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Neuro-fuzzy network modeling versus fractional model identification of a thermal process

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***Abstract*— This paper deals with the modeling and identification of heat transfer process. This diffusive phenomena exhibit complex dynamics varying with the operating frequency band. Two models are investigated: a fractional order state space model and a neural fuzzy networks. Numerical simulations on a thermal benchmark are performed in order to analyze the two models fitting ability to approximate the diffusive phenomena.**

***Index Terms*— Modeling, identification, Heat transfer process, fractional order system, neural fuzzy network.**

I. INTRODUCTION

Modeling and identification of a complex dynamical system is an important task for its simulation, control design, prediction and fault diagnosis. Various non linear model structures have been proposed in the literature such as Voltera series, Wiener and Hammerstein models, neural networks and fuzzy logic based models [1], [2], [3]. On the other hands, studies of diffusive phenomena, such as thermal [4], electrochemical [5]...exhibit complex fractional behaviour characterized by long memory transients and infinite dimensional structure, and the use of classical integer models is inappropriate; thus, a further class based on the fractional derivative called fractional models has been developed for their modeling.

The well known heat transfer process is addressed in this paper. Different approaches based on linear fractional systems have been proposed for its modeling [6], [7], although, they give satisfactory approximation in the time domain, for some methods, a priori choice of 0.5 on the fractional order is required [4], [8]. Another major limitation evolves, when temperature ranges in a wide interval, causing that certain thermophysical parameters of the medium (conductivity and diffusivity) to vary and depend on the temperature [9]. In this context, the process internal behaviour contains unknown non-linearities, difficult to model and not fully understood, which can be described qualitatively by a set of rules. Thus, the use of intelligent computational methods such as neural networks and fuzzy logic based models may be attractive. They offer the advantages of the learning and adaptation mechanisms to adjust the model parameters, while conventional approaches tend to fail or become cumbersome.

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Artificial Neural Networks have gone through a rapid development and grown past the experimental stage to become implemented in a wide range of engineering applications [9]. On the basis of supplied training data the neural network learns (trains) the relationship between the process input and output, and these weighted nodes communicate values to attain the desired process behaviour [3], [10], [11]. However, neural network are notoriously difficult to train, especially when the system dynamics involve many changing factors such as unstructured uncertainties, and the membership functions have to be adjusted, to composite for these effects. A solution to overcome these drawbacks is the powerful approach of neural fuzzy network. The approximation of the complex process behaviour is performed using multiple models with fuzzy transitions, based on neural network structures. The inclusion of a compensatory operator allows to take into account the varying parameters governing the thermal diffusive phenomena [12], [13], [14].

The aim of this paper is to present two methods for the modeling of the heat transfer process. The first one is based on parsimonious fractional state space models, with no a priori choice on the fractional order, and the second one uses the neural fuzzy networks to reproduce the thermal process behaviour. These methods are applied to a thermal benchmark and their fitting ability to approximate the diffusive phenomena is analyzed.

The paper is organized as follows: Section 2 recalls the heat transfer process properties, and its fractional behavior is pointed out. In section 3, some preliminaries on the fractional derivative and fractional models are introduced, and the identification method for fractional systems is presented. Section 4, is dedicated to the modeling with neural fuzzy network. In Section 5, the thermal benchmark is described, and the simulation results obtained using the fractional model and the neuro-fuzzy network are analyzed. Finally, we conclude this paper with the evaluation of the two methods efficiency, and some remarks on future research

II. DYNAMICAL BEHAVIOUR OF A THERMAL PROCESS

One dimension heat transfer is governed by the following partial differential equation [4], [16] :

$$\begin{cases} \partial T(x,t)/\partial t = \alpha_c(\partial^2 T(x,t)/\partial x^2), 0 < x < \infty,\ t > 0 \\ \lambda_c(\partial T(x,t)/\partial x) = \phi(x,t) \text{ when } x = 0,\ t > 0 \\ T(x,t) = 0 \text{ when } 0 \le x < \infty,\ t = 0 \end{cases} \tag{1}$$

