Two Order Sliding Fuzzy Type-2 Control Based on Integral Sliding Mode for MIMO Systems

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Abstract

A higher order sliding fuzzy type-2 controller scheme for an *nth* order multi-input multi-output (MIMO) nonlinear uncertain perturbed systems is proposed in the paper. To overcome the constraint on the knowledge of the system model, local models related to some operating points were used to synthesize a nominal fuzzy type-2 global model. The controller uses integral sliding mode concept and contains two parts. Adaptive fuzzy type-2 systems have been introduced to generate the Super Twisting signals to avoid both the chattering and the constraint on the knowledge of upper bounds of both disturbances and uncertainties. These fuzzy type-2 systems are adjusted on-line by adaptation laws deduced from the stability analysis in Lyapunov sense. The advantages of the method are that its implementation is easy, the time convergence is chosen in advance and the robustness is ensured. Simulation results of two link robot manipulator are presented to illustrate the tracking performances of the method.

Keywords: Higher order sliding mode, MIMO nonlinear systems, Integral sliding mode, Fuzzy logic type-2, T-S fuzzy systems, Lyapunov stability.

1. Introduction

In the last few years, a markable attention has been paid to another type of fuzzy system called Type-2 fuzzy logic system (T2FLS). In T2FLS, the uncertainty is represented using a function, which is it self a Type-1 fuzzy number. The member ship functions of T2 fuzzy sets are three dimensional and include a foot print of uncertainty (FOU) with a new third dimension of Type-2 fuzzy sets. An FOU provides additional degrees of freedom that make it possible to directly model and handle uncertainties and consequently, T2FLS has the potential to out perform T1FLS [2] in such cases. Such advantage is quite important if we know that modeling of linguistic information and decision making represents the main application of FLS.

The sliding mode control (SMC) is a powerful method to control high-order nonlinear dynamic systems operating under uncertain conditions [3][4]. The technique consists of two stages. First, a sliding surface, to which the controlled system trajectories must belong, is designed according to some performance criterion. Then, a discontinuous control is designed to force the system state to reach the sliding surface such that a sliding mode occurs on this manifold. When sliding mode is realized, the system exhibits robustness properties with respect to parameter perturbations and external matched disturbances [3]. In spite of claimed robustness properties, high frequency oscillations of the state trajectories around the sliding manifold known as chattering phenomenon [3][4] are the major obstacles for the implementation of SMC in a wide range of applications. A number of methods have been proposed to resolve this problem: among them, the boundary layer solution [3][5], tangent hyperbolic [6] and observer-based solution [4]. This methods can eliminate chattering effect but a finite steady state error will exist [7] and requires a compromise between the level of performance tracking and a start-up control [8]. In [7][8] the modified switching signal (with saturation) is substituted by an adaptive fuzzy system. However, the convergence of the corresponding adaptive algorithm depends directly on the choice of the initial values of the adaptive fuzzy system.

The recently developed concept of higher order sliding modes (HOSM) [9] which is the generalization of so-called first order sliding modes or classical sliding mode control has been presented in the literature as solution to the chattering phenomenon. In HOSM control, the control acts on higher derivatives of the sliding variable. For example, the case of second order sliding modes corresponds to the control acting on the second derivative of the sliding variable, namely $\ddot{s}$, and the sliding set is defined as $s = \dot{s} = 0$, where s denotes the sliding variable. Several algorithms have been presented for SISO systems in the literature [10]. In [11], Bartolini have also extended their SISO algorithm for a class of nonlinear MIMO systems. However, the algorithm still requires singular values of the sliding variable (maxima, minima or flex points) to be detected online. Nevertheless, the common drawback of these approaches is the calculation of sliding gains used in the control law. Indeed, they are chosen to satisfy the finite time convergence condition, which requires the well-knowledge of the system parameters. In [12] and [13], the optimal control gains are computed using type-2 adaptive systems.

The aim of this paper is to present an arbitraryorder sliding mode controller for uncertain MIMO nonlinear systems. The main objective of this new approach is to propose a controller for which the implementation is easy, the convergence time is finite and well-known in advance and the computation of the control gains of the discontinuous part will be simplified. The design uses the integral sliding mode concept [14]. Actually, the problem of the HOSMC of SISO minimum-phase uncertain systems is formulated in input-output terms only (as in [15]) through the differentiation of the sliding variable s, and is equivalent to the finite time stabilization of an rth order input-output linear dynamics with bounded uncertainties. The control strategy presented in the sequel, whose basic idea has been introduced in [16], contains two parts: the discontinuous one forces the establishment of a sliding mode on the integral sliding manifold, and ensures the robustness with respect to bounded uncertainties, throughout the entire response of the system. The second part, obtained through optimal feedback control over finite time interval with fixed final states [17], is used to stabilize to zero in finite time the rth order input output dynamics without uncertainties. In [18], this part is based on close-loop control and the forcing term δ is used only for $t \in [0, t_f]$. The computation of the control gains will be simplified by introducing adaptive fuzzy type-2 systems in the second SMC. Their updating is performed using adaptation laws derived from the study of stability. Since the considered MIMO system is nonlinear and uncertain, we propose to use Takagi-Sugeno (TS) fuzzy system to construct its nominal model. For this, we define some local models around some operating points, which will be used as conclusion of this TS system. The paper is organized as follows: section 2 is dedicated to the formulation and the investigation of the MIMO system tracking problem. In section 3, the different parts of the control design are described. In section 4, we present the synthesis of the proposed sliding mode fuzzy type-2 interval controller. Simulation example to demonstrate the performance of the proposed method is provided in section 5. Section 6 gives the conclusions of this paper.

