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Fault Detection for Uncertain Delayed Switching Discrete-time Systems: A Cone Complementarity Approach

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Abstract 1: In this paper, fault detection problem for uncertain discrete-time switching systems with delay is studied. Sufficient conditions of building an observer are obtained by using multiple Lyapunov function. These condition are worked out in a new way, using cone complementarity technique, to obtain new LMIs with slack variables and multiple weighted residual matrices. The obtained results are applied on a numerical example with all the entries of the same sizes.

Key-words: Switching systems, delay, uncertain systems, arbitrary switching sequence, fault detection, observer, multiple Lyapunov function, LMI, cone complementarity.

I. INTRODUCTION

Switched systems are a class of hybrid systems encountered in many practical situations which involve switching between several subsystems depending on various factors. Generally, a switching system consists of a family of continuous-time subsystems and a rule that supervises the switching between them. For example, many process in the chemical and pharmaceutical industries operate following batches, composed of different operations that are carried out in sequence. This changes discontinuously the dynamics of the operation [21]. In manufacturing, hybrid switched systems are found in steel rolling mills [17], used for producing thin metal sheets, following several steps based on pressing the metal strip with rolling cylinders: the dynamics are known to change at each pass due to the variation in thickness [12]. Many other examples can be found in the automotive industry, in aircraft and air traffic control, and many other fields.

Two main problems are widely studied in the literature according to the classification given in [11], [9]: The first one, which is the one solved in this work, looks for testable conditions that guarantee the asymptotic stability of a switching system under arbitrary switching rules [13], [3], [4], [5], [7], while the second is to determine a switching sequence that renders the switched system asymptotically stable (see [20] and the reference therein). Following the first approach, [10] investigate the problem of designing a switching compensator for a plant switching amongst a (finite) family of given configurations (A_i, B_i, C_i).

A main problem which is always inherent to all dynamical systems is the possibility of the presence of actuator faults or sensor faults saturations. Even for linear systems, this problem has been an active area of research for many years [16], [1], [24]. The study of this problem was extended to switching systems (see [23], [19], [2] and the references therein). In [23], a switching discrete-time uncertain system with state delays is considered. The design method is based on the construction of a filter and a fault estimation. This approach leads to a big number of matrices to be determined. In [8], an observer is built to detect the fault when it occurs. In this work, the main condition is worked out by introducing slack variables in the main inequality. In the present work, we treat the same class of systems as in [23] but with a Luenberger observer as in [8]. The obtained condition of asymptotic stability in presence of fault based on the H_∞ technique is then worked out in a simple way to obtain new LMIs. The proposed LMIs are different from the ones obtained in [19] and [8]. The idea is the use of the well known cone complementarity technique. However, an optimization problem is necessary to find the required solutions. In order to avoid this way, a new and simple technique is proposed. The keystone of this technique is to look for matrix S satisfying $-P^{-1} < -S$ by introducing slack variables only in this inequality. Besides the introduction of slack variables X_i, the residual signal is designed with multiple matrices V_i. The applicability of the obtained results on a numerical example shows the usefulness of the observer which can work with success even in presence of input entry, bounded unknown input and fault of the same sizes while filter based approaches may not success in this case. This new technique is less conservative than the one presented in [8].

This paper is organized as follows: Section 2 deals with the problem statement while Section 3 presents some preliminary results on fault detection problem. The main results of this paper are developed in Section 4 together with an illustrative example.

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II. PROBLEM FORMULATION

Consider the following delayed discrete-time switching system:

$$\begin{aligned} x(k+1) &= \mathcal{A}_\alpha x(k) + \mathcal{A}_{d\alpha} x(k-\tau) + \mathcal{B}_\alpha u(k) + E_\alpha d(k) + \\ & F_\alpha f(k) \qquad (1) \\ y(k) &= C_\alpha x(k) + C_{d\alpha} x(k-\tau) + D_\alpha u(k) + N_\alpha d(k) + \\ & M_\alpha f(k) \end{aligned}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the control, $y \in \mathbb{R}^p$ is the output, τ is the delay, $d \in \mathbb{R}^g$ is an external unknown input, $f \in \mathbb{R}^q$ is the fault and α a switching rule which takes its values in the finite set $\mathcal{I} := \{1, \dots, N\}$, $k \in \mathbb{Z}_+$. Each subsystem α is called a mode. Matrices $\mathcal{A}_i, \mathcal{A}_{di}, \mathcal{B}_i$ has the same following structure: $\mathcal{A}_i = A_i + \Delta A_i(k), \mathcal{A}_{di} = A_{di} + \Delta A_{di}(k), \mathcal{B}_i = B_i + \Delta B_i(k)$, with the uncertainty terms are norm-bounded according to:

$$[\Delta A_i(k) \;\; \Delta A_{di} \;\; \Delta B_i(k)] = L_i W(k) [H_{1i} \;\; H_{2i} \;\; H_{3i}]. \qquad (2)$$

