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Improved Stabilizability Condition for Time Delay Systems with Saturating Control

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Abstract— **This paper is devoted to the stabilization of time-delay systems containing saturating actuators. Improved delay-dependent stabilizability conditions are derived. These conditions are given under LMI formalism. Hence, stabilizing memoryless state feedbacks are easily deduced. An illustrative example is given to present the application of this method to overcome some conservativness that aroses in previous works.**

Index Terms— **Time-delay systems, Delay Dependent stabilizability Condition, saturating control, Linear matrix Inequalities (LMI).**

I. INTRODUCTION

In recent years, many research works have been reported on the presence of limitations in certain variables within given sets. In particular, the problem of constrained and/or saturating system's variables is of continuing interest. These constraints arise from physical or technological limitations and/or linearization approximations of inherently nonlinear systems. Hence, the problem of controling constrained and/or saturating input and/or state for linear dynamical systems attracts a lot of research work. Many approaches can be used to solve such problems, including the positive invariance concept [1], [2], [3] and the references therein; the small and high gain concept [16]; and the l_1 optimization concept [8]. More recently, a new approach dealing with the problem of saturation arises that is more adequate for use with the Linear Matrix Inequalities (LMI) tool has been proposed [5], [12], [13]....

On the other hand, time delays in dynamical systems is well known to be a source of performance degradation and even instability [19]. This issue, i.e. control of time-delay systems, has also attracted the effort of automatic control community and a lot of research work to develop stability and stabilizability conditions for time-delay systems [4], [15], [9] ... .

Combining the two problems in one system is an attempt to approach a real system where both saturating actuators and time delay may be present. Hence, some results on control of linear systems with both time-delay and saturating actuators can be found in the literature. One may cite [7], [11], [23], [20]. These works are mainly based on the measure matrix, complex Lyapunov equations or even Razmukhin's approach. In [22], the problem of stabilizing time-delay systems with saturating control is studied using a Lyapunov-Krasovskii technique. Writing the saturating system as a convex combination of some linear systems is the cornerstone of this approach. The difficulty of applying such results resides in the need for a prior knowledge of a lower bound for the degree of saturation that enables the saturating system to be written as convex combination [22]. Furthermore, the set of initial conditions that ensures closed loop asymptotic stability is computed from one more LMI constraint which may be restrictive. In [6], the writing of the saturating system as a convex combination of some delay systems [12] enables the use of Lyapunov Krasovskii and Lyapunov Razmukhin functionals to derive memoryless stabilizing state feedbacks. An estimate of the attraction domain is also given. Using the same methodolgy for writing the saturating system, but looking for uniform boundedeness and with an additive external perturbation, some conditions are derived in [21] for designing memoryless state feedback. Furthermore, an estimate of a safe set of initial conditions is studied and an optimization problem is given to enlarge such a set.

In this work, we use the same approach as in [6], [21] to write the saturated delay system as a convex combination of some delay systems [12], where the difficulty of the lower bound is overcome. The results obtained in the former are delay independent and may

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be conservative when the bounds of the delay are known. In [17], partitioning of the delay interval is used to overcome some previous conservativness from previous proposals in the literature. Here, an improved delay-dependent criteria [24] is used. This criteria is first extended to controlled delay systems and hence delay-dependent stabilizability conditions are derived for non saturating systems. As a main result, this condition is extended to saturating delay systems. The obtained LMI's seem less conservative as the delay bound information is used.

The paper is organized as follows: Section 2 is reserved for presenting the problem as studied thereafter. Section 3 is devoted to some preliminary results which consist of presenenting an improved delay-dependent criteria together with the convex combination writing of the saturating delay systems. The main results, which consists first of extending the stability criteria to a stabilizability condition and second extended to the case of delay system with saturating input, are given in section 4. Results are derived under LMI formulism to enable the synthesis of stabilizing memoryless state feedbacks. Illustrative examples are presented to show the effectiveness of the approach.