2. Problem Formulation

Consider the following nth order MIMO uncertain nonlinear dynamic system:

$$\begin{cases} \underline{x}^{(n)} = F(X) + G(X)\underline{u} + \underline{d} \\ \underline{y} = \underline{x} \end{cases} \tag{1}$$

where $X^T = \left[\underline{x}^T, \underline{\dot{x}}^T, ..., \left(\underline{x}^{(n-1)}\right)^T\right] = \left[\underline{x}_1^T, \underline{x}_2^T, ..., \underline{x}_n^T\right]$ is the overall measurable state vector, $F(X) \in \Re^n$ is a vector of nonlinear continuous functions and $G(X) \in \Re^{n\times n}$ is a matrix of $n \times n$ nonlinear continuous functions. $\underline{y}$ and $\underline{u} \in \Re^{n\times 1}$ are respectively the output and the input of the system. $\underline{d}$ represents the external disturbances vector. We assume that $F(X)$ and $G(X)$ can be written as the sum of their nominal functions and an unknown bounded uncertainties:

$$\begin{aligned} F(X) = F_0(X) + \Delta F(X); ||\Delta F(X)|| < \Delta_F \\ G(X) = G_0(X) + \Delta G(X); ||\Delta G(X)|| < \Delta_G \end{aligned} \tag{2}$$

where Δ_F and Δ_G are two positive constants. Replacing (2) into (1), we obtain:

$$\begin{cases} \underline{x}^{(n)} = F_0(X) + G_0(X)\underline{u} + \underline{D} \\ \underline{y} = \underline{x} \\ \underline{D} = \Delta F(X) + \Delta G(X)\underline{u} + \underline{d} \end{cases} \tag{3}$$

We assume that the system is always controllable so $G(X)^{-1}$ exists and does not equal to zero.
The nominal model of the system can be obtained by identification or by approximation using a fuzzy system. Here we consider the second case, because it allows us to exploit linguistic information from the expert human. The approximation can be made either using a classical TS fuzzy system or an adaptive one. This later can certainly gets good results but requires a long computing time when we consider more than two inputs. Moreover, the linearization techniques allows to transform the dynamics of the system into local models around some operating points. A fuzzy dynamic model has been proposed by Takagi and Sugeno to represent a nonlinear system [19]. The TS fuzzy model is a piecewise interpolation of several linear models through membership functions. The fuzzy model is described by fuzzy $IF - THEN$ rules and will be employed here to deal with the control design problem for the nonlinear system [19][20]. However, as known in literature, the knowledge used to construct T1FL rules are uncertain. This uncertainty leads to obtain rules whose premises or consequences are uncertain, which can create uncertainty in the membership function. Since a T2FS can take in account this uncertainties, we propose to extend the dynamic model given in [20], by introducing the concept of type-2 fuzzy logic. Then, the nominal type-2 fuzzy model related to the i^{th} rule will be given as follows:
IF $\underline{x}$ is $\tilde{H}_1^i$ and $\underline{\dot{x}}$ is $\tilde{H}_2^i$ and..... and $\underline{x}^{(n-1)}$ is $\tilde{H}_n^i$
THEN

$$\underline{x}^{(n)} = A_i X + B_i \underline{u} \tag{4}$$

Where $\tilde{H}_j^i, (j = 1, 2, ..., n)$ is the j^{th} type-2 fuzzy interval set of the i^{th} rule. For a given pair $(X, \underline{u})$, the fuzzy nominal model resulting appears as a weighted average of local models. If we use the product as an interference engine, the method of center set for the reduction type and center of gravity for defuzzification, the output fuzzy system will be giving by:

$$\underline{x}^{(n)} = \frac{\sum_{i=1}^{r} w^i \left[A_i X + B_i \underline{u}\right]}{\sum_{i=1}^{r} w^i} \tag{5}$$

If we denote by $F_0(X) = \begin{bmatrix} \frac{\sum_{i=1}^{r} w^i A_i X}{\sum_{i=1}^{r} w_i} \end{bmatrix}$ and $G_0(X) = \begin{bmatrix} \frac{\sum_{i=1}^{r} w^i B_i}{\sum_{i=1}^{r} w^i} \end{bmatrix}$, then the fuzzy nominal model will be giving by:

$$\underline{x}^{(n)} = F_0(X) + G_0(X)\underline{u} \tag{6}$$

where the variable w^i is the firing interval.
The control objective is to fulfil the constraint of the sliding variables vector $\underline{s}(X,t) = 0$ in finite time and to keep it exactly by discontinuous feedback control, and to ensure that the system out-put vector $\underline{y}$ follows a reference trajectory $\underline{y}_d$. The set:
$\underline{s}^r = \{X | \underline{s}(X,t) = \underline{\dot{s}}(X,t) = ... = \underline{s}^{r-1}(X,t) = 0\}$ called "rth order sliding set", is non-empty and locally is an integral set in the Filippov sense, the motion on $\underline{s}^r$ is called "rth order sliding mode" with respect to the sliding variables s_i. The rth order SMC approach allows the finite time stabilization to zero of the sliding variables s_i and their $r-1$ first time derivatives by defining a suitable discontinuous control functions. $\underline{s}^r$ in function of the systems variables and the control $\underline{u}$, it is given by:

$$\underline{s}^r = \phi(.) + \gamma(.)\underline{u} \tag{7}$$

r means also the number of times to derive the sliding surface to appear explicitly the control.
For SISO systems, Laghrouche assumes in [18] that: $\phi(.)$ and $\gamma(.)$ are bounded uncertain functions, and the sign of the control gain $\gamma(.)$ is constant and strictly positive. Thus, there exist $K_m \in \mathrm{R}^{+*}$, $K_M \in \mathrm{R}^{+*}$, $C_0 \in \mathrm{R}^{+}$ such that $0 < K_m < \gamma < K_M$, $|\phi| \leq C_0$. This assumption implies that this result is considered local. In section (4) , we propose a method to avoid this restriction which makes the result global. The rth order SMC with respect to the sliding variables s_i for each subsystem is equivalent to the finite time stabilization of [18]:

