Equivalent formulations for the Profitable Vehicle Routing Problem with Multiple Trips

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Abstract— **In this paper we address the issue of different mathematical models for the Profitable Vehicle Routing Problem with Multiple Trips (PVRPMT). Surprisingly, this problem has received little attention in the literature in spite of its importance in practice. The PVRPMT arises when each vehicle performs several routes during the workday under a strict time limit on route duration and only a subset of customers can be served due to the limited size fleet of vehicles. The customers' choice is based on their profitability (profit minus travelling cost). Two strategies of sub tours elimination constraints will be presented and for each strategy two different cases are distinguished.**

I. Introduction

The Vehicle Routing Problem (VRP) is a well-known problem studied by researchers in Operational Research. It deals with finding the optimal routes of delivery or collection from a depot to a number of customers by a fleet of vehicles, while satisfying some constraints. The solution of a VRP is a set of minimum cost routes which fulfill the customers' requirements. Several operational constraints are considered in practical applications. In this paper, we focus on a new variant called the PVRPMT, which can be very practical especially at an operational context where daily drivers schedules must be achieved with a fixed vehicle fleet and short distance distribution networks. The routes can be constructed by only a subset of customers for lack of means or for insufficiency of the offer and several routes can be assigned to the same vehicle. This variant represents the combination of two known variants such that: the Vehicle Routing Problem with Multiple Trips or with multiple uses of vehicles (VRPMT or VRPM) and the Profitable Vehicle Routing Problem (PVRP).

To the best of our knowledge, this problem is studied only by N. Azi et al [1] but with time window. An exact approach is generated based in column generation and branch and price algorithm. In what concerns the VRPMT, the first research that explicitly addresses the multiple trips case was made by Salhi [2]. Fleischmann [3] proposed a modification of the algorithm of Clarke and Wright [4] and used the bin packing heuristic Best Fit Decreasing (BFD) [5] to assign the routes to the vehicles. Taillard *et al.* [6] proposed a three-phase algorithm. In the first phase, a set of routes satisfying capacity constraints is constructed by a tabu search heuristic. Next, these routes are combined into complete VRP solutions. Finally, the routes of each solution are assigned to the vehicles solving a bin packing problem. A constructive and improvement heuristics were proposed by Brandao and Mercer [7]. To compare with the benchmark of Taillard *et al.* [6], Brandao and Mercer [8] modified their heuristic to solve the classical VRPMT. Their approach is based on the nearest neighbor rule and an insertion criterion to assign customers to routes within vehicles. Golden *et al.* [9] adopted the approach of Taillard *et al.* [6] to solve a VRPMT using the minimax objective. Petch and Salhi [10] developed a multi-phase constructive heuristic. This heuristic integrates in part the approach of Taillard *et al.* [6] as initial solution and that of Brandao and Mercer [7] as an improvement phase. Lin *et al.* [11] studied a variant of a multi-objective problem that integrates the multiple uses of vehicles and the location of deposits. The proposed heuristic is composed of a tracking phase and a routing phase. Olivera and Viera [12] put forward an interesting implementation of adaptive memory search for the VRPMT. This is based on the same principle of Taillard *et al.* [6] with some incorporated enhancements. Initial VRP solutions are constructed by a sweep-based algorithm. The memory is then constructed by the top solutions up to a certain memory size. A bin packing procedure is adopted to pack the routes into vehicles while introducing some refinements based on reducing the driver overtime. Battar *et al.* [13] consider the PVRPMT with time window. A two-phase heuristic is proposed. The first step is to determine a list of roads by vehicle routing problem with time window heuristics. During the second phase a set of routes are aggregated into multiple routes based on a bin packing heuristic.