Matrices $A_i, Ad_i, B_i, E_i, \;\; F_i, C_i, Cd_i, D_i, N_i, M_i, L_i, H_{1i}, H_{2i}, H_{3i}$ are of appropriate size constant known matrices. It is assumed that:

- Each time only one subsystem is active.
- The switching rule is not known *a priori* but $\alpha(k)$ is available at each sampling time k.
- $W(k)^T W(k) \leq \mathbb{I}$.

The second assumption corresponds to practical implementations where the switched system is supervised by a discrete-event system or operator allowing for $\alpha(k)$ to be known in real time. Upon introducing the indicator function:

$$\xi(k) = [\xi_1(k), \dots, \xi_N(k)]^T \qquad (3)$$

where $\xi_i(k) = 1$ if the switching system is in mode i and $\xi_i(k) = 0$ if it is in a different mode, one can write the switching system (1) as follows:

$$\begin{aligned} x(k+1) = \sum_{i=1}^{N} \xi_i(k) [&\mathcal{A}_i x(k) + \mathcal{A}_{di} x(k-\tau) + \mathcal{B}_i u(k) \\ & + E_i d(k) + F_i f(k)]. \\ y(k) = \sum_{i=1}^{N} \xi_i(k) [&C_i x(k) + C_{di} x(k-\tau) + D_i u(k) \\ & + N_i d(k) + M_i f(k)] \end{aligned} \qquad (4)$$

In this work, we are interested by the synthesis of an observer of this class of systems in order to detect a default when it occurs in the switching system. For this, consider the following switching observer:

$$\begin{aligned} \hat{x}(k+1) = \sum_{i=1}^{N} \xi_i(k) [&A_i \hat{x}(k) + A_{di} \hat{x}(k-\tau) \\ & + B_i u(k) + K_i (y(k) - \hat{y}(k))]. \\ \hat{y}(k) = \sum_{i=1}^{N} \xi_i(k) [&C_i \hat{x}(k) + C_{di} \hat{x}(k-\tau) + D_i u(k)] \end{aligned} \qquad (5)$$

In this structure, only matrices K_i are to be designed.

Remark 2.1: As assumed in the second assumption, the switching rule is not known *a priori* but $\alpha(k)$ is available at each sampling time k. This means that one can synchronize the switch of the observer with the switch of the system. In this case, the problem to have the system and the observer evolving in different modes can not occur as it arises in the continuous case studied in [2].

The residual of this observer is defined as:

$$r(k) = \sum_{i=1}^{N} \xi_i(k) V_i (y(k) - \hat{y}(k)) \qquad (6)$$

Matrices V_i are to be computed. It is worth noting that the proposed structure of the residual is different from the classical one used in [2] where only one matrix V is looked for. Defining the observer error by $e_k = x_k - \hat{x}_k$ leads to:

$$\begin{aligned} e_{k+1} = \sum_{i=1}^{N} \xi_i(k) [&(A_i - K_i C_i) e_k + (A_{di} - K_i C_{di}) e_{k-\tau} \\ & + (E_i - K_i N_i) d_k + (F_i - K_i M_i) f_k + \Delta A_i(k) x_k \\ & + \Delta A_{di}(k) x_{k-\tau} + \Delta B_i(k) u_k]. \end{aligned}$$

Define an augmented state vector as: $\tilde{x}_k = [x_k^T \;\; \hat{x}_k^T \;\; e_k^T]^T$ and an augmented input vector as $w_k = [u_k^T \;\; d_k^T \;\; f_k^T]^T$. The corresponding dynamical system is then derived.

$$\begin{aligned} \tilde{x}_{k+1} &= \sum_{i=1}^{N} \xi_i(k) [\tilde{\mathcal{A}}_i \tilde{x}_k + \tilde{\mathcal{A}}_{di} \tilde{x}_{k-\tau} + \tilde{\mathcal{B}}_i w_k] \qquad (7) \\ r_k &= \sum_{i=1}^{N} \xi_i(k) [\tilde{C}_i \tilde{x}_k + \tilde{C}_{di} \tilde{x}_{k-\tau} + \tilde{D}_i w(k)], \qquad (8) \end{aligned}$$