$$\begin{cases} \dot{z}_i = z_{i+1} \\ \dot{z}_r = \phi(.) + \gamma(.)u \end{cases} \tag{8}$$

with $1 \leq i \leq r-1$ and z=$[z_1 \;\; z_2 \;\; ...z_r]^T := [s\dot{s}...s^{(r-1)}]^T$. The generalisation to MIMO systems of (8) is given by:

$$\begin{cases} \dot{Z}_i = Z_{i+1} \\ \dot{Z}_r = \underline{\phi}(.) + \underline{\gamma}(.)\underline{u} \end{cases} \tag{9}$$

with $1 \leq i \leq r-1$ and

$$Z = \begin{bmatrix} z_{11} & ... & z_{r1} \\ z_{12} & ... & z_{r2} \\ \vdots & \vdots & \vdots \\ z_{1p} & ... & z_{rp} \end{bmatrix} = \begin{bmatrix} s_1 & \dot{s}_1 & ... & s_1^{r-1} \\ s_2 & \dot{s}_2 & ... & s_2^{r-1} \\ \vdots & \vdots & \vdots & \vdots \\ s_p & \dot{s}_p & ... & s_p^{r-1} \end{bmatrix}$$

$\underline{\phi}(.)$ and $\underline{\gamma}(.)$ are vector and matrix of bounded functions.

3. Control Design

The control law is composed of two parts. The first one, named ideal control [14], is continuous and stabilizes (9) in finite time at the origin when there are no uncertainties. In fact, this control part is used in order to generate trajectories which are tracked by the system. The second part, named integral sliding mode control, provides the complete compensation of uncertainties for $t \geq 0$ and ensures to reach the control objectives.

3.1. Continuous Control Design

Consider system (9) which can be rewritten as:

$$\begin{cases} \dot{Z}_i = Z_{i+1} \\ \dot{Z}_r = \underbrace{\underline{\phi}(.) + [\underline{\gamma}(.) - I]\underline{u}} + \underline{u} \end{cases} \tag{10}$$

where I $(p \times p)$ is an identity matrix, for $1 \leq i \leq r-1$, it yields:

$$\begin{cases} \dot{Z}_i = Z_{i+1} \\ \dot{Z}_r = \beta(.) + \underline{u} \end{cases} \tag{11}$$

Let us define $\underline{u} = \underline{u}_0 + \underline{u}_{dis}$, with $\underline{u}_0$ being the ideal control vector, and $\underline{u}_{dis}$ being the integral sliding mode control vector. Consider now the particular case $\beta(.) = 0$. Then, as no control part $\underline{u}_{dis}$ is necessary to compensate the uncertainties, the control law $\underline{u}$ becomes $\underline{u} = \underline{u}_0$. One gets

$$\dot{Z} = AZ + B\underline{u}_0 \tag{12}$$

where $A^T = [a_1, ..., a_p]$ and $B^T = [b_1, ..., b_p]$ are defined by

$$a_j = \begin{bmatrix} 0 & 1 & \dots & 0 & \dots \\ \vdots & \ddots & \ddots & \ddots & \\ 0 & \ddots & \ddots & \dots & 1 \\ 0 & \ddots & \ddots & \ddots & 0 \end{bmatrix}_{r_j \times r_j},$$

$b_j^T = \begin{bmatrix} 0 \dots 0 \; 1 \end{bmatrix}_{1 \times r_j} . j = 1, ..., p$
The control objective is to drive the state of (12) to $Z = 0$ at the fixed final time $t = t_f < +\infty$, with $Z(0)$ being a bounded initial state vector. This objective can be done through the use of optimal control laws. The control law $\underline{u}_0$ ensures the minimization of the following criterion:

$$J = \frac{1}{2}\int_0^{t_f} Z^T Q Z + [u_0^2]_j dt \tag{13}$$

with $t_f < +\infty$, $|Z(0)| < +\infty$ and Q is a diagonal matrix of symmetric positive definite matrixes q_j under the fixed final state constraint $Z(t_f) = 0$.
Theorem [17]: Consider the linear system (12) with (A, B) reachable. An elements of the control law $\underline{u}_0$ minimizing the criterion (13) and driving system (12) to $Z(t) = 0$ at $t = t_f$ for an initial bounded condition $Z(0)$ is given by (with $0 \leq t \leq t_f < +\infty$)

$$u_{0j} = -b_j^T M_j z_j(t) + b_j^T \delta_j(t) \tag{14}$$

$j = 1, ..., p$ with $\delta_j(t)$ and M_j are defined by

$$\begin{cases} \dot{\delta}_j = -(a_j^T - M_j B_j B_j^T)\delta_j \\ 0 = M_j a_j + a_j^T M_j - M_j b_j b_j^T M_j + q_j \end{cases} \quad (15)$$

The initial conditions $\delta_j(0)$ of $\delta_j(t)$ are selected in order to satisfy the terminal conditions $z_j(t_f) = 0$. The terms u_{0j} are defined in order to drive in finite time the system (12) to $Z(t) = 0$ at $t = t_f$, then the state variables of (12) converge exactly to the origin at $t = t_f$. The control laws u_{0j} will maintain the required equilibrium states $z_j = 0$ even after the terminal time, i.e. $t > t_f$, provided that the forcing terms $\delta_j(t)$ are removed at the terminal time, i.e. $u_{0j} = -b_j^T M_j z_j$ for $t > t_f$ [17]. Then, in order to reach in a finite time t_f the origin $Z = 0$, and to maintain the system (12) at this point for $t > t_f$, we can use [17]:

$$u_{0j} = \begin{cases} -b_j^T M_j z_j(t) + b_j^T \delta_j(t), 0 \le t_f \\ -b_j^T M_j z_j(t), t > t_f \end{cases} \quad (16)$$