While the literature is rich with research which considers problems of vehicle routing where all the customers are served, only few papers approaches the contrary case. In many practical situations, it is not possible to satisfy the entire costumers' request for lack of means or for insufficiency of the offer. It is thus necessary to privilege the most important customers according to criteria of potentiality in the long terms or recorded effective sales turnover. Obviously, the profit brought by a satisfied and faithful customer can be quantified according to various manners. In addition, the economic value of such a profit must be projected with an appropriate manner with the used time horizon for optimization. It is clear that a customer can be described as profitable or advantageous if it allows a

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direct and punctual benefit relating to the current sales turnover, i.e. realized in the temporal horizon of required optimization, or to a still fictitious sales turnover which can be potentially carried out in the future with a probable commercial reinforcement. The interest in these problems is growing recently (see Toth and Vigo [14]). Dell'Amico *et al.* [15] are the first to prepare the study of the PVRP. They presented the problem which they called the Profitable Tour Problem (PTP) as a variant of the Traveling Salesman Problem. If a vertex (costumer) is left unvisited a given penalty has to be paid. In addition, they considered the Prize-collecting Traveling Salesman Problem (PCTSP) where each vertex is associated with a prize and there is a knapsack constraint which guarantees that a sufficiently large prize must to be collected. Feillet *et al.* [16] were the first to gather and propose a classification of the routing problems which authorize to not serve some customers with the consideration of the profit brought by the visit of each one. The survey carried out by Feillet *et al.* [16] focused on Traveling Salesman Problems (TSP) with profits using a single available vehicle. The objective may be either the maximization of the collected total profit (problem called Orienteering Problem shortened to OP), the minimization of the total traveling costs and penalties associated to unselected costumers (PTP), or the optimization of a combination of both. Feillet *et al.* [17] add the capacity constraint at PTP and Team Orienteering Problem (TOP). For each problem, a lower bound is determined via a column generation procedure. Then, two tabu search heuristics are developed. Recently, Feillet *et al.* [18] studied the Undirected Capacitated Arc Routing Problem with profits (UCARPP). The objective is to find a set of routes that satisfy the maximum duration and capacity constraints to maximize the total collected profit.

In this paper, we tackle the Profitable Vehicle Routing Problem with Multiple Trips. The issue of different mathematical models for this problem is going to be addressed. The remainder of this paper is organized as follows. In Section 2, we give a detailed theoretical graph description of PVRPMT and we introduce the corresponding proprieties and notations furthermore we determine the complexity of this problem. In Section 3, based on two different strategies of sub tours elimination constraints, we provide four mathematical models of the problem including MILP and 0-1 ILP. Computational study compares the performance of the different models will be proposed in Section 4. Finally, in Section 5, we put some conclusions and discuss future directions of the research.

II. PROBLEM DEFINITION AND NOTATION

A. Problem description and Notation

We consider a complete undirected graph $G = (V,E)$, where $V=\{0,...,n\}$ is the set of vertices and E is the set of edges. Vertex 0 represents a depot where a fleet $k = \{1,...,m\}$ of identical vehicles is based. Each vehicle has a limited capacity Q and a maximum number of trips L. An edge $(i,j)\in E$ represents the possibility to travel from customer i to customer j. A non-negative demand q_i, profit p_i, and time service S_i, are associated with each customer i ($p_0=q_0=0$). A symmetric travel time t_{ij} and cost c_{ij} are associated with each edge $(i,j)\in E$ Each vehicle starts and ends its tour at vertex 0, and can visit any subset of customers with a total demand that does not exceed the capacity Q. In addition, there exists a time horizon denoted by T_{max} which establishes the duration of a working day. It is assumed that all parameters are nonnegative integers and the environment is deterministic.

B. Problem definition and proprities

The studied problem in this paper, called PVRPMT, consists on determining a set of routes and to assign each route to one vehicle where each vehicle can be used by several routes while respecting the time horizon capacity. The objective is to maximize the difference between the total collected profit and the cost of the total traveled distance. Note that the following properties: (1) the optimal solution may be composed by a sub set of customers. (2) the number of trips in the optimal solution does not exceed $L*m$. (3) the customers' demand in the same route does not exceed Q. (4) the duration of routes assigned to the same vehicle does not exceed T_{max}. (5) the profit associated at each customer is fixed and can be collected by any vehicle since the workday.

