

Evaluating Consulting Firms Using a Centroid Ranking Approach based Fuzzy MCDM Method

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Abstract

The purpose of this paper is to evaluate and select consulting firms by suggesting a centroid ranking approach based fuzzy multiple criteria decision making (MCDM) method, where ratings of alternatives versus qualitative criteria and the different importance weights among criteria are assessed in linguistic values represented by fuzzy numbers. Criteria are categorized to qualitative, benefit quantitative and cost quantitative ones. Membership functions of the final fuzzy evaluation values can be developed through α -cuts and interval arithmetic. The ranking approach of Euclidean distance based on centroid points is applied to determine the ranking order of all alternatives. Formulas for the ranking procedure can be clearly derived. Finally, a numerical example is used to demonstrate the computational process of the suggested model.

Keywords: Consulting firms, Fuzzy MCDM, Centroid, Ranking

1. Introduction

To survive in tight competition in today's business world, a company usually develops a new product which is different from, or better than, that of its competitors. A crucial factor such as pricing must be determined when introducing a new product to the market because it is very sensitive to customers. Wrong pricing strategy for a new product developed from a heavy investment can lead a company into loss or even bankruptcy. However, determining the best pricing strategy for a new product is difficult and many factors must be considered. To cautiously deal with this problem, many companies usually ask for consulting firm service. Thus evaluating and selecting a suitable consulting firm becomes an important issue.

A consulting firm is a firm of experts providing professional advice to an organization for a fee. A consulting firm consists of consultants who are experts in their field. For some global consulting firms, their employees represent from many nationality. Usually, a consulting firm provides its service which is in core business discipline, from marketing to operations; but there are consulting firms which not only provide business service but politics as well.

Many criteria must be considered when evaluating consulting firms, some of them are qualitative, such as reputation, some are quantitative, such as firm size;

moreover, criteria may have different importance. Therefore, how to comprehensively aggregate these criteria and importance weights becomes a critical issue in effectively evaluating consulting firms. Some relevant works have been studied in the evaluation of consulting firms. Creplets, et al.^[13] (2001) analyzed theoretically and empirically the differences between consultants and experts in the framework of the knowledge-based economy in order to introduce the central concepts of epistemic community and community of practice. However they did not talk detail about the other criteria that are supposed to be considered by a consulting firm such as the implementation cost and its knowledge. Wang and Chen^[21] (2006) presented how human inputs (top management, users, and external consultants) are linked to communication effectiveness and conflict resolution in the ERP consulting process, as well as the effects of these factors on the quality of the system implemented. Their findings indicated that top management support indirectly enhances ERP system quality through its positive effect on conflict resolution in the consulting process. The results also showed that high user support enhances communication effectiveness; however, communication effectiveness does not influence conflict resolution and ERP system quality. This paper lacks formulas for completing decision making. Altman^[3] (2008) indicated what should company consider and give guidelines in choosing the right consultancy. This article gives clear understanding about what is a consulting firm, but is short of quantitative process in aggregating the needed criteria. In 2009, Saremi, et al.^[19] used Nominal Group Technique (NGT) in deciding criteria for selecting the best consultant firm. They helped company to choose the best consultant for TQM selection under fuzzy TOPSIS, where qualitative and quantitative criteria are considered. But all criteria, even implemented cost, are regarded as qualitative one. Cebeci and Ruan^[7] (2007) provided an analytical tool to select the best quality consultant providing the most customer satisfaction, where fuzzy analytic hierarchy process was applied to compare these consultancy firms. But, all the attributes are qualitative.

To resolve the above problems in evaluating consulting firms, this paper suggests a centroid ranking approach based fuzzy MCDM method, where both subjective (or qualitative) and objective (or quantitative) criteria are considered. In the proposed model, ratings of alternatives versus qualitative criteria and the importance weights of all the criteria are assessed in