3.2. Integral Sliding Manifold

The basic idea consists in determining a sliding manifold such that the state trajectories start on this manifold at the initial time $t = 0$, which induces a sliding mode without reaching phase [3]. One has (for $1 \le i \le r_j - 1$)

$$\begin{cases} \dot{Z}_i = Z_{i+1} \\ \dot{Z}_r = \beta(.) + \underline{u}_0 + \underline{u}_{dis} \end{cases} \quad (17)$$

$\underline{u}_{dis}$ is a discontinuous function designed in order to exactly reject the perturbation $\beta(.)$ throughout the entire response of the system. In order to reach this objective, the integral sliding mode control [14][4] is used. Let $\sigma \in$ R the sliding manifold defined as

$$\sigma = \sigma_0 + \zeta \quad (18)$$

where $\sigma_0^T = [\sigma_{01}, ..., \sigma_{0p}]$ may be designed as the linear combination vector of the state variables of (17). The term $\zeta^T = [\zeta_1, ..., \zeta_p]$ induces the integral terms and will be determined below. In order to determine the motion equations on the sliding manifold, the equivalent control method [14] is used. The time derivative of σ is given by:

$$\dot{\sigma} = \frac{\partial \sigma_0}{\partial Z}(AZ + B\underline{u}_0) + \frac{\partial \sigma_0}{\partial Z_r}(\underline{u}_{dis} + \beta(.)) + \dot{\zeta} \quad (19)$$

A sufficient condition ensuring $\dot{\sigma} = 0$ for $t \ge 0$ is to choose the equivalent control $\underline{u}_{eq}$ of $\underline{u}_{dis}$:

$$\underline{u}_{eq} = -\beta(.) \quad (20)$$

and $\dot{\zeta} = -\frac{\partial \sigma_0}{\partial Z}(AZ + B\underline{u}_0)$, $\zeta(0) = -\sigma_0(Z(0))$, where $\zeta(0)$ is determined based on the requirement $\sigma(0) = 0$.

In this work, to simplify the implementation of the method we consider the special case

$$\sigma_0 = Z_r \longrightarrow \dot{\sigma} = \underline{u} + \beta(.) + \dot{\zeta} \quad (21)$$

Then, condition (20) holds if

$$\dot{\zeta} = -\underline{u}_0, \zeta(0) = -Z_r(0) \quad (22)$$

3.3. Discontinuous Control Design

The control law $\underline{u}_{dis}$ is designed to ensure that the sliding motion on the sliding manifold is guaranteed for $t \ge 0$ in spite of uncertainties and disturbances. Due it's relative simplicity of implementation and their robustness, Super-Twisting algorithm is among the 2-sliding algorithms which are widely used [21], it is given by:

$$\underline{u}_{dis} = -\underline{u}_1 - \underline{u}_2 \quad (23)$$

$$\dot{\underline{u}}_1 = \begin{bmatrix} \dot{u}_{11} \\ ... \\ \dot{u}_{1p} \end{bmatrix} = \begin{bmatrix} \alpha_1 sign(\sigma_1) \\ ... \\ \alpha_p sign(\sigma_p) \end{bmatrix} \text{ and}$$

$$\underline{u}_2 = \begin{bmatrix} u_{21} \\ ... \\ u_{2p} \end{bmatrix} = \begin{bmatrix} \beta_1 |\sigma_1|^{(\frac{1}{2})} sign(\sigma_1) \\ ... \\ \beta_p |\sigma_p|^{(\frac{1}{2})} sign(\sigma_p) \end{bmatrix},$$

where α_j and β_j $(j = 1, ..., p)$ are the Super Twisting control gains, tuned such that the η-attractivity condition is satisfied:

$$\sigma\dot{\sigma} \le -\eta|\sigma|, \eta > 0 \quad (24)$$

The aim of our study is to determine a 2-order sliding modes controller, so that the system output $\underline{y}$ follows a reference trajectory $\underline{y}_d$ and the tracking error vector $\underline{e} = \underline{y} - \underline{y}_d$ converges to zero in presence of uncertainties and disturbances. For this, we consider the following sliding surface [5]:

$$\sigma_0 = \sum_{k=0}^{n-1} \frac{(n-1)!}{k!\,(n-k-1)!} \left(\frac{\partial}{\partial t}\right)^{(n-k-1)} \Lambda^k \underline{e} \quad (25)$$

$\Lambda = diag[\lambda_i]$ is a nth order diagonal matrix of positive constants λ_i, $i = 1, ..., n-1$. The derivative of σ_0 can be expressed by:

$$\dot{\sigma_0} = \delta_{\sigma_0} - \underline{y}_d^{(n)} + F_0(X) + G_0(X)\,\underline{u} + \underline{D} \quad (26)$$

which has the same form as given by equation (7), with $r = 1$, $\phi(.) = \delta_{\sigma_0} - \underline{y}_d^{(n)} + F_0(X) + \underline{D}$, $\gamma(.) = G_0(X)$ and $\delta_{\sigma_0} = \sum_{k=1}^{n-1} \frac{(n-1)!}{k!(n-k-1)!} \left(\frac{\partial}{\partial t}\right)^{(n-k-1)} \Lambda^k \dot{\underline{e}}$.