Let us consider a plane surface of thickness L, where the finite medium is homogeneous (thermal conductivity λ_c, diffusivity α_c), x being the abscissa of the measurement slot in the medium. The boundary conditions are:

$$\begin{cases} \phi(0,t) = u(t) \\ \phi(L,t) = T(L,t)/R \end{cases}$$

A dynamical model, linking the temperature $T(x,t)$ and the heat flux $\phi(x,t)$ at the diffusion interface ($x = 0$) is described by the transfer function:

$$H_p(s) = \frac{\tanh\left(L\sqrt{\frac{s}{\alpha_c}}\right)}{S_m\lambda_c\sqrt{\frac{s}{\alpha_c}}} \tag{2}$$

$$\lim_{s\to\infty} H_p(s) = \frac{1}{S_m\lambda_c}\frac{\sqrt{\alpha}_c}{s^{0.5}} \quad \lim_{s\to 0} H_p(s) = \frac{L}{S_m\lambda_c}\frac{1}{1+\frac{L^2}{\alpha_c}s} \tag{3}$$

At low frequencies an integer model is sufficient to describe the diffusion interface, while, for high frequencies, it behaves like a fractional integrator of order 0.5; thus fractional models are appropriate to describe the diffusive interface behaviour in this frequency band [4], [7]. Also, from the study of thermodynamics, it can be shown that some thermophysical coefficients are directly related to specific state vector components, thus are time varying parameters which induce non linear uncertainties. In order to circumvent this difficulty, two models can be considered: a fractional model with at least two fractional orders to composite with the effects of the varying parameters with the frequency band [23]–[25], and a neural-fuzzy network known to be able to picture the unknown uncertainties of dynamical systems.

III. Fractional order models

Fractional calculus is the generalization of integration and differentiation to fractional (non integer) order α. In this study, we adopt the following Caputo definition for fractional derivative, more appropriate for real processes description [15]–[18].

$${}_0^C D^\alpha f(t) = D^\alpha f(t) = \frac{1}{\Gamma(r-\alpha)}\int_0^t \frac{f^{(r)}(\tau)}{(t-\tau)^{(\alpha-r+1)}}d\tau \tag{4}$$

where $r-1 < \alpha < r$, $r \in \mathbf{N}$, $D^\alpha f(t)$ is the fractional derivative of order α of the function $f(t)$ and Γ is the Euler Gamma function given by:

$$\Gamma(\beta) = \int_0^\infty \nu^{\beta-1}e^{-\nu}d\nu \tag{5}$$

A fractional single input, single output (SISO), linear, time invariant (LTI) system can be described by the following fractional order differential equation: [16], [18].

$$y(t) + \sum_{i=1}^{n} a_i\, D^{\alpha_i} y(t) = \sum_{i=1}^{m} b_i\, D^{\beta_i} u(t) \tag{6}$$

where: $a_i, b_i \in \mathbf{R}$, $\alpha_i, \beta_i \in \mathbf{R}^+$, $u(t) \in \mathbf{R}$ is the input and $y(t) \in \mathbf{R}$ is the output; in the particular case where, the fractional orders are multiple of the same real number α, the system is called a commensurate order system. Assuming that the system is relaxed ($y(t) = u(t) = 0$ for $t \le 0$), Laplace transforms of $D^{\alpha_i}y(t)$ and $D^{\beta_i}u(t)$ are respectively $s^{\alpha_i}Y(s)$ and $s^{\beta_i}U(s)$, where $Y(s)$ and $U(s)$ stand respectively for the transforms of $y(t)$ and $u(t)$.
Applying Laplace transform to Equation (6) yields to:

$$Y(s) + \sum_{i=1}^{n} a_i\, s^{\alpha_i}\, Y(s) = \sum_{i=1}^{m} b_i\, s^{\beta_i}\, U(s) \tag{7}$$

The fractional system transfer function is then deduced:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{\sum_{i=1}^m b_i\ s^{\beta_i}}{1+\sum_{i=1}^n a_i\ s^{\alpha_i}} \tag{8}$$