C. Problem complexity:

Olivera and Viera [12] proved that the VRPMT is NP-hard such as the PVRP [16]. The studied problem represents the combination of two NP-hard problems. So, we can prove that it is NP-hard. In addition to that, PVRPMT is a generalization of the VRP: any VRP instance can be transformed to an equivalent PVRPMT, setting $M=m\times L$ and $V'=V\backslash\ \{V''\}$ with V'' represents the set of unvisited customers. As the VRP is an NP-hard problem [19], the PVRPMT is also NP-hard.

D. Illustrative example

As stated before, a solution for the PVRPMT instance can be presented by a set of routes, which satisfies the capacity constraint and the workday time limit and which combines the visited customers, and a set of unvisited customers whose demand cannot be satisfied due to scarcity of ressources. Figure 1 illustrates a solution representation in the case of 15 customers and three vehicles.

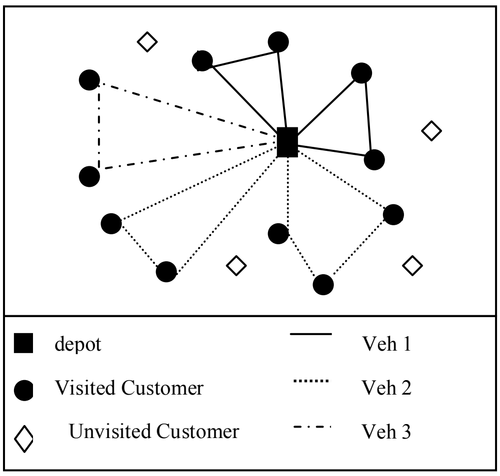

Figure 1: An illustration of a solution representation of a PVRPMT

III. A MATHEMATICAL MODELS FOR THE PVRPMT

The design of the VRP solution stands against the presence of the sub tours. The most classical constraint and the most used in the literature can be written in this way:

$$\sum_{i\in S}\sum_{j\neq i} x_{ij}^{kl} \leq |S|-1 \quad with\ S\subset\{1,...,n\}; 2\leq |S| \leq (n-1)$$

this formulation is exponential and influences strongly the resolution time. In this section, we propose four mathematical formulations for the PVRPMT. The difference between these formulations is based on the choice of the decision variables and the strategy used to eliminate the sub tours. The different indices, parameters and decision variables are given by table1.

TABLE I. NOMENCLATURE

Indices
i, j : customer index k : vehicle index l : trip index t: order index
Variables
$x_{ij}^{kl}=\begin{cases}1\ if\ (i,j)\ is\ assigned\ to\ the\ vehicle\ k\ during\ the\ trip\ l\\0\ Otherwise\end{cases}$ $\delta^{kl}=\begin{cases}1\ if\ the\ vehicle\ k\ is\ used\ during\ the\ trip\ l\\0\ Otherwise\end{cases}$ $y_{it}^{kl}=\begin{cases}1\ if\ i\ is\ the\ t^{th}\ visited\ customer\ in\ the\ trip\ l\\0\ Otherwise\end{cases}$ U_i : variable associated to customer i used to reformulate the sub-tour elimination constraints
Parameters
c_{ij} : cost associated with the edge (i, j) t_{ij}: time to traverse edge (i, j) Q : capacity of the vehicle m : number of available vehicles L: the maximum number of trips can be made by one vehicle n: number of vertices T_{max}: working day time limitation q_i: demand of customer i p_i: profit collected at customer i S_i: service time at customer i M : a big positive number

A. 0-1 Integer Lineaire Formulation (0-1 ILP)

We start with the idea to specify for each assigned customer his order in the trip. This idea has been applicated in the scheduling problem. For each job, we determine the position of the job in the sequence. We use the visit order of customer i in the trip in order to eliminate the invalid tours. Accordingly, we eliminate the sub set S and the associated constraints. A new decision variable y_{it}^{kl} is added which informs about the visit order of customer i in the trip l assigned to the vehicle k.

Based on the choice of the principal variable in the formulation, we can distinguish two different mathematical models.