Since the system is relative degree 1, the seconde derivative of σ_0 is given by:

$$\ddot{\sigma}_0 = \varphi + \varrho\dot{\underline{u}} \quad (27)$$

with $\varphi = \dot{\delta}_{\sigma_0} - \underline{y}_d^{(n+1)} + \dot{F}_0(X) + \dot{\underline{D}} + \dot{G}_0(X)\,\underline{u}$ and $\varrho = G_0(X)$. In order to satisfy the constraint (24), Levant consider the system (27) without D and its derivative and gave the a sufficient conditions [21], which are generalized here for the MIMO systems:

$$\begin{cases} \Delta_j > 0, |\varphi_j| \le \Delta_j, 0 < \Gamma_{m_j} \le \varrho \le \Gamma_{M_j} \\ \alpha_j > \frac{\Delta_j}{\Gamma_{m_j}},\ \beta_j \ge \frac{4\Delta_j}{\Gamma_{m_j}^2} \frac{\Gamma_{M_j}(\alpha_j + \Delta_j)}{\Gamma_{m_j}(\alpha_j - \Delta_j)} \end{cases} \quad (28)$$

In steady state, we will have $\sigma = 0$, choosing these sufficient conditions allow us easily to satisfy the condition (24) at startup. However, for optimal choice of α_j and β_j in the approaching phase, the knowledge of the $\underline{D}$ and $\underline{\dot{D}}$'s upper bounds are required. Nevertheless even if the problem of determining these upper bounds is resolved, the presence of the function $sign\,(\sigma_j)$ causes a chattering phenomenon [12][13]. Even if its order is less important than in the classical SMC, it stills residual.The objective of the proposed approach in the next section, is to use the same control structure with smooth control signal without reducing the tracking performances.

4. Sliding Mode Fuzzy Type-2 Controller

In dealing with MIMO process, the most obvious difference compared to SISO appears in the complicated interactions among the parameters, which makes MIMO process difficult in both of parameters estimation and adjustment. This complexity explains why some of the traditional control approaches in the SISO system are not suitable for direct implementation on the MIMO case. In this section, in order to obtain a smooth global control, the switching signals u_{1j} and u_{2j} will be replaced by control signals from adaptive type-2 fuzzy systems. The adaptation laws of adjustable parameters of the fuzzy systems are deduced from the stability study of the closed-loop process. This adaptation in addition smoothing the control will enable us to better anticipate disturbances by providing us at each time the optimal values ($\underline{\alpha}^*$ and $\underline{\beta}^*$) of $\underline{\alpha}$ and $\underline{\beta}$ respectively. Where $\underline{\alpha} = [\alpha_1, \alpha_2, ..., \alpha_p]^T$ and $\underline{\beta} = [\beta_1, \beta_2, ..., \beta_p]^T$.

4.1. Proposed Type-2 FLS

Both adaptive systems used to approximate the control gains have the same structure as given by (2). The antecedent type-2 Membership functions $\mu_{\tilde{F}_i^j}$ will be fixed as given in figure (2). The consequent sets which are the adjustable parameters will be type-1 centroids.
The output fuzzy systems are given by:

$$Y_{tr} = [y_l, y_r] = \int_{\theta^1} ... \int_{\theta^M} \int_{w^1} ... \int_{w^M} 1/\frac{\sum_i^M w^i \theta^i}{\sum_i^M w^i} \quad (29)$$

In the case of type-2 fuzzy systems, the universal approximation property is retained with an additional phase of reduction type. Since each set on the right-hand side of (29) is an interval type-1 set, hence Y_{tr} is an interval type-1 set. So, to find the type-reduced set Y_{tr}, we just need to compute the two end points of these intervals. The maximum value of y is y_r and its minimum value is y_l,$\forall y \in Y_{tr}$, y can be represented by:

$$y = \frac{\sum_i^M w^i \theta^i}{\sum_i^M w^i} \quad (30)$$

The y_r points are associated solely with y_r^i, in same way y_l is associated solely with y_l^i. In the centre of sets, Karnik and Mendel [22] have shown that the two end points of Y_{tr}, y_r and y_l^i depend only on a mixture $\underline{w}^i$ or $\overline{w}^i$ of values, since $w^i \in W^i = [\underline{w}^i, \overline{w}^i]$. In this case, each y_r and y_l can be represented as a vector of fuzzy basis functions expansion:

$$y_l = \frac{\sum_{i=1}^M w_l^i \theta_l^i}{\sum_{i=1}^M w_l^i} = \sum_{i=1}^M \theta_l^i \xi_l^i \quad (31)$$

where w_l^i denotes the firing strength membership grade (either $\overline{w}^i$ or $\underline{w}^i$) of the i^{th} rule contributing to the left-most point y_l and $\xi_l^i = \frac{w_l^i}{\sum_{i=1}^M w_l^i}$ are the components of the first FBF vector ξ_l of y. Similarly [12]:

$$y_r = \frac{\sum_{i=1}^M w_r^i \theta_r^i}{\sum_{i=1}^M w_r^i} = \sum_{i=1}^M \theta_r^i \xi_r^i \quad (32)$$

To obtain a crisp outputs from the T2FLS y, we must defuzzify the type-reduced set Y_{tr}. Since this type-reduced set is an interval set, therefore, the defuzzified output of y will be the average of y_l and y_r, that is:

$$y = \frac{y_l + y_r}{2} \quad (33)$$

Replacing (31) and (32) into (33), we obtain:

$$y = \frac{\underline{\theta}_l^T \underline{\xi}_l + \underline{\theta}_r^T \underline{\xi}_r}{2} = \underline{\theta}^T \left[\frac{\underline{\xi}_l + \underline{\xi}_r}{2}\right] = \underline{\theta}^T \underline{\xi} \quad (34)$$

Where $\underline{\xi} = \frac{\underline{\xi}_l + \underline{\xi}_r}{2}$ is the average FBF of y. In order to compute y_l and y_r we use the Karnik and Mendel iterative algorithm [22].