linguistic values represented by fuzzy numbers. Many methods have been proposed to solve fuzzy MCDM problems. A review and comparison of many of these methods can be found in Chen and Hwang^[9] (1992), Carlsson and Fullèr^[6] (1996), Ribeiro^[18] (1996) and Triantaphyllou and Lin^[20] (1996). Some recent applications can be found in Chen^[8] (2001), Chou^[10] (2007), Chou et al.^[11] (2006) and Ö nüt et al.^[17] (2009). In most fuzzy MCDM problems, the final evaluation values of alternatives are still fuzzy numbers and these fuzzy numbers need a proper ranking approach to defuzzify them into crisp values for decision making. Many approaches for ranking fuzzy numbers have been studied. A review of many of these approaches can be found in Bortolan and Degani^[5] (1985), Chen and Huang^[9] (1992), and Wang and Kerre^[22] (2001). Some recent methods can be seen in Abbasbandy and Asady^[1] (2006), Abbasbandy and Hajjari^[2] (2009), Asady^[4] (2010), Liu and Han^[16] (2005), Wang and Lee^[23] (2008) and Yong et al.^[25] (2006). In spite of advantages, some of these methods are computational complex and difficult to implement and none of them can satisfactorily rank fuzzy numbers in all cases. Moreover, the above works do not provide connection by formulae between the final evaluation values and the ranking procedure, limiting their applicability and computation efficiency.

To resolve these problems, this paper suggests applying the ranking approach of centroid from Wang et al.^[24] (2006) to obtain the Euclidean distance to defuzzify all the final fuzzy evaluation values in order to complete the model. Formulae can be clearly developed to present the ranking procedure. Finally, a numerical example demonstrates the computational process of the suggested model.

2. Fuzzy set theory

Fuzzy set theory was first introduced by Zadeh^[27] in 1965. It is to deal with problems involving fuzzy phenomena. Also, it has been considered as a useful tool of modeling language represented by fuzzy numbers to approximate system in which fuzzy phenomena exists. Some basic concepts in fuzzy set theory are briefly introduced in the following sections, which can also be seen in the author's previous work^[12].

2.1. Fuzzy set

The fuzzy set A can be expressed as^[15]:

$$A = \{(x, f_A(x)) \mid x \in U\} \quad (1)$$

where U is the universe of discourse, x is an element in U , A is a fuzzy set in U , $f_A(x)$ is the membership function of A at x . The larger $f_A(x)$, the stronger the grade of membership for x in A .

2.2. Fuzzy numbers

A real fuzzy number A is described as any fuzzy subset of the real line R with membership function f_A which

possesses the following properties^[14]:

- (a) f_A is a continuous mapping from R to $[0,1]$;
- (b) $f_A(x) = 0, \forall x \in (-\infty, a]$;
- (c) f_A is strictly increasing on $[a, b]$;
- (d) $f_A(x) = 1, x \in [b, c]$;
- (e) f_A is strictly decreasing on $[c, d]$;
- (f) $f_A(x) = 0, \forall x \in [d, \infty)$;

where $a \leq b \leq c \leq d$, A can be denoted as $[a, b, c, d]$. The membership function f_A of the fuzzy number A can also be expressed as:

$$f_A(x) = \begin{cases} f_A^L(x), & a \leq x \leq b \\ 1, & b \leq x \leq c \\ f_A^R(x), & c \leq x \leq d \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

where $f_A^L(x)$ and $f_A^R(x)$ are left and right membership functions of A , respectively.

2.3. α -cut

The α -cuts of fuzzy number A can be defined as^[15]:

$$A^\alpha = \{x \mid f_A(x) \geq \alpha\}, \alpha \in [0,1] \quad (3)$$

where A^α is a non-empty bounded closed interval contained in R and can be denoted by $A^\alpha = [A_l^\alpha, A_u^\alpha]$, where A_l^α and A_u^α are its lower and upper bounds, respectively.

2.4. Fuzzy arithmetic operations

Given fuzzy numbers A and B , $A, B \in R^+$, the α -cuts of A and B are $A^\alpha = [A_l^\alpha, A_u^\alpha]$ and $B^\alpha = [B_l^\alpha, B_u^\alpha]$, respectively. By interval arithmetic, some main operations of A and B can be expressed as follows^[15]:

$$\begin{aligned} (A \oplus B)^\alpha &= [A_l^\alpha + B_l^\alpha, A_u^\alpha + B_u^\alpha] \\ (A \ominus B)^\alpha &= [A_l^\alpha - B_u^\alpha, A_u^\alpha - B_l^\alpha] \\ (A \otimes B)^\alpha &= [A_l^\alpha \cdot B_l^\alpha, A_u^\alpha \cdot B_u^\alpha] \\ (A \oslash B)^\alpha &= \left[\frac{A_l^\alpha}{B_u^\alpha}, \frac{A_u^\alpha}{B_l^\alpha} \right] \\ (A \otimes r)^\alpha &= [A_l^\alpha \cdot r, A_u^\alpha \cdot r], r \in R^+ \end{aligned} \quad (4)$$

