop real-time management strategies.

In this paper, an overview of the existing literature on the problem of predicting water flow in the main sections of an urban drainage network as an effect of rainfall is proposed, a critical view on some open issues is presented and some novel ideas towards a grey-box approach to the problem are presented and discussed. Finally some experimental results obtained by estimating linear and nonlinear parameter-varying models on the basis of experimental data are illustrated.

II. Overview of modelling approaches

A. Conceptual models

In this Section, a short presentation of the most widely used conceptual models is given, together with their parameter tuning procedures. In particular, the main step of the tuning procedure is the estimation of the Instantaneous Unit Hydrograph (IUH), i.e., the impulse response of the rainfall-runoff system.

A.1 Classical reservoir models

This model is based on the hypothesis that the urban drainage network can be effectively considered as a linear tank, i.e., it can be described as a linear system with an impulse response of the form

$$g(t) = \frac{1}{k} e^{-t/k}. \tag{1}$$

So doing, only the tank characteristic time k is needed, which can be defined as follows [14], [12] :

$$k = \frac{W}{Q_r} = \frac{W_r + W_o}{Q_r}, \tag{2}$$

where W is the storage total volume which is the sum of W_r, the total water volume in the pipes when they are totally full and of W_o, which is the water volume stored on the catchment surface ; Q_r is the flow at the catchment closure section when it is totally full. A major drawback of this approach is that the implicit assumption of instantaneous propagation of critical events drives to overestimate the reservoir constant k and consequently to dangerously underestimate the critical flow in the closure section [7].

Many methods based on experimental data and network design parameters have been proposed in the literature for the estimation of the reservoir constant. In particular, in this work, the following estimation formulas have been chosen for comparison with estimates obtained from black-box models :

– Pedersen formula [25]

$$k = 3.458(Ln)^{0.6} i^{-0.4} P^{-0.3} \tag{3}$$

– Desbordes formula [10]

$$k = 5.07 A^{0.18} L^{0.15} \theta^{0.21} (1 + I)^{-1.9} (100P)^{-0.36} h^{-0.07} \tag{4}$$

– Ciaponi-Papiri formula [8]

$$k = 0.5 A^{0.351} S_r^{-0.290} d^{0.358} I)^{-0.163} \tag{5}$$

In addition the Italian method and the Revised Italian Method have been considered [2], [22]. In all the preceding formulae A is the area of the catchment ; I is the fraction of impervious ground on total catchment area ; L is the main pipe length ; n is Manning's roughness coefficient (see [7]) ; P is the mean catchment slope ; θ, h, i are the duration, the height and the intensity of the rain ; S_r is the percent mean slope of the sewer network ; d is the density of drainage, defined as the ratio between the total network length [m] and the catchment area [ha].

A.2 Time-area model

This model [12] is based on the hypothesis that the flow phenomena are more relevant than the accumulation ones. So, it is possible to compute a characteristic travel time that fully characterises the dynamic phenomena in the catchment. The estimated time is known as the concentration time T_c and it is defined as the sum of two time intervals :

– the network time t_r, which is the time that a water drop takes to run along the longest pipe in the network, when it is completely full ;
– the entry time t_e, which is the time that a water drop takes to enter the sewer network from outside.

The network time t_r can be analytically obtained knowing the water speed at full pipe and the length of the longest hydraulic path in the network. The travel speed at full pipe flow is computed using the Chézy formula [7] :

$$V = K_S \left(\frac{D}{4} \right)^{2/3} \sqrt{s} \tag{6}$$

where K_S is the Strickler's roughness coefficient ; D is the pipe diameter ; s is the pipe mean slope. The entry time t_e should be estimated empirically.

The concentration time is usually estimated by the IUH through a least squares fitting of its integral (i.e., the system step response called the "S-shaped hydrograph" from its typical shape) with a three segment broken line. The concentration time estimate is the difference between the start-stop abscissa of the middle line segment.