In the particular case of a commensurate order system, Equation (8) can be written under the form:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{\sum_{i=1}^m b_i\ (s^{\alpha})^i}{1+\sum_{i=1}^n a_i\ (s^{\alpha})^i} \tag{9}$$

The fractional (LTI) system can also be described by the fractional state space model defined by [19], [20]:

$$\begin{cases} D^{(\alpha)}(x) = A\,x + B\,u \\ \\ y = C\,x + D\,u \end{cases} \tag{10}$$

where:

$$D^{(\alpha)}(x) = \left[\ D^{\alpha_1}x_1, \quad D^{\alpha_2}x_2 \quad \ldots \quad D^{\alpha_n}x_n\ \right]^T \tag{11}$$

with: $x \in \mathbf{R}^n$, $u \in \mathbf{R}$, $y \in \mathbf{R}$, and the matrices A, B, C, D of appropriate dimensions.
For the commensurate case all states $x_i(t)$ are differentiated to the same fractional order α, with:

$$D^{(\alpha)}(x) = D^\alpha(x) = D^\alpha \left[\ x_1 \quad x_2 \quad \ldots \quad x_n\ \right]^T \tag{12}$$

A. Numerical simulation of a fractional model

The numerical simulation of a fractional model in the time domain is a major difficulty, since an analytical expression of the model output is often not simple to obtain. Different approaches have been proposed in the literature [6], [19], [21], [22], depending on the way the fractional operator is modelled. In this paper, the simulation is carried out by an indirect approach based on the bounded fractional integrator [6]. This last is approximated by a recursive distribution of poles and zeros (number of cells related to the quality of the approximation) on a limited frequency range : $[\omega_b, \omega_h]$, where it has a fractional behavior, and outside this interval, it acts as a conventional integrator. We used $N_C = 30$ cells, on the frequency band $[10^{-5}, 10^5]$.

B. Fractional system identification

Given the fractional model state space representation (10), without loss of generality, the controllable canonical form (13) can be considered, with the matrices A, B, C, D defined as follows:

$$A = \begin{bmatrix} 0 & 1 & \dots & 0 \\ \vdots & & & \\ 0 & 0 & \dots & 1 \\ -a_1 & \dots & -a_{n-1} & -a_n \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} c_1 & c_2 & \dots & c_n \end{bmatrix} \qquad D = \begin{bmatrix} d \end{bmatrix} \tag{13}$$

The objective of the identification consists in estimating not only the state space model matrices A, C, D, but also the non integer order $(\alpha) = [\alpha_1 \dots \alpha_n]$.
The vector θ of parameters to be estimated is given by:

$$\theta = \begin{bmatrix} \tilde{\theta} & (\alpha) \end{bmatrix} = \begin{bmatrix} -a_1 & \dots & c_1 & \dots & c_n & d & (\alpha) \end{bmatrix} \tag{14}$$

The model output being nonlinear in θ, an output error method based on a nonlinear programming algorithm has been developed in preceding work [23]–[25], for the fractional case. The sampled data set is composed of K observations $\{u_k, y_k^*\}$ with $t = kT_e$ (T_e being the sampling period).
Let $\hat{\theta}$ be the estimate of the exact parameters vector θ, the output prediction error is given by: $\varepsilon_k = y_k^* - \hat{y}_k$. Where $\hat{y}_k$ is the corresponding output estimate. The optimal value of $\hat{\theta}$ is obtained by minimizing the quadratic criterion using Marquardt algorithm [26], [27]: $J(\theta) = \frac{1}{2}\sum_{k=1}^{K} \varepsilon_k^2$.

$$\theta^{(i+1)} = \theta^{(i)} - \left\{ \left[J''_{\theta\theta} + \lambda I \right]^{-1} J'_{\theta} \right\}_{\hat{\theta}=\theta^{(i)}}$$

$J'_{\theta} = -2 \left(\sum_{k=1}^{K} \varepsilon_k \, \sigma_{k/\theta} \right)$ is the gradient