Notations

For $X \in \Re^{n\times n}$ a real valued matrix of dimension (n,n), X^T denotes its transpose and $X = X^T > 0$ means X symmetric positive definite. $'*'$ stands for symmetric parts of symmetric matrices. $\lambda_{max}(X)$, $\lambda_{min}(X)$ note respectively, the maximum and minimum eigenvalues of X. I_n, is the identity matrix of dimension (n,n). For a function

$$\begin{array}{ccc}\Phi(), [-h,0] & \to & \Re^n, \\ t & \to & \Phi(t)\end{array} \quad \|\Phi(t)\|_c = \underset{-h\le t\le 0}{sup} \|\Phi(t)\|$$

where the norm $\|\ \|$ stands for the euclidean norm or the induced matrix norm. Furthermore, we define $\mathcal{C}_{n,h}$ as the Banach space of continuous vector functions mapping $[-h\ \ 0]$ into $\mathbb{R}^n$ with the topology of uniform convergence. $co\{\}$ denotes the convex hull of $\{\}$. $x_t \in \mathcal{C}_{n,\tau}$ denotes the restriction of $x(t)$ to the intervall $[t-\tau,\ \ t]$ translated to $[-\tau,\ \ 0]$.

II. PROBLEM STATEMENT

Let us consider the time-delay saturating control system given by:

$$\left\{\begin{array}{lcl}\dot{x}(t) & = & Ax(t) + A_h x(t-h) + Bsat(u(t)) \\ x(t) & = & \Phi(t) \text{ for } t \in [-h, 0)\end{array}\right. \quad (1)$$

where $x(t)$ is the state of the system, $u(t)$ the control, and h the delay. The function $\Phi(\cdot) \in \mathcal{C}_{n,h}$. A and A_h are known real matrices of appropriate size. The saturation is the standard nonlinearity as :

$$sat(u_i(t)) = sign(u_i(t))\ min(1,\ |u_i(t)|) \quad (2)$$

For the sake of simplicity, we will use the same notation of saturation for scalars and vectors. Assuming that one uses the memoryless state feedback given by:

$$u(t) = Kx(t) \quad (3)$$

the closed loop system becomes

$$\left\{\begin{array}{l}\dot{x}(t) = Ax(t) + A_h x(t-h) + Bsat(Kx(t)) \\ x(t) = \Phi(t) \text{ for } t \in [-h, 0)\end{array}\right. \quad (4)$$

For a given positive definite matrix P, we define the ellipsoid set as:

$$\Omega(P,\ \rho) = \{x \in \mathbb{R}^n / x^T P x < \rho\} \quad (5)$$

The set of admissible controls, where the linear behaviour of the closed loop system is guaranteed, is given by $\pounds(K)$:

$$\pounds(K) = \{x \in \mathbb{R}^n / |k_i x| < 1, k_i \ \ i^{th} \text{ row of } \ K\} \quad (6)$$

Note that here the attraction domain of the origin in this case is given by all initial condition functions ψ such that:

$$S = \{\psi \in \mathcal{C}_{n,h} / x(t,\psi) \longrightarrow 0 \text{ f}or\ \ t \longrightarrow \infty\} \quad (7)$$

The problem we are studying hereafter may be formulated as follows:

Pb : For a given time-delay system represented as in (1), find stabilizing memoryless state feedback K and a set $\mathcal{D} \subset S$ such that the closed loop system (4) is asymptotically stable for all $\phi \in \mathcal{D}$.

III. PRELIMINARIES

In this section, we begin by recalling an improved delay-dependent stability criteria for delay systems presented in [24]. As pointed out by the authors, this criteria is less conservative than some previous works. Hence, it is worth extending such conditions to obtain a stabilizability condition and also to the case of the saturating system. Besides, a result enabling the staurating delay system to be written as a convex combination of a set of 2^m delay systems is presented.

Let us consider the autonomous delay system without saturation given by :

$$\left\{\begin{array}{lcl} \dot{x}(t) & = & Ax(t)+A_hx(t-h) \\ x(t) & = & \Phi(t) \text{ for } t\in[-h,0) \end{array}\right. \tag{8}$$

Theorem 1: [24] The time-delay system (8) is asymptotically stable for any delay h satisfying $0 < h \leq \bar{h}$ if there exist matrices $P > 0$, $Q > 0$, $Z > 0$, R and W such that the following LMI holds true:

$$\begin{bmatrix} A^TP+PA+R+R^T+Q & PA_h-R+W^T & -\bar{h}R & \bar{h}A^TZ \\ * & -Q-W-W^T & -\bar{h}W & \bar{h}A_h^TZ \\ * & * & -\bar{h}Z & 0 \\ * & * & * & -\bar{h}Z \end{bmatrix} < 0. \tag{9}$$