A.3 Nash model

In the Nash model [12] the catchment is modelled as a sequence of n linear tanks, all with the same characteristic time k. Using $n \in \mathcal{R}^+$, $k \in \mathcal{R}^+$, a more general model is obtained, with the following, analytically computable, IUH :

$$g(t) = \frac{1}{k\Gamma(n)} \left(\frac{t}{k} \right)^{n-1} e^{\left(-\frac{t}{k}\right)}. \tag{7}$$

Unfortunately the used empirical tuning procedures for k and n generally give unsatisfactory results. The more reliable estimates are obtained using the empirical Ramachanda method [12] :

$$k = 0.0883 S^{0.399} (1 + I)^{-0.662} P^{-0.106} \theta^{0.222} \tag{8}$$

$$n = \frac{0.1547 S^{0.458} (1 + I)^{1.662} P^{-0.267} \theta^{0.371}}{k}, \tag{9}$$

where S is the catchment area ; I is the percentage of imperviousness ; P is the rainfall depth ; θ is the rainfall duration.

B. *Time-invariant black-box models*

B.1 Linear time-invariant models

Both input-output (of the ARX and OE type) and state-space models have been considered in the literature. The former have been identified by means of the classical least squares (LS) method, while for the latter a subspace-based model identification (SMI) algorithm has been applied.

As highlighted in [29], where a detailed analysis using nonlinearity detection techniques on experimental data was carried out, linear time-invariant models alone do not seem capable of capturing the relevant dynamics with sufficient accuracy. However, as a first step of the identification procedure for more complex mode structures it is useful to perform linear time-invariant model identification with a twofold aim : first, LTI models will serve as a basis for model quality and performance comparison ; second, linear models will give useful information for model structure selection and in particular about the time delay between input and output.

B.2 Nonlinear time-invariant models

In order to circumvent the limitations of LTI models, Nonlinear AutoRegressive eXogenous (NARX) models have been also considered. A NARX model represents a nonlinear relationship between the predicted output and the past values of input and output [31], i.e.,

$$y(t) = f(\varphi(t), e(t)) \tag{10}$$

where

$$\varphi(t) = [y(t-1), \ldots, y(t-n_a), u(t-n_k), \ldots, u(t-n_k-n_b)]^T$$

is the regression vector. Identification of NARX models is performed in two steps. The first step is the structure identification and consists in choosing a parametric expression $f(\cdot;\eta)$ for the nonlinear function in (10) and the dimensions of the regression vector, i.e., suitable values for n_a, n_b and n_k. Then, the parameter vector η must be estimated. In the present work, the nonlinear function has been parametrized by means of a multilayer feedforward neural network ([31]). Such a model can be represented as

$$y(t) = \sum_{i=1}^{n_h} \alpha_i \sigma \left(\sum_{j=1}^{n_a+n_b+1} w_{ij}\phi_j(t) + \beta_i \right), \tag{11}$$

where ϕ_j is the j-th element of the regression vector ; w_{ij}, $i = 1, \ldots, n_h$, $j = 1, \ldots, n_a + n_b + 1$, are the parameters connecting the input layer to the hidden layer ; α_i, $i = 1, \ldots, n_h$, are the parameters connecting the hidden layer to the output ; β_i, $i = 1, \ldots, n_h$, are the biases of the neurons ; $\sigma(\cdot)$ is the neuron activation function which is a nonlinear function endowed with suitable mathematical properties so that (11) has good approximation capabilities. The structure of the model (11) is completely defined by the dimension of the regression vector, i.e., the number of past output values n_a and the number of past input values n_b used to predict the model output, together with the dimension of the hidden layer n_h.

C. *Parameter-varying black-box models*

Parameter-varying models have been proposed in the early '90s in the control engineering literature as a paradigm for the formulation of gain-scheduling control system design problems and are now widely used in view of control design using modern robust control theory (see, e.g., [1]). In particular, most of the literature on parameter-varying models focuses on the linear case, i.e., on the so-called Linear Parameter-Varying (LPV) models. The identification of LPV models has been extensively studied in the literature (see, for example [18], [23], [21], [3], [34]).

More recently, in [26], [27] an extension of the LPV model class, the so-called NLPV model structure, which can take into account "hidden" nonlinearities which might appear in the parameter-varying reformulation of a nonlinear system has been proposed. Similar approaches have been proposed in, e.g., [35], [36], [17], [5].