- 0-1 ILP with x_{ij}^{kl} is the principal variable

Here, the second variable y_{it}^{kl} is used just to make the connection between the edge and the vertices. The resulting formulation can be presented as follow:

$$\text{Max} \quad \sum_{i=0}^{n}\sum_{\substack{j=0\\j\neq i}}^{n}\sum_{k=1}^{m}\sum_{l=1}^{L}(p_i-c_{ij})x_{ij}^{kl} \qquad (1.1)$$

Subject to

$$\sum_{\substack{j=0\\j\neq i}}^{n} x_{ij}^{kl} = \sum_{t=1}^{n+1} y_{it}^{kl} \qquad \forall i=0,...,n; k=1,...,m; l=1,...,L \quad (1.2)$$

$$\sum_{t=1}^{n+1}\sum_{k=1}^{m}\sum_{l=1}^{L} y_{it}^{kl} \leq 1 \qquad \forall i=1,...,n \quad (1.3)$$

$$\sum_{i=1}^{n} y_{it}^{kl} \leq 1 \qquad \forall t=1,...,n; k=1,...,m; l=1,...,L \quad (1.4)$$

$$\sum_{t=1}^{n+1} y_{0t}^{kl} \leq 1 \qquad \forall k=1,..,m; l=1,...,L \quad (1.5)$$

$$\sum_{i=0}^{n}\sum_{\substack{j=0\\j\neq i}}^{n} q_i x_{ij}^{kl} \leq Q \qquad \forall k=1,...,m; l=1,...,L \quad (1.6)$$

$$\sum_{i=0}^{n}\sum_{\substack{j=0\\j\neq i}}^{n}\sum_{l=1}^{L}(t_{ij}+S_i)x_{ij}^{kl} \leq T_{\max} \qquad \forall k=1,...,m \quad (1.7)$$

$$\sum_{\substack{i=0\\i\neq h}}^{n} x_{ih}^{kl} - \sum_{\substack{j=0\\j\neq h}}^{n} x_{hj}^{kl} = 0 \qquad \forall h=0,...,n; k=1,..,m; l=1,...,L \quad (1.8)$$

$$x_{ij}^{kl} + y_{it}^{kl} \leq 1 + y_{j,t+1}^{kl} \quad \forall i,j=0..n; t=1..n+1; k=1..m; l=1..L \quad (1.9)$$

$$y_{j1}^{kl} = x_{0j}^{kl} \qquad \forall j=1,...,n; k=1,...,m; l=1,...,L \quad (1.10)$$

$$x_{i0}^{kl} + y_{it}^{kl} \leq 2 - \sum_{t'=t+2}^{n+1} y_{jt'}^{kl} \quad \forall i,j=1..,n; t=1..,n+1; k=1..,m; l=1..,L \quad (1.11)$$

$$M\sum_{j=1}^{n} x_{0j}^{kl} \geq \sum_{i=0}^{n}\sum_{\substack{j=0\\j\neq i}}^{n} x_{ij}^{kl} \qquad \forall k=1,..,m; l=1,...,L \quad (1.12)$$

$$y_{it}^{kl} \in \{0,1\} \quad \forall i=0,..,n; t=1,..,n+1; k=1,..,m; l=1,..,L \quad (1.13)$$

$$x_{ij}^{kl} \in \{0,1\} \qquad \forall i\neq j=0,...,n; k=1,...,m; l=1,...,L \quad (1.14)$$

In this formulation, the objective is to maximize the overall collected profit minus the cost of transportation. The constraint (1.2)-(1.4) guarantee that each customer is visited at most ones. If the route kl exists, it should start and finish in the depot (1.5). (1.6) represent the capacity constraint. The limit duration of a working day is restricted by (1.7). (1.8) represents the flow conservation constraint.

The constraint (1.9) represents the counter over t. (1.10) represents the initialization of the counter and (1.11) stands against the addition of an customer if the route is closed. Each constructed trip should start in the depot (1.12). (1.13) and (1.14) represent the integrity constraints.

