Proceedings of the 2nd International Conference on Systems and Control, Marrakech, Morocco, June 20-22, 2012

Stabilization of uncertain discrete-time switching systems

Ouahiba Benmessaouda and Abdellah Benzaouia

***Abstract*— This paper presents sufficient conditions for the stabilization of constrained switched discrete- time linear systems with polytopic uncertainties. A strategy of conception of switched laws from the solution of Lyapunov-Metzler inequalities is developed. The state variables are supposed to be not accessible so that the feedback strategy depend on given output variables. A numerical example is used to illustrate the proposed techniques.**

keywords: Switched systems, Lyapunov functions, Lyapunov-Metzler inequalities, uncertain parameters.

I. INTRODUCTION

Switched systems are a class of hybrid systems encountered in many practical situations which involve switching between several subsystems depending on various factors. Generally, a switching system consists of a family of continuous-time subsystems and a rule that supervises the switching between them. This class of systems has numerous applications in the control of mechanical systems, the automotive industry, aircraft and air traffic control, switching power converters and many other fields. Two main problems are widely studied in the literature according to the classification given in [6]: The first one, which is the one solved in this paper, looks for testable conditions that guarantee the asymptotic stability of a switching system under arbitrary switching rules, while the second is to determine a switching sequence that renders the switched system asymptotically stable (see [8], [14] and references therein).
A main problem which is always inherent to all dynamical systems is the presence of actuator saturations. Even for linear systems, this problem has been an active area of research for many years. Besides approaches using anti-windup techniques [15] and model predictive controls [9], two main approaches have been developed in the literature: The first is the so-called positive invariance approach which is based on the design of controllers which work inside a region of linear behavior where saturations do not occur (see [5] and the references therein). This approach has already being applied to a class of hybrid systems involving jumping parameters [7]. It has also been used to design controllers for switching systems with constrained control under complete modelling taking into account reset functions at each switch and different system's dimension [4]. The second approach, however allows saturations to take effect while guaranteeing asymptotic stability (see [3], [12] and references therein). The main challenge in these two approaches is to obtain large domains of initial states which ensures asymptotic stability for the system despite the presence of saturations.
This work extends the results obtained [16], [17] and [18] for switched systems without uncertainties nor state constraints.
This paper is organized as follows: Section II deals with the problem presentation and presents some preliminary results. The main results of this paper are given in Section III together with illustrative examples.

Benmessaouda and Benzaouia are with LAEPT-URAC 28, University Cadi Ayyad, Faculty of Science Semlalia, P.B. 2390, Marrakech, Morocco, ouahiba04@gmail.com, benzaouia@uca.ma

II. STATE FEEDBACK

A. Preliminary results

Let us consider the uncertain switching discrete-time linear system described by:

$$x_{k+1} = A_\alpha(q_\alpha(k))x_k \tag{1}$$

where $x_k \in \mathbb{R}^n$, is the state, function $\alpha(k) : \mathbb{N} \longmapsto \mathcal{I}$ is a switching rule taking its values $\alpha(k) = i$ in the finite set $\mathcal{I} = \{1, ..., N\}$ and $q_\alpha(k) \in \Gamma_\alpha \subset \mathbb{R}^{d_\alpha}$ are the bounded uncertainties that affect the system parameters in such a way that

$$A_\alpha(q_\alpha(k)) = A_\alpha + \sum_{h=1}^{d_\alpha} A_{\alpha h} q_{\alpha h}(k) \tag{2}$$

where matrix A_α represent the nominal matrix and $q_{\alpha h}(k)$ the hth component of vector $q_\alpha(k)$:

$$q_\alpha(k) = [q_{\alpha 1}(k) \;\; q_{\alpha 2}(k) \;\; ... \;\; q_{\alpha h}(k) \;\; ... \;\; q_{\alpha d_\alpha}(k)]^T.$$