C.1 Linear parameter-varying black-box models

LPV model identification algorithms are available in the literature both for input/output and state space representations of parametrically-varying dynamics. If, however, the aim of the identification procedure is to eventually work out LPV models in state space form for control design purposes, one should keep in mind that the usual equivalence notions applicable to LTI systems cannot be directly used in converting LPV models from input-output to state space form, as the *time-variability* of LPV systems ought to be taken into account (see, e.g., the discussion in [33]). Bearing this in mind, we focus in this work on state space LPV models, in the form

$$\begin{aligned} x(t+1) &= A(\delta(t))x(t) + B(\delta(t))u(t) \\ y(t) &= C(\delta(t))x(t) + D(\delta(t))u(t), \end{aligned} \tag{12}$$

where $\delta \in \mathbb{R}^{\sim}$ is the parameter vector and $x \in \mathbb{R}^{\ltimes}$, $u \in \mathbb{R}^{\gtrdot}$, $y \in \mathbb{R}^{\lessdot}$. It is often necessary to introduce additional assumptions regarding the way in which $\delta(t)$ enters the system matrices. The most common assumptions are

1. Affine parameter dependence (LPV-A) :

$$A(\delta(t)) = A_0 + A_1\delta_1(t) + \ldots + A_s\delta_s(t) \tag{13}$$

and similarly for B, C and D, and where by $\delta_i(t), i = 1, \ldots, s$ we denote the i-th component of vector $\delta(t)$. This form can be immediately generalised to polynomial parameter dependence.

2. Input-affine parameter dependence (LPV-IA) : this is a particular case of the LPV-A parameter dependence in which only the B and D matrices are considered as parametrically-varying, while A and C are assumed to be constant : $A = A_0$, $C = C_0$.

3. LFT parameter dependence (LPV-LFT) : in this case the plant is assumed to be constituted by the feedback interconnection of an LTI system

$$\begin{aligned} x(t+1) &= \mathcal{A}x(t) + \mathcal{B}_0 w(t) + \mathcal{B}_1 u(t) \\ z(t) &= \mathcal{C}_0 x(t) + \mathcal{D}_{00} w(t) + \mathcal{D}_{01} u(t) \\ y(t) &= \mathcal{C}_1 x(t) + \mathcal{D}_{10} w(t) + \mathcal{D}_{11} u(t) \end{aligned} \tag{14}$$

with a time-varying block which depends on the parameter vector

$$w(t) = \Delta(t)z(t), \quad \Delta(t) = \mathrm{diag}(\delta_1(t)I_{r_1} \ldots \delta_s(t)I_{r_s}), \tag{15}$$

and $w, z \in \mathbb{R}^{\llcorner}$, $r = r_1 + \ldots + r_s$. The elements of the system matrices turn out to be first order rational functions of the elements of the parameter vector if $\mathcal{D}_{00} \neq 0$ and linear functions of the parameter vector if $\mathcal{D}_{00} = 0$.

C.2 Nonlinear parameter-varying black-box models

A Nonlinear Parameter-Varying (NLPV) model is a linear regression model the parameters of which change with time in a nonlinear way. According to [26] the model family has the structure

$$\begin{aligned} y(t) = {} & a_1(t)y(t-1) + \cdots + a_{n_a}(t)y(t-n_a) + \\ & + b_1(t)u(t-1) + \cdots + b_{n_b}(t)u(t-n_b) + e(t), \end{aligned} \tag{16}$$

where n_a is the maximum output lag, n_b is the maximum input lag and the term $e(t)$ incorporates modelling error and disturbance effects. The parameters of equation (16) are time-varying according to the pseudo-affine transformations

$$a_i(t) = a_{i1} + a_{i2}z(t), \quad i = 1, \ldots, n_a \tag{17}$$

$$b_j(t) = b_{j1} + b_{j2} z(t), \quad j = 1, \dots, n_b \tag{18}$$

where $z \in \mathbb{R}$ is a scheduling variable that is assumed to be the output of a nonlinear dynamic parametric model of the form

$$z(t) = f(\psi(t); \eta), \tag{19}$$

where η is a parameter vector and $\psi(t)$ is the regressor of the scheduling model, i.e.,

$$\psi(t) = [y(t-1), \dots, y(t-n_y), u(t-1), \dots, u(t-n_u), \dots \\ \delta(t-1), \dots, \delta(t-n_\delta)]^T. \tag{20}$$