where $\tilde{\mathcal{A}}_i = \tilde{A}_i + \Delta \tilde{A}_i(k)$, $\tilde{\mathcal{A}}_{di} = \tilde{A}_{di} + \Delta \tilde{A}_{di}(k)$ and $\tilde{\mathcal{B}}_i = \tilde{B}_i + \Delta \tilde{B}_i(k)$, with the following matrices:

$$\begin{aligned} \tilde{A}_i &= \begin{bmatrix} A_i & 0 & 0 \\ 0 & A_i & K_i C_i \\ 0 & 0 & A_i - K_i C_i \end{bmatrix}, \\ \tilde{A}_{di} &= \begin{bmatrix} A_{di} & 0 & 0 \\ 0 & A_{di} & K_i C_{di} \\ 0 & 0 & A_{di} - K_i C_{di} \end{bmatrix}, \\ \tilde{B}_i &= \begin{bmatrix} B_i & E_i & F_i \\ B_i & -K_i N_i & -K_i M_i \\ 0 & E_i - K_i N_i & F_i - K_i M_i \end{bmatrix}, \end{aligned} \qquad (9)$$

$$\begin{aligned}
\Delta\tilde{A}_i(k) &= \begin{bmatrix} \Delta A_i(k) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \Delta A_i(k) \end{bmatrix}, \\
\Delta\tilde{A}_{di}(k) &= \begin{bmatrix} \Delta A_{di}(k) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \Delta A_{di}(k) \end{bmatrix}, \\
\Delta\tilde{B}_i(k) &= \begin{bmatrix} \Delta B_i(k) & 0 & 0 \\ 0 & 0 & 0 \\ \Delta B_i(k) & 0 & 0 \end{bmatrix}, \\
\tilde{C}_i &= \begin{bmatrix} 0 & 0 & V_iC_i \end{bmatrix}, \tilde{C}_{di} = \begin{bmatrix} 0 & 0 & V_iC_{di} \end{bmatrix}, \\
\tilde{D}_i &= \begin{bmatrix} 0 & V_iN_i & V_iM_i \end{bmatrix}. \qquad (10)
\end{aligned}$$

The uncertain terms for the augmented system can be again developed as follows: $[\Delta\tilde{A}_i(k) \quad \Delta\tilde{A}_{di}(k)] = \tilde{L}_i\tilde{W}[\tilde{H}_{1i} \;\; \tilde{H}_{2i}], \Delta\tilde{B}_i(k) = \tilde{L}_{3i}\tilde{W}\tilde{H}_{3i}$, where

$$\begin{aligned}
\tilde{L}_i &= \begin{bmatrix} L_i & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & L_i \end{bmatrix}, \\
\tilde{H}_{3i} &= \begin{bmatrix} H_{3i} & 0 & 0 \\ 0 & 0 & 0 \\ H_{3i} & 0 & 0 \end{bmatrix}, \\
\tilde{H}_{si} &= \begin{bmatrix} H_{si} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & H_{si} \end{bmatrix}, \\
\tilde{W}(k) &= diag\{W(k), W(k), W(k)\}, s = 1, 2. \quad (11)
\end{aligned}$$

In this structure, the new input w_k also includes the fault vector. As a consequence, the effect of the fault on the residual will be also reduced. This fact can have a positive impact on the stability of the system.

The objective of this work is to develop a synthesis method based on the H_∞ tool to compute the unknown matrices representing the switching observer gains K_i and the residual gains V_i. One can note that the way followed in this paper is different from the one used in [23] where, a filter and a dynamic of the fault are completely characterized. With the use of a Luenberger switching observer, the number of unknown matrices is lower in our case.

III. PRELIMINARY RESULTS

As used in the literature of fault detection [23], [2], [24], the identification of the fault f_k is not necessary. One can use the following residual criterion:

$$J_k = \left(\sum_{i=k_0}^{k_0+k} r_i^T r_i \right)^{1/2}, \qquad (12)$$

where k_0 and k define the interval of the evaluation window, k being the current sampled time. The size of this window is increasing till is equal to the global horizon. Then, this evaluation function can be compared to a threshold J_0 to conclude if a fault occurs or not, as follows:

$$\begin{aligned}
J_k &> J_0 \Rightarrow \text{Faults} \Rightarrow \text{Alarm} \\
J_k &\leq J_0 \Rightarrow \text{No Faults}
\end{aligned}$$

The threshold criterion can be chosen as indicated in [23] as:

$$J_0 = \sup_{d\in l_2, u\in l_2, f=0,k} J_k \qquad (13)$$

Nevertheless, another evaluation function based on a past receding window can be defined as:

$$J_r = \left(\sum_{i=k-T}^{k} r_i^T r_i \right)^{1/2}, k > T, \qquad (14)$$

where T is the fixed window size and k is the current sampled time.