At this level, one wants to recall that the saturating delay system may be written as the convex combination of 2^m time-delay systems. Without loss of generality, the saturation function is standard and the limitation is taken to the unity. In fact, if the saturation term is different from 1, the B matrix of the system may be scaled to satisfy the unity limitation. On the other hand, consider a vector $v \in \Re^m$ such that $|v| < 1$, the saturation of vector $u \in \Re^m$ can be written as a convex combination of two terms as given in the following lemma:

Lemma 2: [13] For all $u \in \Re^m$ and $v \in \Re^m$ such that $|v_l| < 1, l = 1, ..., m$

$$sat(u) \in co\{D_s u \ + \ D_s^- v\}, \ \ s \in [1, 2^m] \tag{10}$$

where D_s is a diagonal matrix with the diagonal element either 1 or 0 and $D_s + D_s^- = I_n$; in such a case, there exist $\delta_i, i = 1, ..., 2^m, \ \ \sum_{i=1}^{2^m} \delta_i = 1, \ \ \delta_i \geq 0$ such that:

$$sat(u) \ = \ \sum_{i=1}^{2^m} \delta_i (D_i u + D_i^- v) \tag{11}$$

Hence, the closed-loop system can be re-written for all $x \in \pounds(H)$ as:

$$\left\{\begin{array}{lcl} \dot{x}(t) & = & Ax(t)+A_hx(t-h)+ \\ & & +B\sum_{i=1}^{2^m}\delta_i(t)(D_iK+D_i^-H)x(t) \\ x(t) & = & \Phi(t) \text{ for } t\in[-h,0) \end{array}\right.$$

using the convexity property one may write:

$$\left\{\begin{array}{lcl} \dot{x}(t) & = & \sum_{i=1}^{2^m}\delta_i(t)(A+B(D_iK+D_i^-H))x(t) \\ & & +A_hx(t-h) \\ x(t) & = & \Phi(t) \text{ for } t\in[-h,0) \end{array}\right.$$

IV. MAIN RESULT

The stability analysis by the criteria given is firstly extended to controlled systems and a stabilizability condition is derived. Hence, the writing of the saturated delay system as a convex combination of some delay systems enables the improved stabilizability condition established below to be extended to the case of saturating delay systems. These conditions are obtained in LMI form and can be easily studied using the existing toolboxes software of the LMI environment.

As claimed, the delay-dependent stability criteria presented above is only useful for analysis. In what follows, it is extended to controlled systems to enable the synthesis of such stabilizing controllers. Consider the controlled delay system without saturation

$$\left\{\begin{array}{lcl} \dot{x}(t) & = & Ax(t)+A_hx(t-h)+Bu(t) \\ x(t) & = & \Phi(t) \text{ for } t\in[-h,0) \end{array}\right. \tag{12}$$

Proposition 3: If there exist matrices $X > 0$, $\bar{Q} > 0$, $Zm > 0$, $\bar{R}$ and $\bar{W}$ such that the LMI below holds true,

$$\begin{bmatrix} XA^T+AX+BY+Y^TB^T+\bar{R}+\bar{R}^T+\bar{Q} & A_hX-\bar{R}+\bar{W}^T & -\bar{h}\bar{R} & \bar{h}XA^T+\bar{h}Y^TB^T \\ * & -\bar{Q}-\bar{W}-\bar{W}^T & -\bar{h}\bar{W} & \bar{h}XA_h^T \\ * & * & -2\bar{h}X+\bar{h}Zm & 0 \\ * & * & * & -\bar{h}Zm \end{bmatrix} < 0 \tag{13}$$

then the time-delay system (12) is asymptotically stable for any delay h satisfying $0 < h \leq \bar{h}$. Furthermore, the controller is given by $K = YX^{-1}$.