In order to formulate and solve the identification problem using the NLPV model family, note that equations (16)-(18) can be written as

$$y(t) = \theta^T \begin{pmatrix} \varphi(t) \\ \varphi(t) f(\psi(t); \eta) \end{pmatrix}, \tag{21}$$

where $\theta \in \mathbb{R}^{\nvDash\ltimes\supset+\nvDash\ltimes}$ is

$$\theta = [a_{11}, \dots, a_{n_a 1}, b_{11}, \dots, b_{n_b 1}, a_{12}, \dots, a_{n_a 2}, b_{12}, \dots, b_{n_b 2}]^T \tag{22}$$

and

$$\varphi(t) = [y(t-1), \dots, y(t-n_a), u(t-1), \dots, u(t-n_b)]^T.$$

The identification problem is to find an estimate $(\hat{\theta}, \hat{\eta})$ of the parameters of the model

$$\hat{y}(t) = \sum_{j=1}^{n} \theta_j \phi_j(t, \eta) \tag{23}$$

where $n = 2n_a + 2n_b$, θ_j indicates the jth component of the vector θ and $\phi_j(t, \eta)$ is the jth component of the vector $\boldsymbol{\phi}(t, \eta) \in \mathbb{R}^{\nvDash\ltimes\supset+\nvDash\ltimes}$ defined as follows, consistently with equation (21)

$$\boldsymbol{\phi}(t, \eta) = [y(t-1), \cdots, y(t-n_a),\ u(t-1), \cdots, u(t-n_b), \\ y(t-1) f(\psi(t); \eta), \cdots, y(t-n_a) f(\psi(t); \eta), \\ u(t-1) f(\psi(t); \eta), \cdots, u(t-n_b) f(\psi(t); \eta)]^T. \tag{24}$$

The parameters (θ, η) must be estimated by minimising the cost function

$$J(\theta, \eta) = \sum_{i=1}^{N} \left(y_i - \sum_{j=1}^{n} \theta_j \phi_j(t_i, \eta) \right)^2, \tag{25}$$

which can be easily rewritten as

$$J(\theta, \eta) = \| \mathbf{y} - \boldsymbol{\Phi}(\eta) \theta \|_{\mathbf{2}}^{\mathbf{2}} \tag{26}$$

where

$$\Phi(\eta) = \begin{pmatrix} \boldsymbol{\phi}^T(t_1, \eta) \\ \boldsymbol{\phi}^T(t_2, \eta) \\ \vdots \\ \boldsymbol{\phi}^T(t_N, \eta) \end{pmatrix}. \tag{27}$$

and $\mathbf{y} = [\mathbf{y_1}, \mathbf{y_2}, \dots, \mathbf{y_N}]^{\mathbf{T}}$, where y_i is the measurement at time step t_i is a set of N output data. Equations (23)-(26) define a Separable Least Squares (SLS) problem, the structure of which has been first investigated in [13]. In this framework, the solution of the SLS problem for the NLPV model family has been proposed in [27].

D. *In perspective : grey-box modelling*

The development of control-oriented mathematical models of physical systems is a complex task, which implies a combination of prior knowledge about the physics of the system with information coming from experimental data (leading to the so-called problem of grey-box modelling, see, e.g., [4]), in view of the application of the model to control systems analysis and design. As discussed in [20], the critical issue in the development of an approach to control-oriented grey-box modelling lies in the integration of methods and tools for physical systems modelling and simulation with methods and tools for parameter estimation. Furthermore, if the eventual application of the model is control system analysis and design, the mathematical structure of the model has to be compatible with methods and tools for such problems.