Let convex sets Γ_i have μ_i vertices $\nu_{i\kappa}, \kappa = 1, \ldots, \mu_i$ so that for every $q_i \in \Gamma_i$, one can write $q_i = \sum_{\kappa=1}^{\mu_i} \beta_{i\kappa}\nu_{i\kappa}$ with $\sum_{\kappa=1}^{\mu_i} \beta_{i\kappa} = 1, 0 \leq \beta_{i\kappa} \leq 1$. The consequence of this, is that each matrix $A_i(q_i(k))$ can be expressed as a convex combination of the corresponding vertices of the compact set Γ_i as follows:

$$\begin{aligned} A_i(q_i) : &= A_i + \sum_{\kappa=1}^{\mu_i} \beta_{i\kappa} A_i(\nu_{i\kappa}) = \sum_{\kappa=1}^{\mu_i} \beta_{i\kappa} A_{i\kappa}, \\ A_i(\nu_{i\kappa}) &= \sum_{h=1}^{d_i} A_{ih}\nu_{i\kappa h}, \\ A_{i\kappa} &= A_i + A_i(\nu_{i\kappa}), \\ \sum_{\kappa=1}^{\mu_i} \beta_{i\kappa} &= 1, 0 \leq \beta_{i\kappa} \leq 1. \end{aligned} \tag{3}$$

where A_i represents the nominal matrix. Note that the system without uncertainties can be obtained as a particular case of this representation by letting the vertices $\nu_{i\kappa} = 0, \forall i, \forall \kappa$. Consider the system

$$x_{k+1} = A_\sigma(k)(k)x_k \tag{4}$$

with

$$\sigma(k) \in \{1, ..., N\} \tag{5}$$

and

$$A_i(k) = A_i(q(k)) \in \Omega = Co\{A_{i1}, A_{i2}...A_{ip}\} \tag{6}$$

Matrices $A_i(k)$ are not necessarily stable. Matrices A_{ij} represent the vertices of the uncertainty polytope, $j = 1.......p_i$ for the mode i. The goal is to determine the function $u(.) : \mathbb{R}^n \to \{1, ..., N\}$, such that

$$\sigma(k) = u(x_k) \tag{7}$$

makes the equilibrium point $x = 0$ of (4) asymptotically stable.
In this section, some preliminary results are recalled for their use in this work. define the simplex

$$\Lambda := \{\lambda \in \mathbb{R}^N : \sum_{i=1}^{N} \lambda_i = 1, \lambda_i \geq 0\} \tag{8}$$

with the set of positive definite matrices $\{P_1..., P_N\}$,we define the following non quadratic Lyapunov function

$$v(x) = \min_{i=1,...,N} x^T P_i x = \min_{\lambda\in\Lambda}(\sum_{i=1}^{N} \lambda_i x^T P_i x) \tag{9}$$

Where x^T stands for the transpose of x
The Metzler matrices denoted by M are matrices in which all the off-diagonal components are nonnegative. In this paper, we use a particular class of Metzler matrices constituted by all matrices $\Pi \in \mathbb{R}^{N\times N}$ with element π_{ij} such that

$$\pi_{ij} \in [0, 1], \quad \sum_{i=1}^{N} \pi_{ij} = 1 \quad \forall j \tag{10}$$

Theorem 2.1: Assume that there exist $\Pi \in M$ and a set of positive definite matrices $\{P_1, ..., P_N\}$ satisfying the Lyapunov-Metzler inequalities

$$A_{ik}^T(\sum_{j=1}^{N} \pi_{ji} P_j)A_{ik} - P_i < 0 \tag{11}$$

$$i = 1, ..., N; \quad k = 1, ..., p_i$$

the state switching control with

$$u(x_k) = arg \min_{i=1,...,N} x_k^T P_i x_k \tag{12}$$

makes the equilibrium solution of (4) asymptotically stable
The Lyapunov-Metzler inequalities (11) are non linear. The following result gives a simpler stability condition that can be expressed by means of LMIs.