Now, a useful lemmas is recalled.

Lemma 3.1: [22] Given symmetric matrix S and matrices $L, W(k)$ and H of appropriate size, then

$$S + L^T W(k) H + H^T W(k) L < 0,$$

holds for $W(k)^T W(k) \leq \mathbb{I}$ if and only if there exists a scalar $\epsilon > 0$ such that

$$S + \epsilon^{-1} L^T L + \epsilon H^T H < 0,$$

The technique of H_∞ problem for the augmented switching system (7) consists in ensuring the asymptotic stability of the system for $w_k = 0$ and $\tilde{x}_0 \neq 0$ while realizing the following condition for $w_k \neq 0$ and $\tilde{x}_0 = 0$:

$$\sup_{w_k\neq 0, w_k\in l_2[0,\infty)} \frac{\sqrt{r_k^T r_k}}{\sqrt{w_k^T w_k}} < \gamma, \gamma > 0. \qquad (15)$$

This condition is realized if

$$J(\gamma) = \sum_{k=0}^{T_k-1} [r_k^T r_k - \gamma^2 w_k^T w_k] < 0. \qquad (16)$$

To realize condition (16), one has to use a Lyapunov function $V(\tilde{x})$ and look for the condition realizing

$$\begin{aligned}
J(\gamma) &= \sum_{k=0}^{T_k-1} [r_k^T r_k - \gamma^2 w_k^T w_k + \Delta V(\tilde{x})] - V(\tilde{x}(T_k)), \\
&\leq \sum_{k=0}^{T_k-1} [r_k^T r_k - \gamma^2 w_k^T w_k + \Delta V(\tilde{x})] < 0 \qquad (17)
\end{aligned}$$

for any $w_k \in l_2[0,\infty)$ and with $\tilde{x}_0 = 0$. It is obvious that condition (17) is satisfied if $\Delta V(\tilde{x})] < 0$, that is the system is asymptotically stable for $w_k = 0$ and $\tilde{x}_0 \neq 0$.

The first result we recall is a sufficient condition of H_∞ fault detection for the augmented switching system (7) presented by [23].

Lemma 3.2: [23] For a given scalar $\gamma > 0$, system (7) under arbitrary switching is asymptotically stable when $w_k = 0$ and under zero-initial conditions, guarantees the performance index (15) for all $w_k \in l_2[0,\infty)$, if there exists positive definite symmetric matrices P_i and Q_i, $i \in \mathcal{I}$, such that:

$$\Sigma \;=\; \begin{bmatrix} -P_j^{-1} & \tilde{\mathcal{A}}_i & \tilde{\mathcal{A}}_{di} & \tilde{\mathcal{B}}_i & 0 & 0 \\ * & -P_i & 0 & 0 & \tilde{\mathcal{C}}_i^T & \mathbb{I} \\ * & * & -Q_s & 0 & \tilde{\mathcal{C}}_{di}^T & 0 \\ * & * & * & -\gamma^2\mathbb{I} & \tilde{\mathcal{D}}_i^T & 0 \\ * & * & * & * & -\mathbb{I} & 0 \\ * & * & * & * & * & -Q_i^{-1} \end{bmatrix} < 0, \quad \forall (i,j,s) \in \mathcal{I}^3, \tag{18}$$

where $*$ stands for the symmetrical term of the corresponding off-diagonal term.

Remark 3.1: Even system (7) is given with similar notations as in [23] to have the possibility to use Lemma 3.2, the changes of variables (10) are completely different from those taken in [23]. Further, the proof of this result is based on the use of a multiple Lyapunov-Krasovskii functional given by:

$$V(x_k) = \tilde{x}_k^T \left(\sum_{i=1}^{N} \xi_i(k+1)P_i\right)\tilde{x}_k + \sum_{s=k-\tau}^{k-1} \tilde{x}_s^T \left(\sum_{i=1}^{N} \xi_i(k)Q_i\right)\tilde{x}_s \tag{19}$$

With these preliminary results, we are now able to solve the problem of fault detection for switching discrete-time systems by using a switching observer.

IV. MAIN RESULTS

The objective of this section is to work out Inequality (18) to obtain an LMI enabling one to synthesize the switching observer together with its corresponding residual, making possible fault detection of the delayed discrete-time switching system.

The cone complementarity technique introduced by [15] and used in many works as [18] is now used to derive an equivalent lemma.