2.5. Linguistic values

A linguistic variable is a variable whose values are expressed in linguistic terms. Linguistic variable is a very helpful concept for dealing with situations which are too complex or not well-defined to be reasonably described by traditional quantitative expressions^[26]. For example, "importance" is a linguistic variable whose values include UI (unimportant), LI (less important), IM

(important), MI (more important) and VI (very important). These linguistic values can be further represented by triangular fuzzy numbers such as VL=(0.0,0.0,0.25), L=(0.0,0.25,0.5), M=(0.25,0.5,0.75), H=(0.5,0.75,1.0) and VH=(0.75,1.0,1.0).

3. Model establishment

Assume that a committee of k decision-makers (D_t , $t=1\sim k$) is responsible for evaluating m alternatives (A_i , $i=1\sim m$) under n criteria, and criteria are categorized to subjective (C_j , $j=1\sim h$) and objective (C_j , $j=h+1\sim n$), objective/quantitative criteria are further classified into benefit (B) and cost (C).

3.1. Average ratings of alternatives versus subjective criteria

Let $x_{ijt} = (o_{ijt}, p_{ijt}, q_{ijt})$, $x_{ijt} \in R^+$, $i = 1, 2, \dots, m$, $j = 1, 2, \dots, h$, $t = 1, 2, \dots, k$.

$$x_{ij} = \left(\frac{1}{k}\right) \otimes (x_{ij1} \oplus \dots \oplus x_{ijt} \oplus x_{ijk}) \quad (5)$$

where $o_{ij} = \sum_{t=1}^k \frac{o_{ijt}}{k}$, $p_{ij} = \sum_{t=1}^k \frac{p_{ijt}}{k}$, $q_{ij} = \sum_{t=1}^k \frac{q_{ijt}}{k}$, x_{ijt} be the rating assigned to A_i by D_t for C_j . x_{ij} is the averaged rating of A_i under C_j .

3.2. Normalize performance of alternatives versus objective criteria

Herein, quantitative values of alternatives versus objective criteria are assumed to be uncertain and can be expressed in triangular fuzzy numbers. These values have different units and must be normalized. The normalization is completed by a suggested approach from Chu and Lin^[12] (2009), which preserves the property where the ranges of normalized triangular fuzzy numbers belong to $[0, 1]$. Suppose $r_{ij} = (e_{ij}, f_{ij}, g_{ij})$ is the performance of alternative i versus objective criterion j , the normalized value can be denoted as:

$$x_{ij} = \left(\frac{e_{ij} - e_j^*}{d_j^*}, \frac{f_{ij} - e_j^*}{d_j^*}, \frac{g_{ij} - e_j^*}{d_j^*}\right), x \in B \quad (6)$$

$$x_{ij} = \left(\frac{g_j^* - g_{ij}}{d_j^*}, \frac{g_j^* - f_{ij}}{d_j^*}, \frac{g_j^* - e_{ij}}{d_j^*}\right), x \in C \quad (7)$$

where $e_j^* = \min_i e_{ij}$, $g_j^* = \max_i g_{ij}$, $d_j^* = g_j^* - e_j^*$, $i = 1 \sim m$, $j = h+1 \sim n$. Benefit (B) criterion has the characteristics of the larger the better; while cost (C) criterion has the characteristics of the smaller the better.

3.3. Average weights

Let $w_{jt} = (a_{jt}, b_{jt}, c_{jt})$, $w_{jt} \in R^+$, $j = 1, 2, \dots, n$; $t = 1, 2, \dots, k$.

$$w_j = \left(\frac{1}{k}\right) \otimes (w_{j1} \oplus w_{j2} \oplus \dots \oplus w_{jk}) \quad (8)$$

where $a_j = \sum_{t=1}^k \frac{a_{jt}}{k}$, $b_j = \sum_{t=1}^k \frac{b_{jt}}{k}$, $c_j = \sum_{t=1}^k \frac{c_{jt}}{k}$, w_{jt} be the weight assigned by D_t to C_j . w_j is the averaged weight of C_j assessed by k decision-makers.