Theorem 2.2: [18] let $Q_i \geq 0$ be given,assume that there exist P_i and a scalar $0 \leq \gamma \leq 1$ satisfying the Lyapunov Metzler inequalities

$$A_{ik}^T(\gamma P_i + (1-\gamma)P_j)A_{ik} - P_i + Q_i < 0, \tag{13}$$

$j \neq i = 1, ..., N$; $k = 1, ..., p_i$
the state switching control with

$$u(x_k) = arg \min_{i=1,...,N} x_k^T P_i x_k \tag{14}$$

makes the equilibrium solution of (4) globally asymptotically stable

B. Reformulation of Lyapunov function

In the previous section, we are interested in the Lyapunov function:

$$V(x) = \min_{i=1,...,N} x^T P_i x = \min_{\lambda\in\Lambda} x^T P(\gamma)x \tag{15}$$

$$P(\lambda) = \sum_{j=1}^{N} \lambda_j P_i.$$

We now consider the matrices $Q_j = P_j^{-1}$. It was explained in [19] that:

$$Q(\lambda) := P^{-1}(\lambda) \tag{16}$$

and $Q(\lambda)$, $P(\lambda) > 0$ for all $\lambda \in \Lambda$. We can then redefine the Lyapunov function as:

$$V(x) = \min_{\lambda\in\Lambda} x^T P(\gamma)x \tag{17}$$

$$= \min_{\lambda\in\Lambda} x^T \left(\sum_{j=1}^{N} \lambda_j Q_i\right)^{-1} x$$

Theorem 2.3: If there exist $\Pi \in \mathcal{M}$ and a set of positive definite matrices $\{X_1, ..., X_N\}$ satisfying the Lyapunov-Metzler inequalities

$$\begin{bmatrix} X_i & X_i A_{i\kappa}^T \\ * & \gamma X_i + (1-\gamma)X_j \end{bmatrix} > 0, \tag{18}$$

$$j \neq i = 1, ..., N, \quad \kappa = 1, ..., \mu_i$$

then the state switching control with

$$s(x_k) = arg \min_{i=1,...,N} x_k^T P_i x_k \tag{19}$$

$$with \;\; P_i = X_i^{-1};$$

makes the equilibrium solution of (4) globally asymptotically stable.

Proof 1: Assume that at an arbitrary time k, $\sigma(k) = s(x_k) = i$ pour $i \in \{1, ..., N\}$ alors

$$V(x_k) = x_k^T P_i x_k$$

and

$$V(x_{k+1}) = \min_{i=1,...,N} x_k^T A_i^T P_j A_i x_k$$
$$= \min_{\lambda \in \Lambda} x_{k+1}^T \left(\sum_{j=1}^{N} \lambda_j X_j \right)^{-1} x_{k+1}$$
$$= \min_{\lambda \in \Lambda} x_k^T A_i^T \left(\sum_{j=1}^{N} \lambda_j X_j \right)^{-1} A_i x_k$$
$$\leq x_k^T A_i^T \left(\sum_{j=1}^{N} \pi_{ji} X_j \right)^{-1} A_i x(k)$$

The difference in the Lyapunov function then checks:

$$\Delta V \leq x_k^T [A_i^T \left(\sum_{j=1}^{N} \pi_{ji} X_j \right)^{-1} A_i - P_i] x_k \quad (20)$$

We suppose now that the condition (18) is verified, multiply by $\beta_{i\kappa}(k)$ and sum, as $\sum_{\kappa=1}^{\mu_i} \beta_{i\kappa}(k) = 1$, one gets:

$$\begin{bmatrix} X_i & X_i A_i^T \\ * & \gamma X_i + (1-\gamma) X_j \end{bmatrix} > 0, \quad (21)$$

We multiply this inequality by π_{ji}, we sum for all $j \neq i = 1, ..., N$ and we multiply finally by $(1-\gamma)^{-1}$, since $(1-\gamma)^{-1} \sum_{j \neq i=1}^{N} \pi_{ji} = 1$, we obtain:

$$\begin{bmatrix} X_i & X_i A_i^T \\ * & \gamma X_i + \sum_{j \neq i=1}^{N} \pi_{ji} X_j \end{bmatrix} = \begin{bmatrix} X_i & X_i A_i^T \\ * & \sum_{j=1}^{N} \pi_{ji} X_j \end{bmatrix} > 0$$

It is equivalent by Schur complement to

$$X_i - X_i A_i^T \left(\sum_{j=1}^{N} \pi_{ji} X_j \right)^{-1} A_i^T X_i > 0 \quad (22)$$

By pre and post-multiplying the latter by P_i , it follows:

$$A_i^T \left(\sum_{j=1}^{N} \pi_{ji} X_j \right)^{-1} A_i - P_i < 0 \quad (23)$$

The difference of Lyapunov function (20) is then negative along the trajectory of the system controlled by the switching law (19).