- 0-1 ILP with y_{it}^{kl} is the principal variable

In this model, we conserve the constraints which establish the relation between x_{ij}^{kl} and y_{it}^{kl} such in *ILP1* and when it is possible we integrate the variable y_{it}^{kl} in the objective function and in the constraints. The objective is to test the influence of this transformation on the upper bound and to know which expression can conserve much information. The resulting model is as follow:

$$\text{Max} \sum_{i=1}^{n}\sum_{t=1}^{n+1}\sum_{k=1}^{m}\sum_{l=1}^{L} p_i\, y_{it}^{kl} - \sum_{i=0}^{n}\sum_{j\neq i}^{n}\sum_{k=1}^{m}\sum_{l=1}^{L} c_{ij}^{kl}\, x_{ij}^{kl} \quad (2.1)$$

subject to:

$$\sum_{\substack{j=0\\ j\neq i}}^{n} x_{ij}^{kl} = \sum_{t=1}^{n+1} y_{it}^{kl} \qquad \forall i=0,\dots,n; k=1,\dots,m; l=1,\dots,L \quad (2.2)$$

$$\sum_{t=1}^{n+1}\sum_{k=1}^{m}\sum_{l=1}^{L} y_{it}^{kl} \le 1 \qquad \forall i=1,\dots,n \quad (2.3)$$

$$\sum_{i=1}^{n} y_{it}^{kl} \le 1 \qquad \forall t=1,\dots,n; k=1,\dots,m; l=1,\dots,L \quad (2.4)$$

$$\sum_{t=1}^{n+1} y_{0t}^{kl} \le 1 \qquad \forall k=1,\dots,m; l=1,\dots,L \quad (2.5)$$

$$\sum_{i=1}^{n}\sum_{t=1}^{n} q_i\, y_{it}^{kl} \le Q \qquad \forall k=1,\dots,m; l=1,\dots,L \quad (2.6)$$

$$\sum_{i=0}^{n}\sum_{j\neq i}^{n}\sum_{l=1}^{L} t_{ij} x_{ij}^{kl} + \sum_{i=0}^{n}\sum_{t=1}^{n+1}\sum_{l=1}^{L} S_i y_{ik}^{lt} \le T_{\max} \qquad \forall k=1,\dots,m \quad (2.7)$$

$$\sum_{\substack{i=0\\ i\neq h}}^{n} x_{ih}^{kl} - \sum_{\substack{j=0\\ j\neq h}}^{n} x_{hj}^{kl} = 0 \qquad \forall h=0,\dots,n; k=1,..,m; l=1,\dots,L \quad (2.8)$$

$$x_{ij}^{kl} + y_{it}^{kl} \le 1 + y_{j,t+1}^{kl} \quad \forall i,j=0..n; t=1..n+1; k=1..m; l=1..L \quad (2.9)$$

$$y_{j1}^{kl} = x_{0j}^{kl} \qquad \forall j=1,\dots,n; k=1,\dots,m; l=1,\dots,L \quad (2.10)$$

$$x_{i0}^{kl} + y_{it}^{kl} \le 2 - \sum_{t'=t+2}^{n+1} y_{jt'}^{kl} \quad \forall i=1..n; t=1..n+1; k=1..m; l=1..L \quad (2.11)$$

$$M\sum_{j=1}^{n} x_{0j}^{kl} \ge \sum_{i=0}^{n}\sum_{\substack{j=0\\ j\neq i}}^{n} x_{ij}^{kl} \qquad \forall k=1,..,m; l=1,\dots,L \quad (2.12)$$

$$x_{ij}^{kl} \in \{0,1\} \qquad \forall i\neq j=0,\dots,n; k=1,\dots,m; l=1,\dots,L \quad (2.13)$$

$$y_{it}^{kl} \in \{0,1\} \quad \forall i=0,..,n; t=1,..,n+1; k=1,..,m; l=1,..,L \quad (2.14)$$

B. *Mixed Integer Lineair program (MILP)*

To overcome the limitation of the classical sub tours elimination constraint, Miller Tucker and Zemlin [20] propose a new one which is corrected by kara[21]. In this level, we propose an adaptation of these constraints to our problem. For that, we remove the decision variable y_{it}^{kl} and we define a new variable U_i .