3.4. Aggregate the weighted ratings

The aggregation of weighted ratings, G_i , for each alternative is implemented by the concept of simple additive weighting as:

$$G_i = \left(\frac{1}{n}\right) \times [R_{i1} \oplus R_{i2} \oplus \dots \oplus R_{ij} \oplus \dots \oplus R_{in}] \quad (9)$$

where $R_{ij} = x_{ij} \otimes w_j$, $i = 1, 2, \dots, m$, $j = 1, 2, \dots, n$.

Let

$$(o_{ij}, p_{ij}, q_{ij}) = \begin{cases} \left(\frac{e_{ij} - e_j^*}{d_j^*}, \frac{f_{ij} - e_j^*}{d_j^*}, \frac{g_{ij} - e_j^*}{d_j^*}\right), & j \in B, \\ \left(\frac{g_j^* - g_{ij}}{d_j^*}, \frac{g_j^* - f_{ij}}{d_j^*}, \frac{g_j^* - e_{ij}}{d_j^*}\right), & j \in C \end{cases} \quad (10)$$

By applying Eqs. (4) and (10), Eq. (9) can be developed via arithmetic operation of fuzzy numbers as:

$$\begin{aligned} G_i^\alpha &= \left(\frac{1}{n}\right) \sum_{j=1}^n x_{ij}^\alpha \otimes w_j^\alpha \\ &= \left[\frac{1}{n} \sum_{j=1}^n (p_{ij} - o_{ij})(b_j - a_j) \alpha^2 + \right. \\ &\quad \left. \frac{1}{n} \sum_{j=1}^n [o_{ij}(b_j - a_j) + a_j(p_{ij} - o_{ij})] \alpha + \frac{1}{n} \sum_{j=1}^n o_{ij} a_j, \right. \\ &\quad \left. \frac{1}{n} \sum_{j=1}^n (p_{ij} - q_{ij})(b_j - c_j) \alpha^2 + \right. \\ &\quad \left. \frac{1}{n} \sum_{j=1}^n [q_{ij}(b_j - c_j) + c_j(p_{ij} - q_{ij})] \alpha + \frac{1}{n} \sum_{j=1}^n q_{ij} c_j \right] \quad (11) \end{aligned}$$

Let

$$\begin{aligned} I_{i1} &= \frac{1}{n} \sum_{j=1}^n (p_{ij} - o_{ij})(b_j - a_j), \\ J_{i1} &= \frac{1}{n} \sum_{j=1}^n [o_{ij}(b_j - a_j) + a_j(p_{ij} - o_{ij})], \end{aligned}$$

$$I_{i2} = \frac{1}{n} \sum_{j=1}^n (p_{ij} - q_{ij})(b_j - c_j),$$

$$J_{i2} = \frac{1}{n} \sum_{j=1}^n [q_{ij}(b_j - c_j) + c_j(p_{ij} - q_{ij})],$$

$$Q_i = \frac{1}{n} \sum_{j=1}^n o_{ij} a_j, \quad Y_i = \frac{1}{n} \sum_{j=1}^n p_{ij} b_j, \quad Z_i = \frac{1}{n} \sum_{j=1}^n q_{ij} c_j.$$

By applying the above assumption to Eqs (11), we have two equations to solve:

$$I_{i1}\alpha^2 + J_{i1}\alpha + Q_i - x = 0 \quad (12)$$

$$I_{i2}\alpha^2 + J_{i2}\alpha + Z_i - x = 0 \quad (13)$$

The left and right membership functions of G_i can be developed as:

$$f_{G_i}^L(x) = \left\{ -J_{i1} + [J_{i1}^2 + 4I_{i1}(x - Q_i)]^{1/2} \right\} / 2I_{i1}, \quad Q_i \leq x \leq Y_i; \quad (14)$$

$$f_{G_i}^R(x) = \left\{ -J_{i2} - [J_{i2}^2 + 4I_{i2}(x - Z_i)]^{1/2} \right\} / 2I_{i2}, \quad Y_i \leq x \leq Z_i. \quad (15)$$