Lemma 4.1: For a given scalar $\gamma > 0$, system (7) under arbitrary switching is asymptotically stable when $w_k = 0$ and under zero-initial conditions, guarantees the performance index (15) for all $w_k \in l_2[0,\infty)$, if there exists positive definite symmetric matrices P_i and Q_i, S_i, R_i $i \in \mathcal{I}$, such that:

$$\Sigma = \begin{bmatrix} -S_j & \tilde{\mathcal{A}}_i & \tilde{\mathcal{A}}_{di} & \tilde{\mathcal{B}}_i & 0 & 0 \\ * & -P_i & 0 & 0 & \tilde{\mathcal{C}}_i^T & \mathbb{I} \\ * & * & -Q_s & 0 & \tilde{\mathcal{C}}_{di}^T & 0 \\ * & * & * & -\gamma^2\mathbb{I} & \tilde{\mathcal{D}}_i^T & 0 \\ * & * & * & * & -\mathbb{I} & 0 \\ * & * & * & * & * & -R_i \end{bmatrix} < 0,$$

$$\forall (i,j,s) \in \mathcal{I}^3, \tag{20}$$

$$P_i S_i = \mathbb{I}, \tag{21}$$

$$Q_i R_i = \mathbb{I}. \tag{22}$$

Proof: The proof is obvious by letting $S_i = P_i^{-1}$ and $R_i = Q_i^{-1}$. $\square$

The following result we present is an equivalent lemma of Lemma 4.1 which takes account of the uncertainties by using the separation Lemma 3.1.

Lemma 4.2: For a given scalar $\gamma > 0$, system (7) under arbitrary switching is asymptotically stable when $w_k = 0$ and under zero-initial conditions, guarantees the performance index (15) for all $w_k \in l_2[0,\infty)$, if there exists positive definite symmetric matrices P_i, Q_i, S_i and R_i $i \in \mathcal{I}$ and positive scalars ϵ_i, such that:

$$\begin{bmatrix} -S_j & \tilde{A}_i & \tilde{A}_{di} & \tilde{B}_i & 0 & 0 & \tilde{L}_i^T \\ * & -\Gamma_{1i} & \Gamma_{4i} & \Gamma_{5i} & \tilde{\mathcal{C}}_i^T & \mathbb{I} & 0 \\ * & * & -\Gamma_{2is} & \Gamma_{6i} & \tilde{\mathcal{C}}_{di}^T & 0 & 0 \\ * & * & * & -\Gamma_{3i} & \tilde{\mathcal{D}}_i^T & 0 & 0 \\ * & * & * & * & -\mathbb{I} & 0 & 0 \\ * & * & * & * & * & -R_i & 0 \\ * & * & * & * & * & * & -\epsilon_i\mathbb{I} \end{bmatrix} < 0,$$

$$\begin{aligned} \forall (i,j,s) &\in \mathcal{I}^3, \\ P_i S_i &= \mathbb{I}, \\ Q_i R_i &= \mathbb{I}. \end{aligned} \tag{23}$$

where

$$\begin{aligned} \Gamma_{1i} &= P_i - \epsilon_i \tilde{H}_{1i}^T \tilde{H}_{1i} \\ \Gamma_{2is} &= Q_s - \epsilon_i \tilde{H}_{2i}^T \tilde{H}_{2i} \\ \Gamma_{3i} &= \gamma^2\mathbb{I} - \epsilon_i \tilde{H}_{3i}^T \tilde{H}_{3i} \\ \Gamma_{4i} &= \epsilon_i \tilde{H}_{1i}^T \tilde{H}_{2i} \\ \Gamma_{5i} &= \epsilon_i \tilde{H}_{1i}^T \tilde{H}_{3i} \\ \Gamma_{6i} &= \epsilon_i \tilde{H}_{2i}^T \tilde{H}_{3i} \end{aligned} \tag{24}$$

Proof: Using the change of variables of the uncertain terms of the augmented system (11), one can rewrite matrix Σ as follows:

$$\Sigma = \Sigma_0 + \bar{L}_i^T \tilde{W}(k)\bar{H}_i + \bar{H}_i^T \tilde{W}(k)\bar{L}_i \tag{25}$$

with

$$\begin{aligned} \Sigma_0 &= \begin{bmatrix} -P_j^{-1} & \tilde{A}_i & \tilde{A}_{di} & \tilde{B}_i & 0 & 0 \\ * & -P_i & 0 & 0 & \tilde{\mathcal{C}}_i^T & \mathbb{I} \\ * & * & -Q_s & 0 & \tilde{\mathcal{C}}_{di}^T & 0 \\ * & * & * & -\gamma^2\mathbb{I} & \tilde{\mathcal{D}}_i^T & 0 \\ * & * & * & * & -\mathbb{I} & 0 \\ * & * & * & * & * & -Q_i^{-1} \end{bmatrix}, \\ \bar{L}_i &= \begin{bmatrix} \tilde{L}_i & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\ \bar{H}_i &= \begin{bmatrix} 0 & \tilde{H}_{1i} & \tilde{H}_{2i} & \tilde{H}_{3i} & 0 & 0 \end{bmatrix}. \end{aligned} \tag{26}$$

Applying Lemma 3.1 for each subsystem and Schur complement, the Inequality 23 follows readily. In addition, it was shown in [6] that using the separation Lemma 3.1 with ϵ_i instead of ϵ leads to less conservative conditions. $\square$

The following result is obtained by working out Inequality (23) to become an LMI by using a cone complementarity technique introduced by [15] and used in many works as [18].