Proof: Using a memoryless state feedback $u(t) = Kx(t)$ and replacing A by $A + BK$ in (8), the closed loop system becomes an autonomous system. Furthermore, the asymptotic stability condition is true if one replaces A by $A + BK$ in (9). Noting $X = P^{-1}$, $\bar{R} = XRX$, $\bar{W} = XWX$, $\bar{Q} = XQX$, $Zm = Z^{-1}$ and post and premultiplying by $diag\{X, X, X, Z^{-1}\}$ leads to the following LMI:

$$\begin{bmatrix} XA^T+AX+BY+Y^TB^T+\bar{R}+\bar{R}^T+\bar{Q} & A_hX-\bar{R}+\bar{W}^T & -\bar{h}\bar{R} & \bar{h}XA^T+\bar{h}Y^TB^T \\ * & -\bar{Q}-\bar{W}-\bar{W}^T & -\bar{h}\bar{W} & \bar{h}XA_h^T \\ * & * & -\bar{h}XZX & 0 \\ * & * & * & -\bar{h}Zm \end{bmatrix} < 0$$

bearing in mind that the following inequality is true

$$-XZX \leq -2X + Z^{-1}$$

leads to the LMI (13). ■

Taking into account the saturating term at the input of the delay system, the following proposition may be given:

Proposition 4: If there exist matrices $X > 0$, $\bar{Q} > 0$, $Zm > 0$, $\bar{R}$ and $\bar{W}$ such that for $i = 1, \ldots, 2^m$ and $j = 1, \ldots, m$

$$\begin{bmatrix} \Pi_i & A_hX-\bar{R}+\bar{W}^T & -\bar{h}\bar{R} & \bar{h}XA^T+\bar{h}Y^TD_iB^T+\bar{h}G^TD_i^-B^T \\ * & -\bar{Q}-\bar{W}-\bar{W}^T & -\bar{h}\bar{W} & \bar{h}XA_h^T \\ * & * & -2\bar{h}X+\bar{h}Zm & 0 \\ * & * & * & -\bar{h}Zm \end{bmatrix} < 0 \tag{14}$$

$$\begin{bmatrix} 1 & g_j \\ * & X \end{bmatrix} \geq 0, \tag{15}$$

where

$$\Pi_i = XA^T + AX + BD_iY + BD_i^-G + \\ +Y^TD_iB^T + G^TD_i^-B^T + \bar{R} + \bar{R}^T + \bar{Q}$$

then the time-delay system (12) is delay-dependent asymptotically stable for any delay h satisfying $0 < h \leq \bar{h}$. Further the controller is given by $K = YX^{-1}$. Furthermore, the set $\mathcal{L}v(\rho)$ is invariant inside the set of attraction of the system.

Proof: First, let us note $G = HX^{-1}$ and consider as in [24], the Lyapunov-Krasovskii functional given by:

$$V(x_t) = V_1(x(t)) + V_2(x(t)) + V_3(x(t)) \tag{16}$$

$$V_1(x_t) = x(t)^TPx(t),$$

$$V_2(x_t) = \int_{-h}^{0}\int_{t+\beta}^{t} \dot{x}(\alpha)^TZ\dot{x}(\alpha)$$

$$V_3(x_t) = \int_{t-h}^{t} x(\alpha)^TQx(\alpha).$$

Define $\mathcal{L}v(\rho)$ as:

$$\mathcal{L}v(\rho) = \{\ \phi \in \mathcal{C}_{n,h} /\ \ V(\phi(t)) \leq \rho\}. \tag{17}$$

Along same lines as in [24], it follows that

$$\dot{V}(x_t) = \frac{1}{h}\sum_{i=1}^{2^m} \delta_i(t) \int_{t-h}^{t} \xi(t,\alpha)^T\Lambda_i(h)\xi(t,\alpha)d\alpha \tag{18}$$

where $\xi(t,\alpha) = [x(t)^T\ x(t-h)^T\ \dot{x}(\alpha)^T]^T$ and

$$\Lambda_i(h) = \begin{bmatrix} \Gamma_i & A_hX-\bar{R}+\bar{W}^T & -\bar{h}\bar{R} & \bar{h}XA^T+\bar{h}Y^TD_iB^T+\bar{h}G^TD_i^-B^T \\ * & -\bar{Q}-\bar{W}-\bar{W}^T & -\bar{h}\bar{W} & \bar{h}XA_h^T \\ * & * & -2\bar{h}X+\bar{h}Z & 0 \\ * & * & * & -\bar{h}Z \end{bmatrix} \tag{19}$$

where

$$\Gamma_i = XA^T + AX + BD_iY + BD_i^-G + Y^TD_iB^T + \\ +G^TD_i^-B^T + \bar{R} + \bar{R}^T + hA^TZA + \bar{Q}.$$