III. OUTPUT FEEDBACK

Since the access to the state is not always possible, looking for an output feedback is important. In this section, we present, first time, important results from [17].
Given the measure

$$y_k = C_{\sigma(k)} x_k \quad (24)$$

The main result is to determine a switching law of the form:

$$\sigma(k) = s(y(.)) \quad (25)$$

such that the system (4) is asymptotically stable. $s(.)$ is a fonction of $y(.)$ in the sense that $y(k)$ is an input of a filter that manages change in the index switching. We introduce the following full rank switching filter:

$$\hat{x}(k+1) = \hat{A}_{\sigma(k)} \hat{x}(k) + \hat{B}\sigma(k) y(k), \quad \hat{x}(0) = \hat{x}_0 \quad (26)$$

with $(\hat{A}_i, \hat{B}_i)$, $i = 1, 2, ..., N$ are the matrices to be determined. Put (4), (24) and (26) together, we get:

$$\tilde{x}(k+1) = \tilde{A}_{\sigma(k)} \tilde{x}(k), \quad (27)$$

with $\tilde{x}^T = [x^T \ \hat{x}^T]$ and

$$\tilde{A}_i = \begin{bmatrix} A_i & 0 \\ \hat{B}_i C_i & \hat{A}_i \end{bmatrix} \quad (28)$$

The solution to this problem is to determine the matrices $\hat{A}_i$ and $\hat{B}_i$ and a switching law such as the expanded system is globally asymptotically stable. To do this, only the switching laws depending on $\hat{x}(.)$ are allowed. The structure of the Lyapunov function is designated so that the switching law depends only on $\hat{x}(.)$. We then consider:

$$\tilde{P}_i = \begin{bmatrix} X & V \\ V^T & \hat{X}_i \end{bmatrix}, \quad detV \neq 0 \quad (29)$$

Note then that:

$$arg \min_i \tilde{x}(k)^T \tilde{P}_i \tilde{x}(k) = arg \min_i \hat{x}(k)^T \hat{X}_i \hat{x}(k) \quad (30)$$

Therefore, the solution is to find a stabilizing switching law of the form:

$$s(y(.)) = arg \min_{i=1,...,N} \hat{x}(k)^T \hat{X}_i \hat{x}(k) \quad (31)$$

The following theorem is proposed by [17] to solve this problem.

Theorem 3.1: Suppose there is a Metzler matrix Π, a positive definite matrix X, L_i matrices and positive matrices Z_i such as the following matrix inequalities are satisfied for all $i = 1, 2, ..., N$:

$$Z_i > A_i^T \left(\sum_{j=1}^{N} \pi_{ji} Z_j \right) A_i \quad (32)$$

and

$$\begin{bmatrix} X - A_i^T X A_i - A_i^T L_i C_i - C_i^T L_i A_i & C_i^T L_i^T \\ * & X \end{bmatrix} > 0 \quad (33)$$

then, the stabilizing law

$$s(y(.)) = arg \min_{i=1,...,N} \hat{x}(k)^T Z_i \hat{x}(k) \quad (34)$$

stabilizes asymptotically the system with a observer form for the filter as follows:

$$\hat{B}_i = -X^{-1}L_i \quad (35)$$

$$\hat{A}_i = A_i - \hat{B}_i C_i \quad (36)$$

Remark 3.1:

- There is no difficulty to get the versions of this theorem, associated to the modified Lyapunov-Metzler inequalities appearing in theorem (2.2), the results follow the same pattern.
- The bloc V of matrix
$$\tilde{P}_i = \begin{bmatrix} X & V \\ V^T & \hat{X}_i \end{bmatrix} \quad (37)$$
is taken equal $-X$, the solution is so conservatif.