Theorem 4.1: For a given scalar $\gamma > 0$, system (7) under arbitrary switching is asymptotically stable when $w_k = 0$ and under zero-initial conditions, guarantees the performance index (15) for all $w_k \in l_2[0,\infty)$, if there exists positive definite symmetric matrices P_i and Q_i, S_i and R_i and positive scalars ϵ_i, $i \in \mathcal{I}$, such that:

$$\begin{bmatrix} -S_j & \tilde{A}_i & \tilde{A}_{di} & \tilde{B}_i & 0 & 0 & \tilde{L}_i^T \\ * & -\Gamma_{1i} & \Gamma_{4i} & \Gamma_{5i} & \tilde{C}_i^T & \mathbb{I} & 0 \\ * & * & -\Gamma_{2is} & \Gamma_{6i} & \tilde{C}_{di}^T & 0 & 0 \\ * & * & * & -\Gamma_{3i} & \tilde{D}_i^T & 0 & 0 \\ * & * & * & * & -\mathbb{I} & 0 & 0 \\ * & * & * & * & * & -R_i & 0 \\ * & * & * & * & * & * & -\epsilon_i\mathbb{I} \end{bmatrix} < 0,$$

$$\forall (i,j,s) \in \mathcal{I}^3, \tag{27}$$

$$\begin{bmatrix} S_i & \mathbb{I} \\ \mathbb{I} & P_i \end{bmatrix} \geq 0 \tag{28}$$

$$\begin{bmatrix} R_i & \mathbb{I} \\ \mathbb{I} & Q_i \end{bmatrix} \geq 0, \tag{29}$$

$$\forall (i,j,s) \in \mathcal{I}^3,$$

where matrices Γ_* are given by (24). **Proof:** The result follows readily by using Schur complement to rewrite equivalently inequalities (21)-(22) as follows: $S_iP_i \geq \mathbb{I}$ is equivalent to $S_i - P_i^{-1} \geq 0$ which can be equivalently written as (28). ☐

This result presents the advantage of computing directly the matrices K_i and V_i. Nevertheless, an optimization problem must be solved to achieve $P_iS_i \simeq \mathbb{I}$ by minimizing iteratively the trace of matrix P_iS_i using an algorithm presented in [15]. This heuristic is based on a linear approximation of $Tr(P_iS_i)$ by $Tr(P_0S_i+S_0P_i)$ where P_0 and S_0 are particular solutions of the LMI constraints (27), (28) and (29).

$$(Pb.1) : \begin{cases} \min_{(S_j,P_i,Q_i,R_i)} \theta \\ s.t. \;\; (27), (28), (29) \\ i,j,s = 1,\ldots,N \end{cases}$$

where θ is a function with multiple objective $\theta = \beta Trace(P_iS_i) + (1-\beta)Trace(Q_iR_i), 0 < \beta < 1$.

In order to avoid this optimization, another technique can be presented by the following result.

Corollary 4.1: For a given scalar $\gamma > 0$, system (7) under arbitrary switching is asymptotically stable when $w_k = 0$ and under zero-initial conditions, guarantees the performance index (15) for all $w_k \in l_2[0,\infty)$, if there exists positive definite symmetric matrices P_i and Q_i, S_i and R_i, matrices X_i, Z_i and positive scalars ϵ_i, $i \in \mathcal{I}$, such that:

$$\begin{bmatrix} -S_j & \tilde{A}_i & \tilde{A}_{di} & \tilde{B}_i & 0 & 0 & \tilde{L}_i^T \\ * & -\Gamma_{1i} & \Gamma_{4i} & \Gamma_{5i} & \tilde{C}_i^T & \mathbb{I} & 0 \\ * & * & -\Gamma_{2is} & \Gamma_{6i} & \tilde{C}_{di}^T & 0 & 0 \\ * & * & * & -\Gamma_{3i} & \tilde{D}_i^T & 0 & 0 \\ * & * & * & * & -\mathbb{I} & 0 & 0 \\ * & * & * & * & * & -R_i & 0 \\ * & * & * & * & * & * & -\epsilon_i\mathbb{I} \end{bmatrix} < 0, \tag{30}$$