$J''_{\theta} = -2 \left(\sum_{k=1}^{K} \sigma_{k/\theta} \, \sigma_{k/\theta}^T \right)$ is the Hessian

$\sigma_{k/\theta} = \partial \hat{y}(k,\theta)/\partial\theta$ is the output sensitivity function

λ is the monitoring parameter

(15)

Fundamentally, this algorithm is based on the calculation of the gradient and Hessian, themselves dependant on the numerical integration of the sensitivity functions. A multivariable fractional order state space model developed in [24], [25] to compute the sensitivity functions with respect to $\tilde{\theta}$, for the last component of θ (the fractional order (α)), its sensitivity is calculated numerically [28].

IV. Neural fuzzy network

Fuzzy modeling expresses qualitatively the system characteristics by using fuzzy reasoning. The complex process behavior is approximated by multiple linear models with fuzzy transitions; fuzzy sets are used to describe the continuous domains of input and output variables by dividing these domains into a small number of overlapping regions which constitute the so-called linguistic values. In each region, a simple model is formulated, thus establishing a link between the model input domain and the output domain, regardless of their analytical dependence by a set of rules: "IF a set of conditions is satisfied THEN a set of conclusions is inferred". A neural-fuzzy system realize the process of fuzzy reasoning using the structure of neural-networks (NNs) and express the parameters of fuzzy reasoning through the weights of neural-networks. To better describe the concept for this neural-fuzzy system, we use the term neural-fuzzy inference (NFI), which can automatically identify the fuzzy rules and tune the membership functions by modifying the connection weights of the NNs using a self-learning algorithm [14], [29].

Assume that a fuzzy system consists of m fuzzy rules, each of which has an input data vector $x : x = (x_1 \dots, x_n)$ and the one dimensional output data vector y. Then the k^{th} model rule (FR^k) is expressed as :

$$FR^k : If\ x_1\ is\ {A_1}^k \dots\ x_i\ is\ {A_i}^k \dots\ and\ x_n\ is\ {A_n}^k \quad Then\ y\ is\ B^k \tag{16}$$

Where ${A_i}^k$ represents a linguistic term or a fuzzy set characterized by a membership function denoted $u_{{A_i}^k}(x_i)$ for $i = 1, 2, 3, \dots n$ and $k = 1, 2, 3, \dots m$, and composing the condition rule part; B^k is the conclusion rule part. The fuzzy membership functions of ${A_i}^k$ and B^k are Gaussians, defined by (17) and (18) respectively [30], [31].

$$u_{{A_i}^k} = exp\left[-\left(\frac{x_i - a_i^k}{\sigma_i^k} \right)^2 \right] \tag{17}$$

$$u_{{B_i}^k} = exp\left[-\left(\frac{y - b^k}{\delta^k} \right)^2 \right] \tag{18}$$

Our problem is to adjust the parameters (a_i, σ_i, b, δ) associated with the rules to better match the given data. In this purpose, the neural fuzzy network is based on a compensatory fuzzy operator, which adaptively adjust the membership functions, and also optimize the convergence of the fuzzy reasoning [13], [32].

Let's consider the compensatory fuzzy neurones $C(x_1, x_2)$ which allow making a relatively compromised decision for the situation between the worse case and the best case, by choosing an appropriate compensatory degree γ, with x_1 being the pessimistic input and x_2 the optimistic input:

$$C(x_1, x_2) = x_1^{1-\gamma} {x_2}^{\gamma} \tag{19}$$

Where $\gamma \in [0, 1]$. Then $u_{{A_i}^k * \dots * {A_n}^k}(x)$ is defined with a compensatory form using the pessimistic operation u^k and the optimistic operation ν^k as follows:

$$u_{{A_i}^k * \dots * {A_n}^k}(x) = (u^k)^{1-\gamma} (\nu^k)^{\gamma} \tag{20}$$

where $u^k = \prod_{i=1}^{n} u_{{A_i}^k}(x_i) \quad and \quad \nu^k = \left[\prod_{i=1}^{n} u_{{A_i}^k}(x_i) \right]^{\frac{1}{n}}$