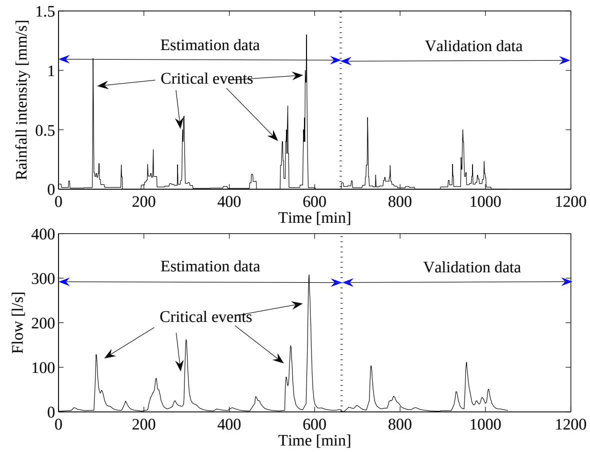
In [11] an approach has been proposed to bridge the gap between physical and control/estimation-oriented system modelling, by automatically deriving standard model structures used in identification and control starting from O-O models of nonlinear plants. As far as O-O modelling is concerned, the Modelica modelling language has been considered, as it allows to describe the plant dynamics in a very general, natural and user-friendly way ; the proposed concepts and algorithms can be applied to any a-causal, equation-based modelling language without any substantial change. On the other hand, the LFT model structure has been considered as the target for identification and control applications. Indeed, LFTs are a widely used model description formalism both in modern control [37], [15] and in identification [18], [9], [16]. Though no results specific for the problem of modelling the RRR have been obtained yet, it is expected that significant benefit might be achieved by means of a highly automated grey-box modelling approach such as the one described in the previous paragraphs (see the cited reference for details).

III. Experimental results using LPV models

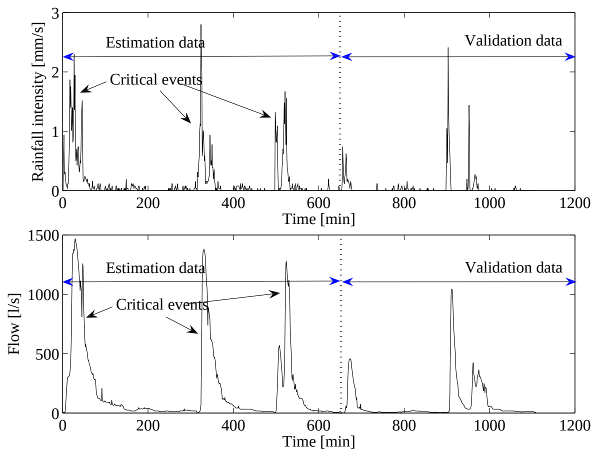
A. *Urban drainage networks and measurement setup*

Two catchments are studied in this paper : the first one is in Munkerisparken (Denmark) and the second one is in Fort Lauderdale (USA). Rainfall-runoff data collected in these two drainage networks are widely used in the hydraulic engineering literature as a benchmark to validate the mathematical models designed to predict flow in the catchment pipes ([22], [29]).

Figure 1 shows the measurements available for model identification, for the Munkerisparken (top) and the Fort Lauderdale (bottom) catchments. These have been divided into an estimation set and a validation set (see [28] for details on the catchments characteristics and data collection issues). Notice that the rainfall (input) is characterized by sharp peaks, the so-called "critical events". As a consequence, a similar behaviour can be noticed in the flow measurements (output) ; these are higher in the Fort Lauderdale catchment which is larger and has a larger impervious area than the Munkerisparken one. It is worth noting that it is a very important criterion for the evaluation of the model to consider the quality of the prediction the amplitude of flow peaks following rain critical events.



(a)



(b)

Fig. 1. Input/output measurements used for model identification : 1(a) Munkerisparken catchment ; 1(b) Fort Lauderdale catchment.

B. Linear parameter-varying models

LPV models have been identified both for the Munkerisparken and the Fort Lauderdale data, again relying on the preliminary analysis performed on ARX models for the main structure selection issues and based of the simple choice $\delta(t) = u(t)$ for the parameter vector. In particular, considering the Relative Square Error (RSE)

$$J_N(\hat{\theta}) = \frac{\sum_{t=1}^{N}(y(t) - \hat{y}(t;\hat{\theta}))^2}{\sum_{t=1}^{N} y^2(t)}, \tag{28}$$

as a performance criterion, where $y(t)$ is the flow measurement at time t, $\hat{y}(t;\theta)$ is the model flow simulation at time t using the estimated parameter vector $\hat{\theta}$ and N is the number of the samples available for the criterion evaluation, ARX (AutoRegressive eXogenous) models have been estimated, using the Least Squares method. As is well known (see, e.g., [19]) the structure of an ARX model is defined by the three parameters n_a (the number of past output values used to predict the model output), n_b (the number of past input values used to predict the model output) and n_k, i.e., the "pure" time delay between input and output. Since these three parameter completely define the model structure, the model is often referred to as ARX(n_a,n_b,n_k). The linear model structures have been chosen according to the Akaike Information Criterion (AIC, see, again, [19]). Quite surprisingly, the AIC provides the same model structure for both networks, so ARX(4,2,5) models have been estimated for both the available data sets. Figure **??** shows a comparison between the validation data set and the simulations provided by the two ARX estimated models. The optimal value of the performance index is $J = 0.0686$ for the Munkerisparken drainage network and $J = 0.0942$ for the Fort Lauderdale network.