$$\begin{bmatrix} -X_i - X_i^T + P_i & -X_i - X_i^T + \mathbb{I} \\ * & -X_i - X_i^T + S_i \end{bmatrix} < 0 \tag{31}$$

$$\begin{bmatrix} -Z_i - Z_i^T + Q_i & -Z_i - Z_i^T + \mathbb{I} \\ * & -Z_i - Z_i^T + R_i \end{bmatrix} < 0, \tag{32}$$

$$\forall (i,j,s) \in \mathcal{I}^3,$$

where matrices Γ_* are given by (24).

Proof: The idea is to realize inequalities $-P_i^{-1} < -S_i$ and $-Q_i^{-1} < -R_i$. The first inequality is equivalent by Schur complement to:

$$\begin{bmatrix} -P_i^{-1} & -\mathbb{I} \\ * & -S_i^{-1} \end{bmatrix} < 0 \tag{33}$$

By pre multiplying this inequality by $diag\{X_i^T, X_i^T\}$ and post multiplying by $diag\{X_i, X_i\}$, where X_i is any non singular matrix, it follows equivalently:

$$\begin{bmatrix} -X_i^TP_i^{-1}X_i & -X_i^TX_i \\ * & -X_i^TS_i^{-1}X_i \end{bmatrix} < 0. \tag{34}$$

The following development

$$(X_i^T - P_i)P_i^{-1}(X_i - P_i) = X_i^TP_i^{-1}X_i - X_i - X_i^T + P_i > 0, \tag{35}$$

leads to

$$-X_i^TP_i^{-1}X_i < -X_i - X_i^T + P_i \tag{36}$$

$$-X_i^TX_i < -X_i - X_i^T + \mathbb{I}. \tag{37}$$

Replacing (36)-(37) one obtains (31). A similar development is used to obtain (32). Inequalities $-P_i^{-1} < -S_i$ and $-Q_i^{-1} < -R_i$ are then used to majorate Inequality (18) to obtain LMIs (30)-(32). ☐

Example 4.1: Consider the following numerical example similar to the one studied in [23] with modified default matrices. The used matrices are divided by 10 to be closed to the perturbation matrices. Besides, the perturbation d_k is taken 100 times the one of [23].

Mode 1:

$$\begin{aligned} A_1 &= \begin{bmatrix} 0.2 & -0.1 \\ 0 & 0.4 \end{bmatrix}, A_{d1} = \begin{bmatrix} 0.1 & 0 \\ 0.1 & 0.3 \end{bmatrix}, \\ B_1 &= \begin{bmatrix} 0.1 \\ 0.3 \end{bmatrix}, E_1 = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}, \\ F_1 &= \begin{bmatrix} 0.13 \\ 0.16 \end{bmatrix}, C_1 = \begin{bmatrix} 0.1 & 0 \end{bmatrix}, C_{d1} = \begin{bmatrix} 0 & 0.1 \end{bmatrix}, \\ L_1 &= \begin{bmatrix} 0.01 \\ 0.1 \end{bmatrix}, H_{11} = \begin{bmatrix} 0.1 & 0.01 \end{bmatrix}, \\ H_{21} &= \begin{bmatrix} 0.01 & 0.01 \end{bmatrix}, D_1 = 1, N_1 = 1.1, M_1 = 1.4, \\ H_{31} &= 0.01. \end{aligned}$$

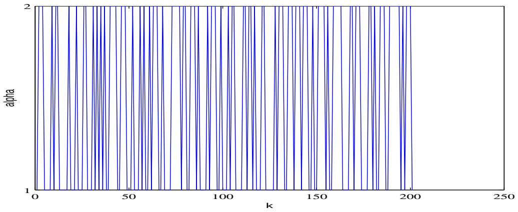

Fig. 1. The evolution of the switching rule α .

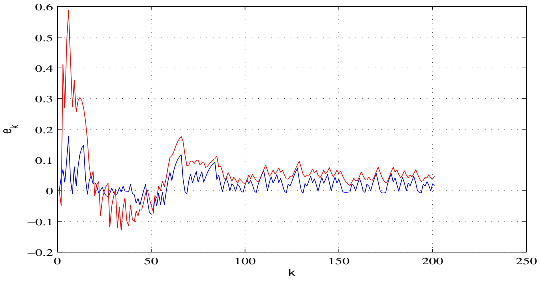

Fig. 2. The evolution of the observer error e_k.