4.2. Proposed Control Synthesis

In this subsection, we study the stability and the robustness of the closed loop system using the adaptive fuzzy systems to replace $\underline{u}_1$ and $\underline{u}_2$. The terms of the high frequency control components will be given by:

$$\underline{\hat{u}}_1(\sigma) = \Theta_1^T \Psi t_c \quad (35)$$

$$\underline{\hat{u}}_2(\sigma) = (diag|\sigma|^{\left(\frac{1}{2}\right)})\Theta_2^T \Psi \quad (36)$$

Where $\Theta_1^T = [\underline{\theta}_{11}, \underline{\theta}_{12}, ..., \underline{\theta}_{1p}]$,$\Theta_2^T = [\underline{\theta}_{21}, \underline{\theta}_{22}, ..., \underline{\theta}_{2p}]$, $\Psi = diag[\xi_i]$ and $diag|\sigma|^{\left(\frac{1}{2}\right)} =$

$diag\left[|\sigma_j|^{\left(\frac{1}{2}\right)}\right]$,$(j = 1, ..., p)$, t_c is the convergence time, it is chosen to be $t_c = t_f$. $\underline{\hat{u}}_1(\sigma)$ and $\underline{\hat{u}}_2(\sigma)$ are two vectors of fuzzy systems defined by (34), whose only input is the value of the sliding variables σ_j, and the outputs give the optimal values of $\underline{\alpha}$ and $\underline{\beta}$ respectively [13], but without the integral sliding mode concept, t_c is only estimated.
The proposed control law is given by:

$$\underline{u} = \underline{u}_0 + \underline{u}_{eq} - \underline{\hat{u}}_1(\sigma) - \underline{\hat{u}}_2(\sigma) \tag{37}$$

To study the stability we consider the Lyapunov equation:

$$V(t) = \frac{1}{2}\sigma^T\sigma + \frac{1}{2\gamma_1}\tilde{\Theta}_1^T\tilde{\Theta}_1 + \frac{1}{2\gamma_2}\tilde{\Theta}_2^T\tilde{\Theta}_2 \tag{38}$$

Where $\tilde{\Theta} = \Theta - \Theta^*$, Θ^* is the optimal vector values of Θ, γ_1 and γ_2 are two positive training constants. So, the time derivative of (38) is:

$$\dot{V}(t) = \sigma^T\dot{\sigma} + \frac{1}{\gamma_1}\tilde{\Theta}_1^T\dot{\Theta}_1 + \frac{1}{\gamma_2}\tilde{\Theta}_2^T\dot{\Theta}_2 \tag{39}$$

By replacing the proposed control into the derivative of the sliding surface, we obtain

$$\begin{aligned} \dot{\sigma} &= -\underline{\hat{u}}_1(\sigma) - \underline{\hat{u}}_2(\sigma) - \underline{D} \\ &= -\underline{\hat{u}}_1(\sigma) - \underline{\hat{u}}_1^*(\sigma) + \underline{\hat{u}}_1^*(\sigma) - \underline{\hat{u}}_2(\sigma) - \underline{\hat{u}}_2^*(\sigma) + \\ &\quad \underline{\hat{u}}_2^*(\sigma) - \underline{D} \end{aligned} \tag{40}$$

Where $\underline{\hat{u}}_2^*(\sigma) = (diag|\sigma|^{\frac{1}{2}})\underline{\Theta}_2^{*T}\Psi$ and $\underline{\hat{u}}_1^*(\sigma) = \underline{\Theta}_1^{*T}\Psi t_c$, then the derivative of the sliding surface can be given by:

$$\dot{\sigma} = -\left(\Theta_1 - \Theta_1^*\right)^T \Psi t_c - (diag|\sigma|^{\left(\frac{1}{2}\right)})\left(\Theta_2 - \Theta_2^*\right)^T \Psi - \underline{\hat{u}}_2^*(\sigma) - \underline{\hat{u}}_1^*(\sigma) - \underline{D} \tag{41}$$

Substituting (41) into (39), we obtain:

$$\dot{V}(t) = -\sigma^T\left(\underline{\hat{u}}_2^*(\sigma) + \underline{\hat{u}}_1^*(\sigma)\right) + \frac{\tilde{\Theta}_1^T}{\gamma_1}\left(\dot{\Theta}_1 - \gamma_1\sigma^T\Psi t_c\right) + \frac{\tilde{\Theta}_2^T}{\gamma_2}\left(\dot{\Theta}_2 - \gamma_2\sigma^T(diag|\sigma|^{\left(\frac{1}{2}\right)})\Psi\right) - \sigma^T\underline{D} \tag{42}$$

If we choose the following adaptation laws:

$$\dot{\Theta}_1 = \gamma_1\sigma^T\Psi t_c \tag{43}$$

$$\dot{\Theta}_2 = \gamma_2\sigma^T(diag|\sigma|^{\left(\frac{1}{2}\right)})\Psi \tag{44}$$

we obtain

$$\dot{V} = -\sigma^T\underline{D} - |\sigma|^T\left(\underline{\alpha}_1^* t_c + \underline{\beta}_2^*|\sigma|^{\left(\frac{1}{2}\right)}\right) \leq 0 \tag{45}$$

Thus, the proposed control law guaranties both robustness and stability of the closed loop system.