Substituting u^k and ν^k in (20), we get:

$$u_{A_i{}^k * \cdots * A_n{}^k}(x) = \left[\prod_{i=1}^{n} u_{A_i{}^k}(x_i) \right]^{1-\gamma+\frac{\gamma}{n}} \quad (21)$$

The general architecture of a compensatory neural fuzzy network is represented in Fig.1. It has five functional layers:

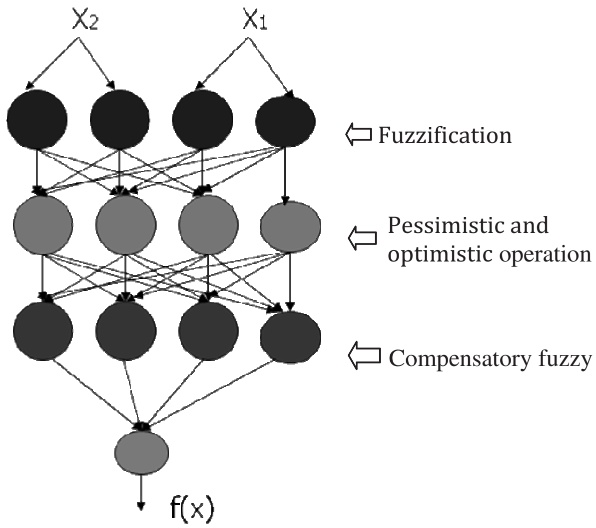

Fig. 1. Compensatory neural fuzzy network architecture

layer 1 is the input layer, layer 2 is the fuzzification layer which performs the mapping of the inputs x_i into the fuzzy sets A_i with degree y, for $y = u_{A_i}(x_i)$, layer 3 is the pessimistic-optimistic operation layer, layer 4 is the compensatory operation layer, and layer 5 is the defuzzification layer, which generates the value $y = f(x)$, based on the inputs x_i and the weights $\omega_i{}^k$ of the output membership function.

For simplicity, we defined $\eta = (1 - \gamma + \frac{\gamma}{n})$, then a typical defuzzifier is given by (22) [13]:

$$f(x) = \frac{\sum_{k=1}^{m} b^k \delta^k \prod_{i=1}^{n} \left\{ \exp\left[-\eta \left(\frac{x_i - a_i^k}{\sigma_i^k} \right) \right] \right\}}{\sum_{k=1}^{m} \delta^k \prod_{i=1}^{n} \left\{ \exp\left[-\eta \left(\frac{x_i - a_i^k}{\sigma_i^k} \right) \right] \right\}} \quad (22)$$

This compensatory neural fuzzy operator is used for learning the system complex behaviour.

A. Learning Algorithm

The training phase is a predominant shape in the conception of a Neurofuzzy system. Many effective learning algorithms for neurofuzzy systems were developed and many structures of neurofuzzy are proposed. The main steps of the learning procedure [13], [32] are as follows: first we establish the set of carrying out the system information to be analyzed; they constitute the condition and conclusion part of the rule. Each set of variables is characterized by a membership function, and the input-output space is then subdivided into a rule set covering the system's domain. Grades for the attributed membership function of each input-output are calculated based on the compensatory operator. The training algorithm allows to optimally adjust the centers and widths of both input and output membership functions of the neurofuzzy system. The gradient method is used in order to minimise the objective function defined as follows: $E^p = \frac{1}{2}\left[f(x^k) - y^k\right]^2$. The deffuzification formula (22), allows to combine all rules contributions in a weighted form, and to compute the inferred model output $y = f(x)$.

V. Modeling and identification of the thermal process

A. Description of the thermal benchmark

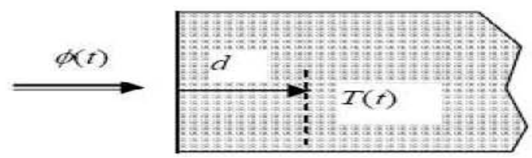

Fig. 2. Heated rod (0-20W heater)

The heat transfer in an aluminium rod of Fig.2 is considered. The input signal is a thermal flux applied at one end, and the output signal is the rod's temperature measured at a distance of d = 0.5 cm from the heated point. As shown by the theory, the thermal process requires a varying parameter model, depending on the frequency band, for its description. In this purpose, two models are investigated to test their ability to describe this diffusion process behaviour.