As in the previous section, to relax the results, we use the Lyapunov function

$$V(x) = \min_{\lambda\in\Lambda} \tilde{x}^T \tilde{P}(\gamma)\tilde{x} = \min_{\lambda\in\Lambda} \tilde{x}^T \left(\sum_{j=1}^{N} \lambda_j \tilde{Q}_i\right)^{-1} \tilde{x} \quad (38)$$

We obtain the following result:

Theorem 3.2: If there exist positive definite matrices X, X_i et Q_i, matrices V and $\hat{B}_i$, a scalaire $0 \le \gamma \le 1$ solution of following LMI's:

$$\begin{bmatrix} \tilde{P}_i & \tilde{A}_i^T \\ \tilde{A}_i & \gamma\tilde{Q}_i + (1-\gamma)\tilde{Q}_j \end{bmatrix} > 0, \quad (39)$$

$$\begin{bmatrix} \tilde{P}_i & \mathbb{I} \\ * & \tilde{Q}_i \end{bmatrix} \ge 0, \quad (40)$$

$\forall (i,j) \in \mathcal{I}^2$, such that $trace(\tilde{P}_i\tilde{Q}_i) = n$, with,

$$\tilde{P}_i = \begin{bmatrix} X & V \\ V^T & X_i \end{bmatrix}, \quad (41)$$

$$\tilde{A}_i = \begin{bmatrix} A_i & 0 \\ \hat{B}_i C_i & A_i - \hat{B}_i C_i \end{bmatrix}, \quad (42)$$

then the switching

$$s(y(.)) = arg \min_{i=1,\ldots,N} \hat{x}(k)^T X_i \hat{x}(k) \quad (43)$$

stabilizes asymptotically the system with a form of observer for the filter as follows:

$$\hat{A}_i = A_i - \hat{B}_i C_i$$

Proof 2: Assume that at an arbitrary time k

$$\sigma(k) = s(x_k) = i, \quad i \in \{1, ..., N\}$$

then

$$V(x_k) = x_k^T \tilde{P}_i x_k$$

and

$$V(x_{k+1}) = \min_{i=1,\ldots,N} x_{k+1}^T \tilde{P}(\lambda) x_{k+1}$$

$$\begin{aligned} &= \min_{\lambda\in\Lambda} x_{k+1}^T \left(\sum_{j=1}^{N} \lambda_j \tilde{Q}_j\right)^{-1} x_{k+1} \\ &= \min_{\lambda\in\Lambda} x_k^T \tilde{A}_i^T \left(\sum_{j=1}^{N} \lambda_j \tilde{Q}_j\right)^{-1} \tilde{A}_i x_k \\ &\le x_k^T \tilde{A}_i^T \left(\sum_{j=1}^{N} \pi_{ji} \tilde{Q}_j\right)^{-1} \tilde{A}_i x_k \end{aligned}$$

The difference in the Lyapunov function then checks:

$$\Delta V(x_k) \le x_k^T \left[\tilde{A}_i^T \left(\sum_{j=1}^{N} \pi_{ji} \tilde{Q}_j\right)^{-1} \tilde{A}_i - \tilde{P}_i \right] x_k$$

We now assume that condition (39) holds, we multiply by Π_{ji}, and sum for all $j \ne i = 1, ..., N$ and finally, multiply by $(1-\gamma)^{-1}$, since $(1-\gamma)^{-1}\sum_{j\ne i=1}^{N} \Pi_{ji} = 1$ we obtain:

$$\begin{bmatrix} \tilde{P}_i & \tilde{A}_i^T \\ * & \gamma\tilde{Q}_i + \sum_{j\ne i=1}^{N} \pi_{ji}\tilde{Q}_j \end{bmatrix} = \begin{bmatrix} \tilde{P}_i & \tilde{A}_i^T \\ * & \Lambda_{ji} \end{bmatrix} > 0,$$

where $\Lambda_{ji} = \sum_{j=1}^{N} \pi_{ji}\tilde{Q}_j$. We can write this inequality, by Schur complement, such as:

$$\tilde{P}_i - \tilde{A}_i^T \left(\sum_{j=1}^{N} \pi_{ji} \tilde{Q}_j\right)^{-1} \tilde{A}_i > 0 \quad (44)$$

This can be used to bound the rate of increase of the Lyapunov function candidate. The inequalities $\tilde{P}_i\tilde{Q}_i = \mathbb{I}$, are forced by minimizing $trace(\tilde{P}_i\tilde{Q}_i)$ with additional constraints $\begin{bmatrix} \tilde{P}_i & \mathbb{I} \\ * & \tilde{Q}_i \end{bmatrix} \ge 0$.