For convenience, Eqs. (14) and (15) of G_i can be denoted as Eq. (16) as follows:

$$G_i = (Q_i, Y_i, Z_i; I_{i1}, J_{i1}; I_{i2}, J_{i2}) \quad (16)$$

The inverse functions of the above two equations, (14) and (15), can be produced as:

$$g_{G_i}^L(y) = I_{i1}y^2 + J_{i1}y + Q_i, \quad 0 \leq y \leq 1 \quad (17)$$

$$g_{G_i}^R(y) = I_{i2}y^2 + J_{i2}y + Z_i, \quad 0 \leq y \leq 1 \quad (18)$$

3.5. Obtain ranking

The centroid ranking method from Wang et al.^[24] (2006) is applied to rank the alternatives and detailed interpretation of centroid equations can be seen in Wang et al.' paper. The centroid on x-axis and y-axis can be shown as follows:

$$x(G_i) = \frac{\int_{-\infty}^{+\infty} x f_{G_i}(x) dx}{\int_{-\infty}^{+\infty} f_{G_i}(x) dx} = \frac{\int_{Q_i}^{Y_i} x f_{G_i}^L(x) dx + \int_{Y_i}^{Z_i} x f_{G_i}^R(x) dx}{\int_{Q_i}^{Y_i} f_{G_i}^L(x) dx + \int_{Y_i}^{Z_i} f_{G_i}^R(x) dx} \quad (19)$$

$$y(G_i) = \frac{\int_0^1 y [g_{G_i}^R(y) - g_{G_i}^L(y)] dy}{\int_0^1 [g_{G_i}^R(y) - g_{G_i}^L(y)] dy} \quad (20)$$

Then we calculate the Euclidean distances from their centroid points to the origin to defuzzify fuzzy numbers. The larger the distance is, the higher priority fuzzy number is ranked.

$$\sqrt{x^2(G_i) + y^2(G_i)} \quad (21)$$

Formulas for centroids on x-axis and y-axis can be developed as follows:

$$\begin{aligned} & \int_{Q_i}^{Y_i} x f_{G_i}^L(x) dx \\ &= \int_{Q_i}^{Y_i} x \left\{ -J_{i1} + [J_{i1}^2 + 4I_{i1}(x - Q_i)]^{1/2} \right\} / 2I_{i1} dx \\ &= \frac{J_{i1}(Q_i^2 - Y_i^2)}{4I_{i1}} + \frac{1}{2I_{i1}} \int_{Q_i}^{Y_i} [J_{i1}^2 + 4I_{i1}(x - Q_i)]^{1/2} x dx \quad (22) \end{aligned}$$

$$\text{Let } [J_{i1}^2 + 4I_{i1}(x - Q_i)]^{1/2} = t$$

$$x = \frac{t^2 + 4I_{i1}Q_i - J_{i1}^2}{4I_{i1}}, \quad dx = \frac{tdt}{2I_{i1}}$$

$$x = Q_i \Rightarrow t = (4Q_i I_{i1} + J_{i1}^2 - 4I_{i1}Q_i)^{1/2} = J_{i1}$$

$$x = Y_i \Rightarrow t = (4Y_i I_{i1} + J_{i1}^2 - 4I_{i1}Q_i)^{1/2}$$

Assume $(4Y_i I_{i1} + J_{i1}^2 - 4I_{i1}Q_i)^{1/2} = Y_i'$, we obtain:

$$\begin{aligned} & \frac{1}{2I_{i1}} \int_{Q_i}^{Y_i} [J_{i1}^2 + 4I_{i1}(x - Q_i)]^{1/2} x dx \\ &= \frac{1}{2I_{i1}} \int_{J_{i1}}^{Y_i'} \frac{t^2 + 4I_{i1}Q_i - J_{i1}^2}{4I_{i1}} \frac{tdt}{2I_{i1}} \\ &= \frac{1}{16I_{i1}^3} \left[\frac{Y_i'^5 - J_{i1}^5}{5} + (4I_{i1}Q_i - J_{i1}^2) \frac{Y_i'^3 - J_{i1}^3}{3} \right] \quad (23) \end{aligned}$$

$$\begin{aligned} & \int_{Q_i}^{Y_i} x \left\{ -J_{i1} + [J_{i1}^2 + 4I_{i1}(x - Q_i)]^{1/2} \right\} / 2I_{i1} dx \\ &= \frac{J_{i1}(Q_i^2 - Y_i^2)}{4I_{i1}} + \frac{1}{16I_{i1}^3} \left[\frac{Y_i'^5 - J_{i1}^5}{5} + (4I_{i1}Q_i - J_{i1}^2) \frac{Y_i'^3 - J_{i1}^3}{3} \right] \quad (24) \end{aligned}$$