A first one is based on a fractional state space model and a second one uses the neural fuzzy network approach based on a compensatory operator, known to take into account the changing factors of a dynamical system. The simulation results are given in the next subsection.

B. Fractional model Identification of the thermal process

The presented identification method is applied to estimate the model parameters that describe the thermal benchmark of Fig. 2. Its input: a pseudo-random binary sequence, and output data are depicted in Fig. 3.

A fractional state space model of dimension two, with

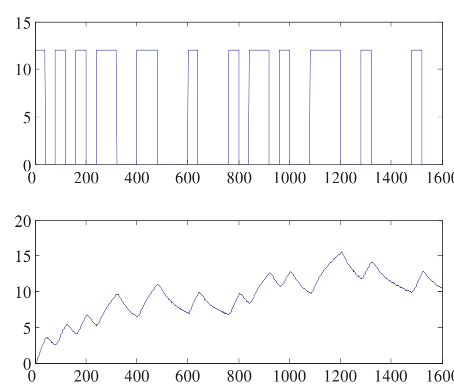

Fig. 3. Thermal process input and output data

$(\alpha) = [\alpha_1\, \alpha_2]$ is used. The estimated model output response

depicted in Fig.4, fits the data with a good accuracy, and the prediction error ($\varepsilon \simeq 0.3$) is plotted in Fig.5. The obtained fractional order is: $(\alpha) = [0.54 \;\; 0.93]$ characterizing the diffusion process is recovered, with a quadratic criterion $J = 5.11$. The use of non commensurate orders ($\alpha_1 \neq \alpha_2$), gives a supplementary degree of freedom that allows to describe the effects of the frequency bands.
This highlights the interest of fractional models to describe the complex heat transfer behaviour.

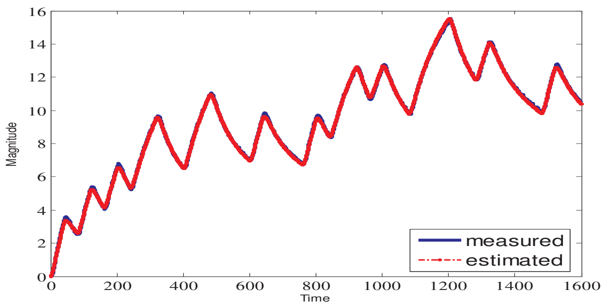

Fig. 4. Thermal process output and its estimated fractional model

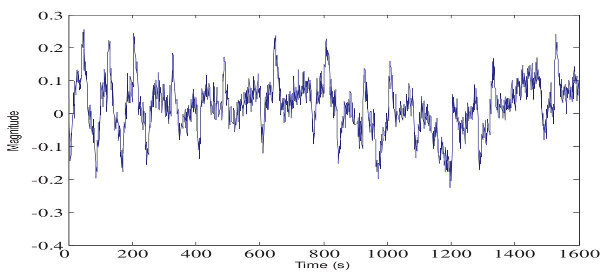

Fig. 5. Model prediction error

C. *Neuro-fuzzy modeling of the thermal benchmark*

This approach is used to build the neuro-fuzzy model of the thermal benchmark, the learning scheme is given in Fig. 6. The input vector of the neuro-fuzzy system consists of the three previous output of the benchmark and the pseudo random binary signal input. From the results depicted in Fig. 7, we see that the proposed neuro-fuzzy output match accurately the thermal process data and the error bound in Fig.8 $\simeq 4 * 10^{-3}$; thus the NFN produces a good performance, and the inclusion of a compensatory operator allows to take into account the unknown non linearities. This way, its potential to model adaptively the thermal process behaviour is verified.