Mode 2:

$$\begin{aligned}
A_2 &= \begin{bmatrix} 0.4 & 0.1 \\ 0.1 & 0.3 \end{bmatrix}, A_{d2} = \begin{bmatrix} 0.1 & 0 \\ 0.2 & 0.1 \end{bmatrix}, \\
B_2 &= \begin{bmatrix} 0.3 \\ 0.2 \end{bmatrix}, E_2 = \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix}, \\
F_2 &= \begin{bmatrix} 0.15 \\ 0.12 \end{bmatrix}, C_2 = \begin{bmatrix} 0 & 0.1 \end{bmatrix}, C_{d2} = \begin{bmatrix} 0.1 & 0 \end{bmatrix}, \\
L_2 &= \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, H_{12} = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}, \\
H_{22} &= \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}, D_2 = 1.1, N_2 = 1.2, M_2 = 1.5, \\
H_{32} &= 0.1, W(k) = \frac{k}{k+1}.
\end{aligned}$$

Using Scilab 5, the LMI (30)-(32) are feasible for $\gamma = 1$. It is worth noting that each time LMIs (30)-(32) are feasible, the corresponding ones with common matrices $P_i = P$ and $Q_i = Q$ are also feasible for this example. The obtained observer and residual gains are given by:

$$K_1 = \begin{bmatrix} 0.1352565 \\ 0.1361103 \end{bmatrix}, K_2 = \begin{bmatrix} 0.1109542 \\ 0.0908712 \end{bmatrix}, \\ V_1 = 0.1170218, V_2 = 0.2774309.$$

The input is a unit step while the fault is generated between $k = 40$ and $k = 60$ with a unit magnitude. The bounded unknown input is $d_k = exp(-0.04k).cos(0.03\pi k)$. Note that here we take the same unknown input, which is a decaying sinusoid, as in [23], however, a random signal would have been a better choice.

Figure 1 presents the switching sequences $\alpha(k)$, while Figure 2 plots the evolution of the observer error e_k. The evolution of the residual r_k is given in Figure 3. Figure 4 presents the evaluation function J_k with fault and free fault together with the threshold function J_0 in dashed line. Figure 5 plots the second evaluation function J_r with $T = 10$. One can notice that the index function J_r with fault case begin its ”take-off” at the same time that the fault occurs, that is at $k = 40$.

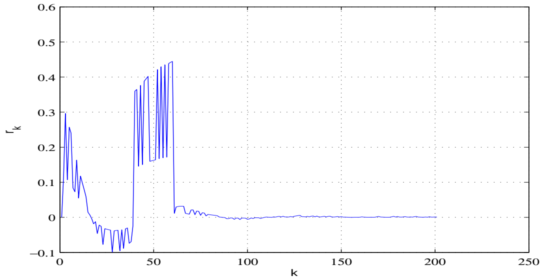

Fig. 3. The evolution of the residual r_k .

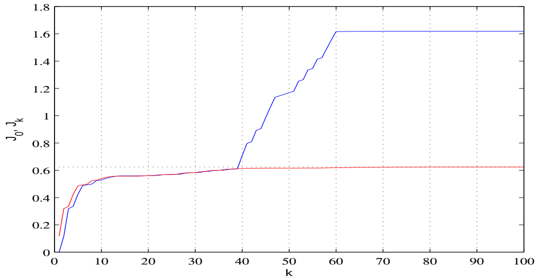

Fig. 4. The evolution of the evaluation function J_k with free fault and fault case.

V. CONCLUSION

In this paper, sufficient conditions for building an observer for discrete-time uncertain switching systems with delay are presented. The obtained condition is worked out in a new way, based on the cone complementarity technique, to obtain new LMIs. Besides the introduction of slack variables X_i, Z_i to realize $-P_i^{-1} < -S_i$ and $-Q_i^{-1} < -R_i$, the residual signal is designed with multiple matrices V_i. The observer is then used in fault detection. A numerical example is

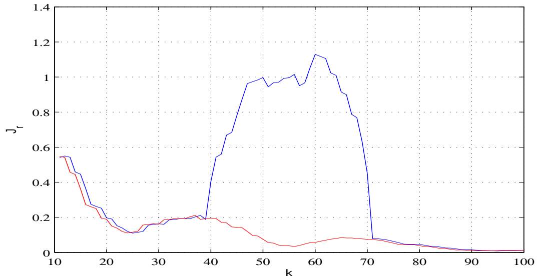

Fig. 5. The evolution of the second evaluation function J_r.

studied to illustrate the applicability of the obtained results. In particular, the proposed observer works with success even in presence of an input entry, a bounded unknown input and a fault of the same sizes. Filter based approaches may not success in this case. This technique allows less conservative results than those presented in [8].

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