Similarly,

$$\begin{aligned} & \int_{Y_i}^{Z_i} x f_{G_i}^R(x) dx \\ &= \int_{Y_i}^{Z_i} x \left\{ -J_{i2} + [J_{i2}^2 + 4I_{i2}(x - Z_i)]^{1/2} \right\} / 2I_{i2} dx \end{aligned}$$

$$= \frac{J_{i2}(Y_i^2 - Z_i^2)}{4I_{i2}} - \frac{1}{16I_{i2}^3} \left[\frac{J_{i2}^5 - Y_i^5}{5} + (4I_{i2}Z_i - J_{i2}^2) \frac{J_{i2}^3 - Y_i^3}{3} \right] \quad (25)$$

$$\begin{aligned} & \int_{Q_i}^{Y_i} f_{G_i}^L(x) dx \\ &= \int_{Q_i}^{Y_i} \left[-J_{i1} + \left[J_{i1}^2 + 4I_{i1}(x - Q_i) \right]^{1/2} \right] / 2I_{i1} dx \\ &= \frac{J_{i1}(Q_i - Y_i)}{2I_{i1}} + \frac{Y_i^3 - J_{i1}^3}{12I_{i1}^2} \quad (26) \end{aligned}$$

$$\begin{aligned} & \int_{Y_i}^{Z_i} f_{G_i}^R(x) dx \\ &= \int_{Y_i}^{Z_i} \left[-J_{i2} + \left[J_{i2}^2 + 4I_{i2}(x - Z_i) \right]^{1/2} \right] / 2I_{i2} dx \\ &= \frac{J_{i2}(Y_i - Z_i)}{2I_{i2}} + \frac{J_{i2}^3 - Y_i^3}{12I_{i2}^2} \quad (27) \end{aligned}$$

By applying Eqs. (22)-(27), Eq. (19) can be produced as:

$$x(G_i) = \frac{J_{i1}(Q_i - Y_i^2) + \frac{1}{16I_{i1}^3} \left[\frac{Y_i^5 - J_{i1}^5}{5} + (4I_{i1}Q_i - J_{i1}^2) \frac{Y_i^3 - J_{i1}^3}{3} \right] + \frac{J_{i2}(Y_i^2 - Z_i^2)}{4I_{i2}} - \frac{1}{16I_{i2}^3} \left[\frac{J_{i2}^5 - Y_i^5}{5} + (4I_{i2}Z_i - J_{i2}^2) \frac{J_{i2}^3 - Y_i^3}{3} \right]}{\frac{J_{i1}(Q_i - Y_i)}{2I_{i1}} + \frac{Y_i^3 - J_{i1}^3}{12I_{i1}^2} + \frac{J_{i2}(Y_i - Z_i)}{2I_{i2}} + \frac{J_{i2}^3 - Y_i^3}{12I_{i2}^2}} \quad (28)$$

Similarly, Eq. (20) can be expressed as:

$$\begin{aligned} & y(G_i) \\ &= \frac{\int_0^1 y \left[(I_{i2}y^2 + J_{i2}y + Z_i) - (I_{i1}y^2 + J_{i1}y + Q_i) \right] dy}{\int_0^1 \left[(I_{i2}y^2 + J_{i2}y + Z_i) - (I_{i1}y^2 + J_{i1}y + Q_i) \right] dy} \\ &= \frac{\frac{I_{i2} - I_{i1}}{4} + \frac{J_{i2} - J_{i1}}{3} + \frac{Z_i - Q_i}{2}}{\frac{I_{i2} - I_{i1}}{3} + \frac{J_{i2} - J_{i1}}{2} + Z_i - Q_i} \quad (29) \end{aligned}$$

Then, Eqs. (28)-(29) are applied to Eq. (21) to produce the Euclidean distances of each fuzzy number. Finally we can rank the priority of segment alternatives based on values of Euclidean distances. Company is suggested to choose alternative candidate which has highest priority as its consulting firm.