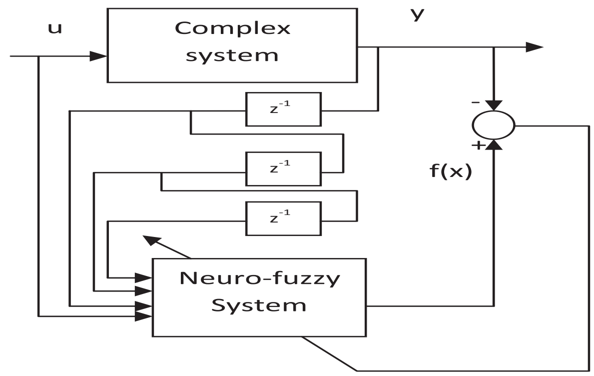

Fig. 6. Learning scheme of a dynamic model

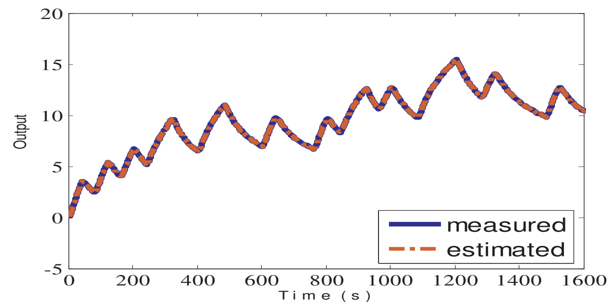

Fig. 7. Thermal process output and its NFN model output

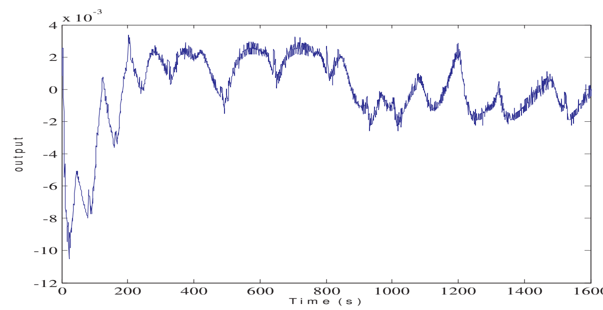

Fig. 8. Neurofuzzy model prediction error

VI. Conclusion

In this paper we have investigated two methods for the heat transfer process modeling. This diffusive phenomena is known to require different models depending on the frequency band considered, thus, a fractional model is necessary for the high frequencies, while for the low frequencies an integer model is sufficient. Another major limitation evolves when the temperature ranges in a wide interval, causing the variation of the thermal process characteristics (in the case of the studied benchmark, the temperature interval is relatively limited). To model these complex dynamics, a first system of fractional order of dimension two (two fractional orders to composite for the frequency effect) has been tested. The model parameters and the fractional orders are estimated, based on an output error identification method. The simulation results show that the model match the thermal data with a good accuracy, and the estimated fractional orders, characteristics of the diffusive process are in adequation with the theory. A second model based on the neural fuzzy network approach has been used; The inclusion of the compensatory fuzzy operator, allows to adjust the membership functions and optimize as well the convergence rate of the algorithm. The simulation results show the efficiency of the neuro-fuzzy model to adapt its internal structure to achieve the fitting of the benchmark behaviour. Thus, we verify that a compensatory neuro-fuzzy system can be a universal approximator that combine verbal power of fuzzy systems with numeric power of neural networks.

The comparison of the two methods, shows that a better performance ($\varepsilon \simeq 4 * 10^{-3}$) is obtained for the neural fuzzy network than that of the fractional model. However, the drawback of this approach, is that the neuro-fuzzy model parameters cannot give any information about the physical system, in opposition to the fractional model that relates its parameters (orders) to the real process characteristics.

The studied benchmark outputs (temperatures) are distributed on a relatively limited range, so that the thermophysical properties (conductivity and diffusivity) of the material can be assumed constant. Thus, a further step will be to test these two models efficiency for an industrial thermal process where the temperatures range in a wide interval.

VII. Acknowledgment

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