Example 3.1: We consider the example of discrete switched system with two modes:

$$A_1 = \begin{bmatrix} 1.0076 & 0.0662 \\ 0.1323 & 0.4122 \end{bmatrix}; \; A_2 = \begin{bmatrix} 0.9893 & 0.1103 \\ -0.2207 & 1.2100 \end{bmatrix};$$

$$C_1 = \begin{bmatrix} 1 & 1 \end{bmatrix}; \qquad C_2 = \begin{bmatrix} 1 & 1 \end{bmatrix};$$

We apply the theorem (3.1), the trajectory of the system obtained in this case is shown in figure (1). The built switching signal $s(x(k))$ is shown in figure (2). The results of theorem (3.2) are presented in the following figure. The corresponding built switching signal $s(x(k))$ is shown in figure (4).

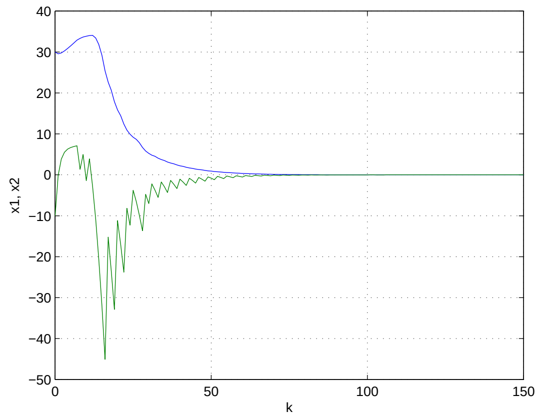

Fig. 1. System trajectry .

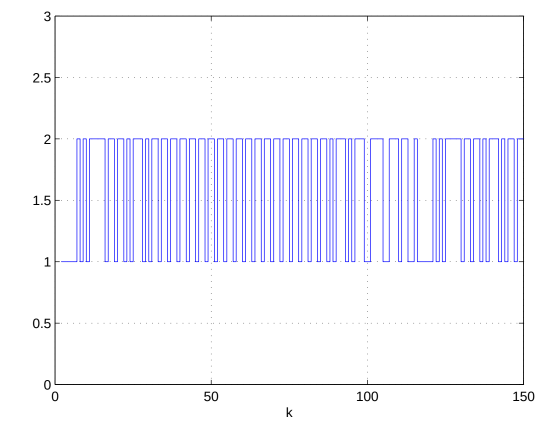

Fig. 2. Switching signal.

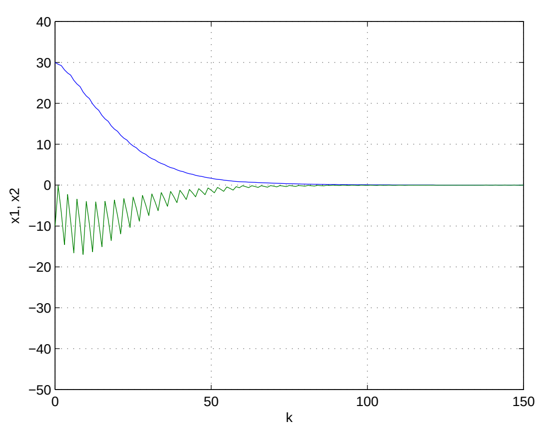

Fig. 3. System trajectry.

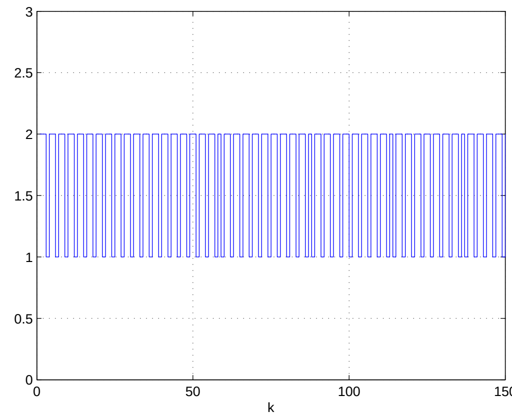

Fig. 4. Switching signal.