Firstly, we present the ordinary model. Then an extension of this model with cut is proposed.

- MILP1 (without cuts)

The formulation is the following:

$$\text{Max} \sum_{i=0}^{n}\sum_{\substack{j=0\\ j\neq i}}^{n}\sum_{k=1}^{m}\sum_{l=1}^{L} (p_i - c_{ij}) x_{ij}^{kl} \quad (3.1)$$

Subject to

$$\sum_{\substack{j=0\\ j\neq i}}^{n}\sum_{k=1}^{m}\sum_{l=1}^{L} x_{ij}^{kl} \le 1 \qquad \forall i=1,\dots,n \quad (3.2)$$

$$\sum_{i=0}^{n}\sum_{\substack{j=0\\ j\neq i}}^{n} q_i x_{ij}^{kl} \le Q \qquad \forall k=1,\dots,m; l=1,\dots,L \quad (3.3)$$

$$\sum_{i=0}^{n}\sum_{\substack{j=0\\ j\neq i}}^{n}\sum_{l=1}^{L} (t_{ij}+S_i) x_{ij}^{kl} \le T_{\max} \qquad \forall k=1,\dots,m \quad (3.4)$$

$$\sum_{\substack{i=0\\ i\neq h}}^{n} x_{ih}^{kl} - \sum_{\substack{j=0\\ j\neq h}}^{n} x_{hj}^{kl} = 0 \qquad \forall h=0,\dots,n; k=1,..,m; l=1,\dots,L \quad (3.5)$$

$$0 \le U_i \le Q \qquad \forall i=0,\dots,n \quad (3.6)$$

$$U_i - U_j + Q\sum_{k=1}^{m}\sum_{l=1}^{L} x_{ij}^{kl} + (Q-q_i-q_j)\sum_{k=1}^{m}\sum_{l=1}^{L} x_{ij}^{kl} \le Q - q_j \quad \forall i\neq j=0,\dots,n \quad (3.7)$$

$$x_{ij}^{kl} \in \{0,1\} \qquad \forall i\neq j=0,\dots,n; k=1,\dots,m; l=1,\dots,L \quad (3.8)$$

The constraint (3.2) guarantees that the customer i is visited at most once during the working day. (3.3) represents the capacity constraint. The limit duration of a working day is restricted by (3.4). (3.5) represents the flow conservation constraint. The adaptation of Miller, Tucker and Zemlin sub tours elimination constraints, as it is modified by Kara, for our problem is given by (3.6) and (3.7). (3.8) represent the integrity constraint.

- MILP2 (with cuts)

For the previous mathematical model (*MILP without cuts*), we add an optional variable δ^{kl} which informs about the used vehicle. So, the correspondent constraints which establish the relation between the two decision variables will be adjoined.

The formulation is the following:

$$\text{Max} \sum_{i=0}^{n}\sum_{\substack{j=0\\ j\neq i}}^{n}\sum_{k=1}^{m}\sum_{l=1}^{L} (p_i - c_{ij}) x_{ij}^{kl} \quad (4.1)$$

subject to

$$\sum_{\substack{j=0\\ j\neq i}}^{n}\sum_{k=1}^{m}\sum_{l=1}^{L} x_{ij}^{kl} \le 1 \qquad \forall i=1,\dots,n \quad (4.2)$$

$$\sum_{i=0}^{n}\sum_{\substack{j=0\\ j\neq i}}^{n} q_i x_{ij}^{kl} \le Q \qquad \forall k=1,\dots,m; l=1,\dots,L \quad (4.3)$$

$$\sum_{i=0}^{n}\sum_{\substack{j=0\\ j\neq i}}^{n}\sum_{l=1}^{L} (t_{ij}+S_i) x_{ij}^{kl} \le T_{\max} \qquad \forall k=1,\dots,m \quad (4.4)$$

$$\sum_{\substack{i=0\\ i\neq h}}^{n} x_{ih}^{kl} - \sum_{\substack{j=0\\ j\neq h}}^{n} x_{hj}^{kl} = 0 \qquad \forall h=0,\dots,n; k=1,..,m; l=1,\dots,L \quad (4.5)$$