5. Illustrative Example

In order to validate the proposed controller, a robot manipulator of two degrees freedom is considered as numerical example. The dynamic model of MIMO system shown in figure (1) is given by the following equation:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \Gamma(q) + \underline{D} \tag{46}$$

$M(q) = \begin{bmatrix} (m_1+m_2)l_1^2 & m_2l_1l_2(s_1s_2+c_1c_2) \\ m_2l_1l_2(s_1s_2+c_1c_2) & m_2l_2^2 \end{bmatrix}$
$C(q,\dot{q}) = m_2l_1l_2(c_1s_2 - s_1c_2)\begin{bmatrix} 0 & -\dot{q}_2 \\ -\dot{q}_1 & 0 \end{bmatrix}$
$G(p) = \begin{bmatrix} -(m_1+m_2)l_1gs_1 \\ -m_2l_2gs_2 \end{bmatrix}$ and $q = [q_1, q_2]^T$.

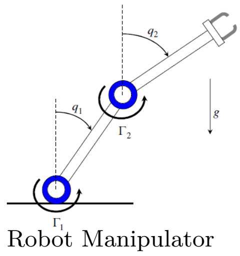

Figure 1: Two Link Robot Manipulator

q_1,q_2 are generalized coordinates, $M(q)$ is the moment of inertia, $C(q,\dot{q})$ includes coriolis centripetal forces, and $G(q)$ is the gravitational force, $\underline{D}$ is the sum of the external perturbation and model uncertainty. Other quantities are link mass
$m_1 = 1(kg)$, $m_2 = 1(kg)$, link length $l_1 = 1(m)$, $l_2 = 1(m)$, angular position q_1, $q_2(rad)$, vector applied torque $\underline{u}(q)(N-m)$, the gravitational acceleration $g = 9.8(m/s^2)$, and short-hand notation $s_i = sin(q_i)$, $c_i = cos(q_i)$, $i = 1, 2$. Let $x_1 = q_1$, $x_2 = \dot{q}_1$, $x_3 = q_2$, and $x_4 = \dot{q}_2$, then (46) can be written as following state-space:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f_1(x) + g_{11}(x)u_1 + g_{12}u_2 + D_1 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = f_2(x) + g_{21}(x)u_1 + g_{22}u_2 + D_2 \\ y_1 = x_1, y_2 = x_2, y_3 = x_3, y_4 = x_4 \end{cases} \tag{47}$$

$$D_1 = (\Delta f_1(x) + \Delta g_{11}(x)u_1 + \Delta g_{12}(x)u_2) + d_1$$

$$D_2 = (\Delta f_2(x) + \Delta g_{21}(x)u_1 + \Delta g_{22}(x)u_2) + d_2$$

We assume that the angular positions x_1, x_3 are constrained within $[-(\pi/2), (\pi/2)]$, as in [20] the T-S fuzzy type-2 nominal model for the system (47) is given by the following nine-rule fuzzy model:
Rule$z$: If x_1 is about $\{0 \vee -\frac{\pi}{2} \vee +\frac{\pi}{2}\}$ and x_3 is about $\{0 \vee -\frac{\pi}{2} \vee +\frac{\pi}{2}\}$ then

$$\begin{cases} \dot{x}(t) = A_zX + B_z\underline{u} \\ y(t) = CX \end{cases}$$

where $X = [x_1, x_2, x_3, x_4]^T$, $\underline{u} = [u_1, u_2]^T$, the functions f_i, g_{ii} and the value of the matrixes A_z, B_z, C are the same given in [20], ($i = 1, 2$ and $z = 1, ..., 9$).
According to the sliding surface (25) and since we have two $2nd$ order subsystems, the sliding surface is given by: $\sigma = [\sigma_1, \sigma_2]^T = [\dot{e}_1 + \lambda_1e_1, \dot{e}_2 + \lambda_2e_2]^T$.
To build the nominal model, we define three type-2 fuzzy interval sets for each variable x_1 and x_2. Similarly to generate the four adaptive fuzzy systems

which allow us to approximate $u_1 = [u_{11}, u_{12}]^T$ and $u_2 = [u_{21}, u_{22}]^T$, we consider five type-2 fuzzy interval sets: Negative, Small Negative, Zero, Small Positive and Positive for each variable s_1 and s_2. In sliding regime, the transition between the approach phase and the sliding phase occurs when the value of this one is zero. Therefore, Hamzaoui and al showed in [8] that more is small the axis of the membership function corresponding to the fuzzy set zero, better are the tracking performances. The membership functions of x_1 and x_3 involved in building the model are given in [13]. Similarly the antecedent sets of the four adaptive systems are obtained by adding an area of uncertainty. Figure (2) (a and b) shows the shape fuzzy sets of the nominal model and the fuzzy adaptive systems. The objective is to con-

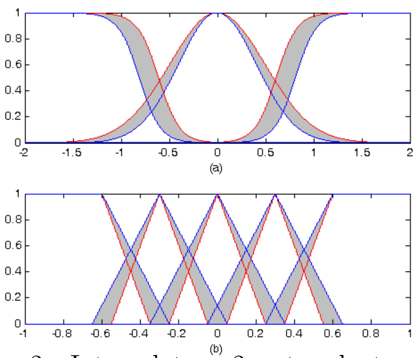

Figure 2: Interval type-2 antecedent membership functions of:(a) x_1 and x_3, (b) σ_1 and σ_2.