Using the Lyapunov Krasovskii theorem [14], for all $x_t \in \mathcal{L}_v(\rho)$, one has to prove that there exist continuous positive scalar non decreasing functions ϵ_1, ϵ_2 and ϵ_3 such that

$$\epsilon_1(\|x_t(0)\| \leq V(x_t) \leq \epsilon_2(\|x_t\|_c) \\ \dot{V}(x_t) \leq -\epsilon_3(\|x_t(0)\|);$$

for the proposed function $V(x_t)$ given by (16), one can write

$$\epsilon_1 \left\|x_t(0)\right\|^2 \leq V(x_t) \leq \epsilon_2 \left\|x_t\right\|_c^2$$

where $\epsilon_1 = \lambda_{min}(P)$ and $\epsilon_2 = \lambda_{max}(P) + h\lambda_{max}(Q) + (h^2/2)(max_{i=1,...2^m} \left\|A + BD_iY + BD_i^-H\right\| + \|A_h\|)\lambda_{max}(Z)$.

On the other hand, from (18), it is possible to write that:

$$\dot{V}(x_t) \leq (-min_{i=1,\ldots,2^m}(\lambda_{min}(-\Lambda_i)) \left\|x_t\right\|^2 \tag{20}$$

■

V. EXAMPLES

Example 1

Consider the non saturating input delay system given by

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}; \quad A_h = \begin{bmatrix} -1 & -1 \\ 0 & -0.9 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

when considering the non saturating case, applying Proposition 3 leads to a less conservative bound delay $\bar{h} = 1.7$ compared to some previous works. Delay bounds together with the stabilizing state feedbacks

Method	$\bar{h}$	K
[10]	1.4	not given
[9]	1.5	$[-58.3 - 294.9]$
Proposition 3	1.7	$[-0.2787 - 2.0635]$

TABLE I

are summarized in Table 1.

Example 2
Let us now consider the saturating input delay system given by (1) taken from [21] where we consider no perturbation and a control bound $u_{max} = 15$, where:

$$A = \begin{bmatrix} -0.2 & 0 \\ 0 & 1 \end{bmatrix}; \quad A_h = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

The non unity of control limitation is easily accomodated by taking

$$B_n = \begin{bmatrix} 0 \\ 15 \end{bmatrix}^T$$

For delays satisfying $0 < h < 1.55$, that is $\bar{h} = 1.55$, LMI's (14) and (15) are found feasible, and this leads to the following data :

$$X = \begin{bmatrix} 0.8758 & 0 \\ 0 & 1.1873 \end{bmatrix};$$

$$\bar{Q} = \begin{bmatrix} 0.8293 & 0 \\ 0 & 1.1471 \end{bmatrix};$$

$$\bar{Z} = \begin{bmatrix} 0.8559 & 0 \\ 0 & 1.7988 \end{bmatrix};$$

$$\bar{R} = \begin{bmatrix} -0.5636 & 0 \\ 0 & -0.2979 \end{bmatrix};$$

$$\bar{W} = \begin{bmatrix} 0.5764 & 0 \\ 0 & 0.2964 \end{bmatrix};$$

the two matrices Y and G are respectively given by

$$Y = [0 \quad -0.1352]$$

$$G = [0 \quad -0.1351]$$

hence the stabilizing memoryless state feedback is given by

$$K = [0 \quad -0.1138]\,. \qquad (21)$$

The figures below show the evolution of the saturating input and the states versus time, respectively, on Figure 1 and Figure 2. Figure 3 is reserved for the trajectory of the system inside the set $\mathcal{L}(G)$ and the ellipsoidal set $\Omega(X, 1)$.

VI. CONCLUSION

This paper is devoted to the elaboration of improved delay-dependent stabilizability conditions for time-delay systems containing saturating actuators. A delay-dependent stability criteria is worked out to derive delay-dependent stabilizability conditions for systems without and with saturating input. Furthermore, the convex writing of the constrained control systems as a combination of a 2^m delay system is used. Hence, the conditions are extended to saturating control delay systems. The results are given under LMI formalism to make the synthesis of stabilizing memoryless state feedbacks for such systems easier. As for delay systems without saturation, the obtained conditions are less conservative and overcome some restrictive problems arising in former works such as bounding the weighted cross product of the state and the delayed state. Illustrative examples, taken from the literature, are treated for both non saturating and saturating delay systems to show the application of the method. Furthermore, the case of the non saturating delay systems is compared to some previous results.