$$U_i - U_j + Q\sum_{k=1}^{m}\sum_{l=1}^{L} x_{ij}^{kl} + (Q-q_i-q_j)\sum_{k=1}^{m}\sum_{l=1}^{L} x_{ij}^{kl} \le Q - q_j \forall k=1,.,m; l=1,.,L \quad (4.7)$$

$$\sum_{i=1}^{n} x_{i0}^{kl} + \sum_{j=1}^{n} x_{0j}^{kl} = 2\delta^{kl} \qquad \forall k=1,..,m; l=1,\dots,L \quad (4.8)$$

$$\delta^{kl} \le \sum_{i=0}^{n}\sum_{\substack{j=0\\ j\neq i}}^{n} x_{ij}^{kl} \qquad \forall k=1,..,m; l=1,\dots,L \quad (4.9)$$

$$\sum_{i=0}^{n}\sum_{\substack{j=0\\ j\neq i}}^{n} x_{hj}^{kl} \le M\delta^{kl} \qquad k=1,..,m; l=1,\dots,L \quad (4.10)$$

$$x_{ij}^{kl} \in \{0,1\} \qquad \forall i\neq j=0,\dots,n; k=1,\dots,m; l=1,\dots,L \quad (4.11)$$

$$\delta^{kl} \in \{0,1\} \qquad \forall k=1,\dots,m; l=1,\dots,L \quad (4.12)$$

In this formulation, we conserve the same constraints (4.1)-(4.7) and (4.11) as well as *MILP1*(*without cuts*). To integrate the new decision variable, three additional constraints are adjoined. (4.8) means that if the route *kl* exists, it should start and finish in the depot. (4.9) guarantees that if the edge (*i,j*) is assigned to the trip *kl*, this latter should be constructed. The opposite case is presented by (4.10). this constraint prohibits the construction of empty route, i.e if the route *kl* is constructed, at least one edge must be assigned at this trip.

IV. Computational Results

The tests will be applied on our own benchmarks. In our benchmark, twenty small size instances are generated randomly with adaptation to obtain interesting instance values according to the reality and the practice. The definition of the paramtresd is given by Chbichib et al. [22].

The algorithm was tested on a Intel Pentium 4 CPU 1.70 GHZ and 504 Mo RAM. The codes are written in C++ and we use in order to test the proposed models, the commercial solver CEPLEX10.0. of ILOG®. All the algorithm was stopped after one hour of computational time. The results are shown in Table 2. The optimal solution obtained by CPLEX is indicated under column f . The "_" means that we can not obtain the optimal solution before the time limit. The column UB represents the solution of the linear relaxed problem (the upper bound). In addition, (%) = (f-UB)/UB× 100.

We observe on Table 2 that the optimal value can be obtained within the time limit just for small size instances for the four mathematical models. For the instances where the number of vertices is superior to 16, the optimal solution cannot be determined. In Table 3, we choose as reference the upper bound of the first mathematical model (*UB1*) and we calculate the deviation of the different upper bounds in comparison with the first. (%*)*w* =(*UBw-UB1*)/*UB1*× 100 .

Different remarks can be taken into account. First by comparing the two strategies: the *0-1 ILP* and the *MILP* it is very clear that the upper bounds obtained by the mixed integer programming are better than those obtained by the 0-1 integer programming for all the tested instances. For the 0-1ILP, the choice of the principal variable has not a great influence on the upper bound (for the majority of the tested instances the upper bounds obtained by the two models are equal) but strongly affects the number of iterations. For the *MILP*, the upper bounds of the mathematical models with cuts are better than those without cuts. So, we can judge that the additional constraints represent valid cuts for our model. To crown all, we can conclude that the *MILP2* (with cuts) represents the best formulation of our problem.