4. Numerical example

This section uses an artificial numerical example to demonstrate the computational process of the suggested model. Suppose a manufacturing company must select a consulting firm to help determine the price for its new product. After preliminary screening, four alternative consulting firms, A_1, A_2, A_3 and A_4 , are chosen for further evaluation. A committee of five decision-makers D_1, D_2, D_3, D_4 and D_5 is formed to conduct the evaluation and selection of the four alternative

consulting firms. Further suppose three benefit qualitative criteria, such as reputation (C_1), technical skills (C_2), and knowledge of business (C_3), as well as three benefit quantitative criteria, such as company size (C_4 , # of employees, unit: hundred), potential profit (C_5 , unit: million \$) and expected growth (C_6 , unit: %), and one cost quantitative criterion, such as cost (C_7 , unit: 10 thousand \$), are considered.

Moreover, suppose that the decision-makers use the linguistic rating set a linguistic rating set $W = \{VL, L, M, H, VH\}$, where VL=Very Low=(0,0.1,0.3), L=Low=(0.1,0.3,0.5), M=Medium=(0.3,0.5,0.7), H=High=(0.5,0.7,0.9), and VH=Very High=(0.7,0.9,1), to evaluate the suitability of consulting firms under each of the qualitative criteria. In addition, the decision-makers employ a linguistic weighting set $W = \{UI, LI, IM, MI, VI\}$, where UI=Unimportant=(0.0,0.0,0.25), LI=Less Important=(0.0,0.25,0.5), IM=Important=(0.25,0.5,0.75), MI=More Important=(0.5,0.75,1.0), and VI=Very Important=(0.75,1.0,1.0), to assess the importance of all the criteria. Moreover, suppose that the suitability ratings of consulting firms versus the five qualitative criteria from the three decision-makers are presented in Table 1, values of consulting firms under quantitative criteria are presented in Table 3, and the importance weights of all criteria are presented in Table 6.

By Eq. (5), the averaged suitability ratings of consulting firm A_i under each qualitative criterion C_j from the decision-making committee can be obtained as shown in Table 2. By Eqs. (6)-(7), normalized values of consulting firm A_i versus each quantitative criterion can be obtained and are displayed in Tables 4-5. Through Eq. (8), the averaged weights of criteria from the decision-making committee can be obtained as presented in Table 7.

Through Eqs. (9)-(16), the membership functions of the final fuzzy evaluation values, $G_i, i=1\sim 4$, can be produced in Table 8. Through Eqs. (17)-(29), centroid points can be produced as shown in Table 9 and the Euclidean distances for the alternative market segments can then be obtained as shown in Table 10..

Table 1. Ratings of consulting firms versus qualitative criteria

A.	C.	D_1	D_2	D_3	D_4	D_5
A_1	C_1	H	M	H	H	VH
	C_2	H	H	VH	H	H
	C_3	H	H	L	M	H
A_2	C_1	L	M	VH	H	H
	C_2	VH	H	VH	VH	H
	C_3	L	VL	M	M	H
A_3	C_1	H	M	VL	M	M
	C_2	M	L	M	L	M
	C_3	L	H	L	VL	VL
A_4	C_1	VH	L	VL	M	VL
	C_2	M	L	M	VL	M
	C_3	VL	L	VL	VL	L

Table 2. Averaged ratings of consulting firms versus qualitative criteria

A.	C.	Averaged ratings
A ₁	C ₁	(0.50, 0.70, 0.88)
	C ₂	(0.54, 0.74, 0.92)
	C ₃	(0.38, 0.58, 0.78)
A ₂	C ₁	(0.42, 0.62, 0.80)
	C ₂	(0.62, 0.82, 0.96)
	C ₃	(0.24, 0.42, 0.62)
A ₃	C ₁	(0.28, 0.46, 0.66)
	C ₂	(0.22, 0.42, 0.62)
	C ₃	(0.14, 0.30, 0.50)
A ₄	C ₁	(0.22, 0.38, 0.56)
	C ₂	(0.20, 0.38, 0.58)
	C ₃	(0.04, 0.18, 0.38)

Table 3. Numerical values of quantitative criteria

C.	A ₁	A ₂	A ₃	A ₄
C ₄	(25,30,37.5)	(8,9.3,10)	(36,43.7,55)	(5,7.5,8.97)
C ₅	(2,3,6)	(6.5,8,12)	(2.8,4,5)	(0.5,1,1.5)
C ₆	(0.2,0.5,1)	(2, 2.5, 3)	(2.25,3,3.5)	(0.5,1,2)
C ₇	(5,7,9)	(6,8,10)	(2,3,5)	(0,1,2)

Table 4. Normalized values of quantitative criteria

C.	A ₁	A ₂
C ₄	(0.4000,0.5000,0.6500)	(0.0600,0.0860,0.1000)
C ₅	(0.1304,0.2174,0.4783)	(0.5217,0.6522,1.0000)
C ₆	(0.0000,0.0909,0.2424)	(0.5455,0.6970,0.8485)
C ₇	(0.1000,0.3000,0.5000)	(0.0000,0.2000,0.4000)

Table 5. Normalized values of quantitative criteria (cont.)