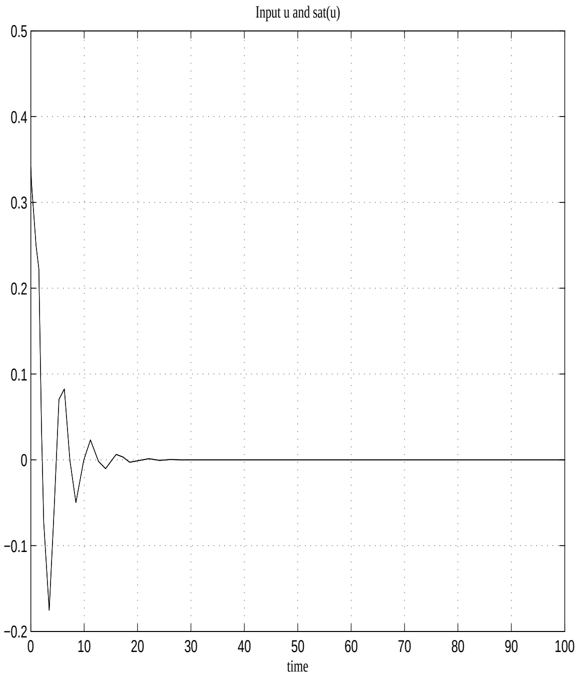

Figure 1: Control's evolution in time.

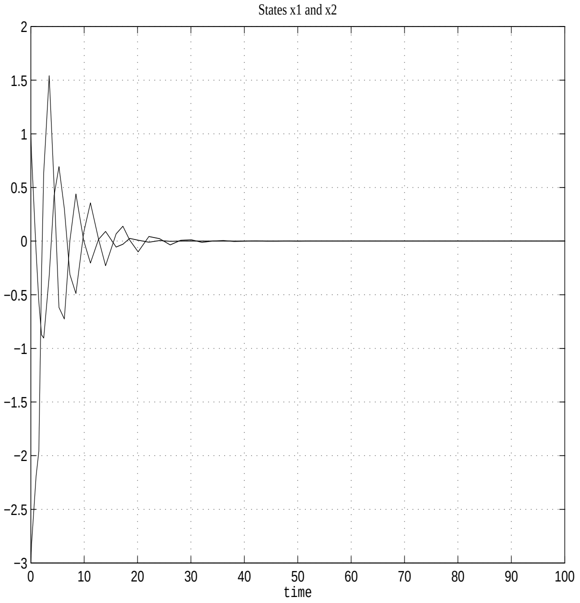

Figure 2: States evolution in time.

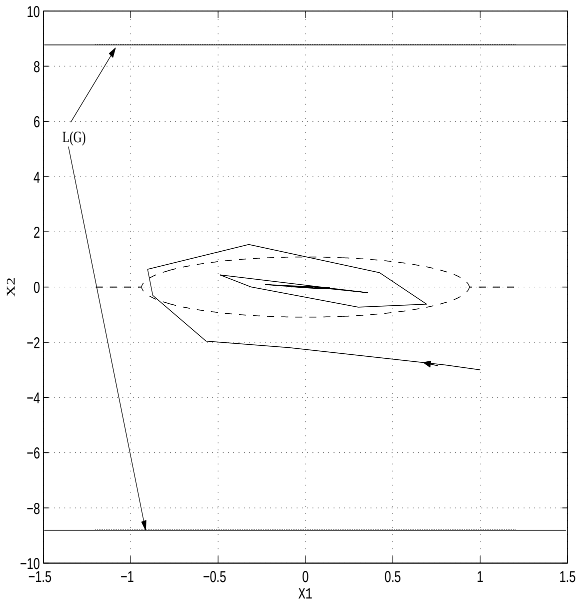

Figure 3: Domain $\mathcal{L}(G)$ and $\Omega(X,1)$ and the system trajectory.

REFERENCES

[1] Benzaouia A. and Hmamed A. (1993), Regulator problem of linear continuous-time systems with constrained control, *IEEE Trans. Aut. Control*, Vol. 38, N^o.10, pp. 1556-1560.