REFERENCES

[1] A. Benzaouia, L. Saydy and O. Akhrif, ”Stability and control synthesis of switched systems subject to actuator saturation”. *American Control Conference, June 30- July 2*, Boston, 2004.

[2] A. Benzaouia, O. Akhrif and L. Saydy, ”Stability and control synthesis of switched systems subject to actuator saturation by output feedback”. 45^{th} *Conference on Decision and Control, December 13-15*, San Diego, 2006.

[3] A. Benzaouia, F. Tadeo and F. Mesquine, ”The Regulator Problem for Linear Systems with Saturations on the Control and its Increments or Rate: An LMI approach”. *IEEE Transactions on Circuit and Systems Part I*, Vol. 53, $N°$. 12, pp. 2681-2691, 2006.

[4] A. Benzaouia, E. DeSantis, P. Caravani and N. Daraoui,” Constrained Control of Switching Systems: A Positive Invariance Approach, Int. J. of Control, Vol. 80, Issue 9, pp.1379-1387, (2007).

[5] F. Blanchini, ”Set invariance in control - a survey”, *Automatica*, Vol. 35, $N°$. 11, pp. 1747-1768, 1999.

[6] F. Blanchini and C. Savorgnan, ”Stabilizability of switched linear systems does not imply the existence of convex Lyapunov functions”. 45^{th} *Conference on Decision and Control, December 13-15*, San Diego, pp.119-124, 2006.

[7] E. L. Boukas, A. Benzaouia. ”Stability of discrete-time linear systems with Markovian jumping parameters and constrained Control”. *IEEE Trans. Aut. Control*. Vol. 47, $N°$. 3, pp.516-520, 2002.

[8] M. S. Branicky, ”Multiple Lyapunov functions and other analysis tools for switched and hybrid systems”, IEEE Automat. Contr., Vol. 43, pp. 475-482, 1998.

[9] E. F. Camacho and C.Bordons, ”Model Predictive Control”', Springer-Verlag, London, 2004.

[10] J. Daafouz, P. Riedinger and C. Iung. ”Stability analysis and control synthesis for switched systems: a switched Lyapunov function approach”. *IEEE Trans. Aut. Control*, Vol. 47, $N°$. 11, pp. 1883-1887, 2002.

[11] L. Hetel, J. Daafouz, and C. Iung. ”Stabilization of Arbitrary Switched Linear Systems With Unknown Time-Varying Delays ”. *IEEE Trans. Aut. Control*, Vol. 51, $N°$. 10, pp. 1668-1674, 2006.

[12] T. Hu, Z. Lin and B. M. Chen, ”An analysis and design method for linear systems subject to actuator saturation and disturbance”. *Automatica*, Vol. 38, pp. 351-359, 2002.

[13] T. Hu, Z. Lin, ” Control Systems with Actuator Saturation: Analysis and Design”', BirkhVauser, Boston, 2001.

[14] D. Liberzon, ”Switching in systems and control”. *Springer*, 2003.

[15] E. F. Mulder, M. V. Kothare and and M. Morari, ”Multivariable anti-windup controller synthesis using linear matrix inequalities”. *Automatica*, Vol. 37, No.9, pp.1407-1416, 2001.

[16] Jos C. geromel and P. Colaneri ”stability and stabilization of discrete time switched system”, *International Journal of Control*, Vol 79, pp 719-728, 2006

[17] P. Colaneri and Jos C. geromel "Output-feedback stabilization of discrete time switched system", *American Control Conference* , 2007
[18] O. Benmessaouda and A. Benzaouia and F. Tadeo "Stabilization of Uncertain State Constrained Discrete-Time Switched Systems", *18th IFAC World Congress* , 2011
[19] T. Hu and Z. lin "Composite Quadratique Lyapunov Function For Constrained Control System" , 2003