C.	A ₃	A ₄
C ₄	(0.6200,0.7740,1.0000)	(0.0000,0.0500,0.0794)
C ₅	(0.2000,0.3043,0.3913)	(0.0000,0.0435,0.0870)
C ₆	(0.6212,0.8485,1.0000)	(0.0909,0.2424,0.5455)
C ₇	(0.5000,0.7000,0.8000)	(0.8000,0.9000,1.0000)

Table 6. Importance weights of criteria

C.	D ₁	D ₂	D ₃	D ₄	D ₅
C ₁	VI	I	I	MI	I
C ₂	VI	MI	MI	VI	VI
C ₃	VI	MI	I	MI	VI
C ₄	LI	UI	LI	LI	UI
C ₅	VI	VI	I	VI	VI
C ₆	I	LI	UI	I	MI
C ₇	VI	VI	I	I	MI

Table 7. Averaged importance weights

C.	Averaged weights
C ₁	(0.40, 0.65, 0.85)
C ₂	(0.65, 0.90, 1.00)
C ₃	(0.55, 0.80, 0.95)
C ₄	(0.00, 0.15, 0.40)
C ₅	(0.65, 0.90, 0.95)
C ₆	(0.20, 0.40, 0.65)
C ₇	(0.50, 0.75, 0.90)

Table 8. Final fuzzy evaluation values

Values	Alternatives			
	A ₁	A ₂	A ₃	A ₄
I _{i1}	0.0364	0.0374	0.0399	0.0277
I _{i2}	0.0289	0.0241	0.0291	0.0266
J _{i1}	0.1382	0.1561	0.1508	0.1
J _{i2}	-0.2595	-0.2562	-0.2614	-0.2029
Q _i	0.1287	0.1645	0.1195	0.0940
Y _i	0.3024	0.3597	0.3102	0.2217
Z _i	0.5330	0.5911	0.5425	0.3980

Table 9. Centroid points

Values	Alternatives			
	A ₁	A ₂	A ₃	A ₄
x	0.3010	0.3198	0.3083	0.2425
y	0.3344	0.3351	0.3347	0.3335

Table 10. Euclidean distance

Values	Alternatives			
	A ₁	A ₂	A ₃	A ₄
Distance	0.4499	0.4632	0.4551	0.4124

Based on Table 10, the alternative candidate (A₂) has highest score; therefore, company should choose A₂ as its consulting firm. This numerical example has clearly demonstrated the computational procedure of the suggested model. A numerical comparison as well as sensitivity analysis may be conducted in the extension of this work in the future for further justification.

5. Conclusions

A centroid ranking approach based fuzzy MCDM method is proposed to evaluate and select consulting firms, where criteria are classified to qualitative, benefit quantitative and cost quantitative ones. In the proposed model, ratings of alternatives versus qualitative criteria and the importance weights of all criteria are assessed in linguistic values represented by fuzzy numbers. Membership functions of the final evaluation values can be developed. The Euclidean distance based on centroid points is applied to defuzzify all the final fuzzy evaluation values to determine the ranking order of alternatives in order to complete the model. Formulas for the two centroid points on horizontal and vertical axes can be clearly developed. Finally, a numerical example has demonstrated the computational process of the suggested model.

The suggested model can also be applied to solve many other fuzzy management problems in a multiple criteria environment. However, when applied, the following problems are worth closer examination:

- (1) The outcome could be different if number of decision makers, linguistic ratings of various alternatives under different qualitative criteria, linguistic weightings for criteria, normalization formula, ... etc. are different.
- (2) A case study is needed to better test the

effectiveness of the suggested model. In addition, the number of criteria can be adjusted upon different cases if needed.

- (3) In the suggested model, the criteria structure is limited to one level. This may not be enough. If some parent criteria contain several sub-criteria and some of these sub-criteria further include several sub-sub-criteria, a hierarchical structure must be needed to describe this situation.

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