[2] Benzaouia A. and Mesquine F. (1994), Regulator problem for uncertain linear discrete time systems with constrained control, *Int. J. of Robust and Nonlinear Control*, Vol. 4, pp. 387-395.

[3] Blanchini F. (1999), Set Invariance in control, *Automatica* , Vol. 35, pp. 1747-1767.

[4] Boukas E.K. and Liu Z. (2002), Detreministic and stochastic time-delay systems. *Birkhauser,* Boston.

[5] Boyd S., El Ghaoui L., Feron E. and Balakrishnan V. (1994), Linear matrix inequalities in systems and control theory. SIAM studies in Appl. mathematics, Philadelphia.

[6] Cao Y.Y. Lin Z. and Hu T. (2002), Satbility analysis of linear time-delay systems subject to input saturation, *IEEE Trans. Circuit and Systems I Fundamental theory and apllications*, Vol. 49, N2, pp. 233-240.

[7] Chen B.S., Wang S.S. and Lu H.C. (1988): Stablization of time-delay systems containing saturating actuators. *Int. J. Control,* Vol. 47, pp 867-881.

[8] Dahleh M.A. and Diaz-Boboillo I.J. (1995), Control of uncertain systems : a linear programming approach, *Englewoods Cliffs, NJ : Prentice Hall*

[9] Fridman E. and Shaked U. (2002), An improved stabilization method for linear time-delay systems. *IEEE Trans. Aut. Control*, Vol. 47, N11, pp. 1931-1937.

[10] Fridman E. and Shaked U. (2003), Delay dependent stability and H_∞ control : constant and time varying delays. *Int. J. of Control,* Vol. 76, pp. 48-90.

[11] Hmamed A., Benzaouia A. and Bensaleh H. (1995), Regulator problem for linear continuous-time delay systems with nonsymmerical constrained control, *IEEE Trans. Aut. Control,* Vol. 40, N9, pp. 1615-1619.

[12] Hu T. and Lin Z. (2001), Control systems with actuator saturation, *Control enginering, Birkhauser*

[13] Hu T., Lin Z. and Chen B.M. (2002), Analysis and design method for linear systems subject to actuator saturation and disturbance. *Automatica*, Vol. 38, pp. 351-359.

[14] Krasovskii N.N. (1963), Stability of motion, *Stanford Univ Press CA*

[15] Lafay J.F. and Conte J. (1995), Analysis and design methods for delay systems. *In Pro. 34th IEEE CDC*, New orleans, pp. 2035-2069.

[16] Lin Z. and Saberi A. (1995), Semi global exponential stabilization of linear discrete time systems subject to input saturation via linear feedbacks, *Systems and Control letters*, Vol. 21, pp. 225-239.

[17] Mesquine F., Benhayoun M., Benzaouia A. and Tadeo F. (2010), Further results on stabilizability for time-delay systems with saturating control. *In Pro. 11th Int. Conf. Sci. and Techniques of Aut. Control & Computer Engineering*, December 2010, Monastir, Tunisia.

[18] Mori T., Fukuma N. and Kuwahara M. (1982): On an estimate of the decay rate for stable linear delay systems. *Int. J. Control*, Vol. 36, pp 95-97.

[19] Niculescu S.I. (2001), Delay effects on stability: A robust control approach, *Springer, London*

[20] Niculescu S.I., Dion J.M. and Dugard L. (1996), Robust stabilization for uncertain time delay systems containing saturating actuators, *IEEE Trans. Aut. Control*, Vol. 41, $N5$ pp. 742-747.

[21] Oucheriah S. (2003), Synthesis of controllers for time delay systems subject to actuator saturation and disturbance, *Journal of Dynamical Systems, Measurmant and Control*, Vol. 125, pp. 244-249.

[22] Tarbouriech S. and Silva J.M.G. (2000), Synthesis of controllers for continuous-time delay systems with saturating controls via LMI's, *IEEE Trans. Aut. Control*, Vol. 45, $N1$, pp. 105-111.

[23] Tissir E. and Hmamed A. (1992): Further results on the stabilization of time delay systems containing saturating actuators. *Int. J. Systems Science*, Vol. 23, pp 615-622.

[24] Xu S. and Lam J. (2005), Improved delay dependent stability criteria for time delay systems. *IEEE Trans. Aut. Control*, Vol. 50, $N3$, pp. 384-387.