trol the state system X to track the reference trajectory $\underline{y}_d = [sin(2t), 2cos(2t), sin(2t), 2cos(2t)]^T$. Suppose that structural uncertainties, which represent the variation in the masses are in the form $dm_j = m_j(0.2sin(2t) + 0.5sin(3t))$, while the external perturbation submitted to the system is given by $\underline{d} = [0, sin(2t) + sin(3t), 0, sin(2t) + sin(3t)]^T$. The system constants are chosen as: $\lambda_1 = 20, \lambda_2 = 18, \alpha_1 = 20, \alpha_2 = 18, \beta_1 = 45, \beta_2 = 35$. Let $z_1 = \sigma_1, z_2 = \sigma_2$, since the subsystems are relative degree one $a_1 = a_2 = [0]$ and $b_1 = b_2 = [1]$. Let $t_f = 0.5s$, the state is initialized at $X(0) = [1, 0, 1, 0]$, which implies that $z_1(0) = z_2(0) = -28$. As mentioned in [17], the initial conditions of δ_1, δ_2 are computed in order that the sliding variable σ_1, σ_2 equal 0 at exactly $t = t_f = 0.5s$. The first step consists in solving Riccati's equation (second line of (15)) for a matrix q_j which is symmetric positive definite. The solution of this equation M_j is a symmetric positive definite matrix. q_1, q_2 are stated as $q_1 = q_2 = [10]$, then one gets $M_1 = M_2 = 3.1623$. The second step consists in determining $\delta_1(0), \delta_2(0)$ such that $z_1(t_f) = z_2(t_f) = 0$, knowing that only initial bounded conditions $z_1(0), z_2(0)$ are known. Then, the initial conditions $\delta_j(0), (j = 1, 2)$ ensuring that $z_j(t_f) = 0$ are derived from $\delta_j(0) = -H_j^{-1} z_j(0)$. H is partial reachability gramien, its computation is given in [18]. We get $\delta_1 = \delta_2 = 32.5907$. Figures 3-4 display the convergence of $\sigma_1, \dot{\sigma}_1, \sigma_2, \dot{\sigma}_2$ to *zero* in exactly $0.5s$. Figures 6 shows a good tracking of the position trajectories. The states converge quickly to their references. Figure (6) displays the applied torque signals which are smooth and free from chattering and abrupt variations.

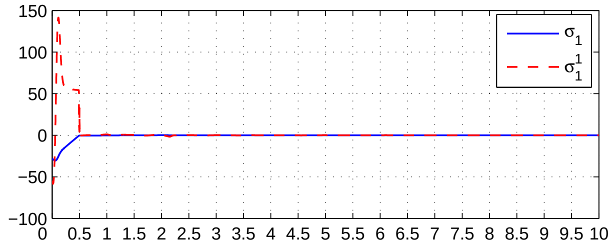

Figure 3: δ_1 and $\dot{\delta}_1$ versus time(s).

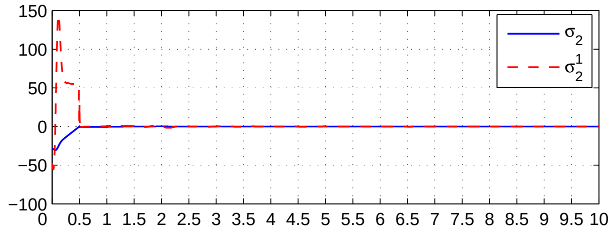

Figure 4: δ_2 and $\dot{\delta}_2$ versus time(s).

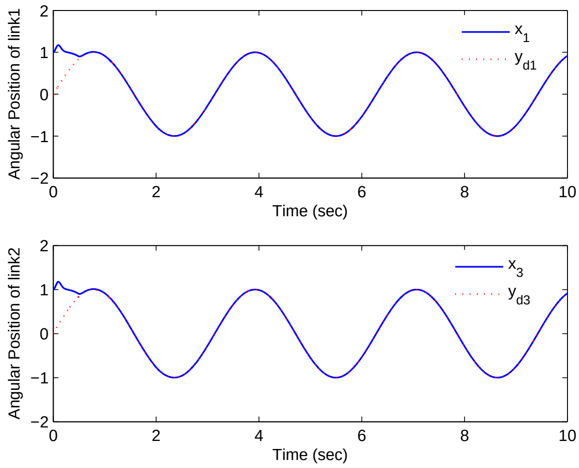

Figure 5: The states x_1,x_3 and their references y_{d1},y_{d3}.

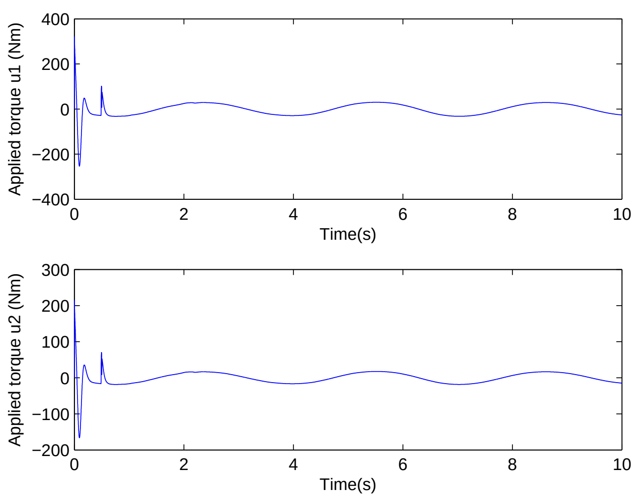

Figure 6: Control input $u_1(Nm)$ and $u_2(Nm)$.

6. Conclusion

In this paper, we have presented a type-2 fuzzy sliding modes controller for a perturbed MIMO uncertain nonlinear systems. The control is generated from a nominal type-2 fuzzy model, who exploits the linear local models of the system. These local models are obtained by linearisation around some operating points. Adaptive interval type-2 fuzzy systems are introduced to calculate the switching control terms in order to overcome some restrictive constraints and to eliminate efficiency the chattering phenomenon. The stability and robustness of the closed loop system are proved analytically. The update of adjustable parameters of the type-2 fuzzy systems are ensured by the adaptation laws derived using Lyapunov theory. The design uses the integral sliding mode concept. The controller is able to steer to zero in finite time the sliding surfaces and their derivatives, the convergence time can be fixed in advance. A simulation example has been presented to illustrate the effectiveness and the robustness of the proposed approach.

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