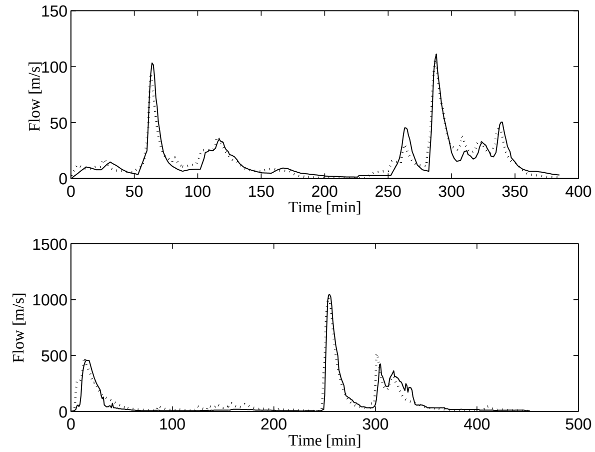


Fig. 2. Comparison between the validation data set and time-varying flow simulation obtained using two LPV models. Top : Munkerisparken catchment ; bottom : Fort Lauderdale catchment.

The results obtained by means of the identified LPV model are illustrated in Figure 2, and correspond to a LPV-IA model for the Munkerisparken data and to a LPV-A one for the Fort Lauderdale data, both defined by choosing the input u as (scalar) parameter δ. The value of the performance index is $J = 0.0591$ for the Munkerisparken drainage network and $J = 0.0970$ for the Fort Lauderdale network. These values are, respectively, 13.8% better and 3% worse than the ones obtained for ARX models.

C. Nonlinear parameter-varying models

In Figure 3 a comparison between the validation data set and time-varying flow simulation obtained using NLPV models applied to the data measured on the two drainage networks is shown. For the identification, the input regressor structure is the same for both the catchments and it has been chosen according to the results obtained in the ARX structure identification. The nonlinear feedback path has been modelled as a neural network, whose input regressor has been set to $n_u = n_y = n_d = 2$ and $n_k = 2$ neurons in the hidden layer have been used. The SLS identification problem is solved by means of an iterative procedure but this always converges very quickly (4 to 6 iterations) to an estimate (no over- or under- parametrization effects have been experienced).

The value of the performance index is $J = 0.0520$ for the Munkerisparken drainage network and $J = 0.0666$ for the Fort Lauderdale Network. These values are, respectively, 24.2% and 29.3% better than the ones obtained for ARX models.

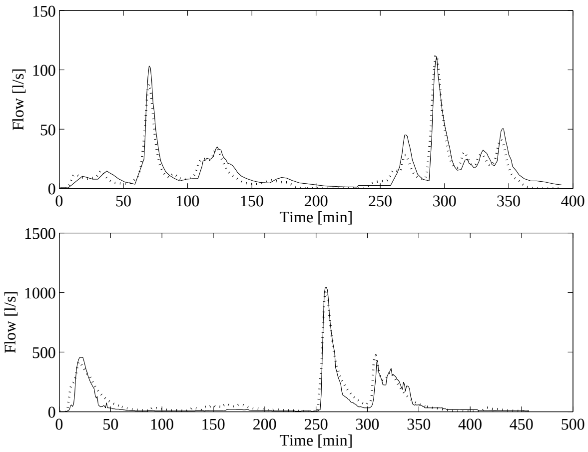


Fig. 3. Comparison between the validation data set and time-varying flow simulation obtained using two NLPV models. Top : Munkerisparken catchment ; bottom : Fort Lauderdale catchment.

IV. Concluding Remarks

The problem of identifying suitable models for the rainfall-runoff relationship of urban catchments has been considered and the performance of parameter-varying models for this problem has been assessed on experimental data and compared with the one achieved by linear and nonlinear ARX models.

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