V. Conclusion

In this paper, we describe a new variant of the vehicle routing problem namely the Profitable Vehicle Routing Problem with Multiple Trips. Two different strategies of sub tours elimination constraints are used and for each strategy two different cases are defined. So, four mathematical models are obtained. Experimental study shows that the mixed integer programming gives an upper bound smaller. The additional constraints in *MILP2* represent valid cuts. We can prove that *MILP2* (with cuts) constitutes the better formulation of our problem and it can be taken as reference in a future work. The strategy of sub tours elimination

TABLE II. NUMERICAL RESULTS

Instance	ILP 1				ILP 2				MILP 1				MILP 2			
	f	*CPU*	*UB1*	*%*	*f*	*CPU*	*UB2*	*%*	*f*	*CPU*	*UB3*	*%*	*f*	*CPU*	*UB4*	*%*
inst001	544,78	0,75	672,52	19,0	544,78	1,47	670,78	18,8	544,78	0,22	662,51	17,8	544,78	0,27	637,79	14,58
inst002	844,77	6,84	979,69	13,8	844,77	6,42	979,69	13,8	844,77	0,66	957,59	11,8	844,77	0,53	957,59	11,8
inst003	1780,4	19,2	2312,05	23,0	1780,43	11,13	2311,61	23,0	1780,43	2,48	2323,86	23,4	1780,43	1,72	2186,67	18,6
inst004	607,73	760	700,05	13,2	607,73	180	699,575	13,1	607,73	7,63	689,1	11,8	607,73	1,95	684,71	11,2
inst005	1932,6	259	2750,85	29,7	1932,59	608,9	2750,85	29,7	1932,04	101,5	2710,68	28,7	1932,04	7,5	2710,12	28,7
inst006	2123,9	333	2553,53	16,8	2123,94		2553,53	16,8	2123,94	42,16	2472,93	14,1	2123,94	19,1	2444,65	13,1
inst007	2310,9	3532	2969,87	22,2	2310,94	3032	2969,87	22,2	_	_	2931,8	_	2310,94	108	2927,51	21,1
inst008	_	_	2418,52	_	_	_	2418,53	_	_	_	2351,2	_	2226,08	113	2346,48	5,1
inst009	_	_	3729,85	_	_	_	3729,85	_	_	_	3604,08	_	_	_	3602,7	_
inst010	_	_	3133,43	_	_	_	3133,44	_	_	_	3043,2	_	_	_	3039,55	_
inst011	_	_	4867,48	_	_	_	4867,48	_	_	_	4761,82	_	3546,04	1665	4720,95	24,9
inst012	_	_	5694,49	_	_	_	5694,49	_	_	_	5610,11	_	_	_	5609,83	_
inst013	_	_	6208,70	_	_	_	6208,7	_	_	_	6100,37	_	_	_	6060,13	_
inst014	_	_	5577,53	_	_	_	5577,53	_	_	_	5539,02	_	_	_	5531,54	_
inst015	_	_	3209,90	_	_	_	3209,9	_	_	_	3166,23	_	_	_	3160,23	_
inst016	_	_	6892,66	_	_	_	6892,66	_	_	_	6758,41	_	_	_	6741,72	_
inst017	_	_	2793,37	_	_	_	2793,37	_	_	_	3938,65	_	_	_	3913,93	_
inst018	_	_	3288,96	_	_	_	3288,81	_	_	_	2697,68	_	_	_	2686,73	_
inst019	_	_	4416,35	_	_	_	4415,71	_	_	_	3267,14	_	_	_	3258,2	_
inst020	_	_	4416,94	_	_	_	4415,71	_	_	_	4372 ,4	_	_	_	4347,68	_

TABLE III: COMPARISON BETWEEN THE UB

Instance	(%*)1	(%*)2	(%*)3	(%*)4
inst001	0	-0,259	-1,488	-5,164
inst002	0	0,000	-2,256	-2,256
inst003	0	-0,019	0,511	-5,423
inst004	0	-0,068	-1,564	-2,191
inst005	0	0,000	-1,460	-1,481
inst006	0	0,000	-3,156	-4,264
inst007	0	0,000	-1,282	-1,426
inst008	0	0,000	-2,784	-2,979
inst009	0	0,000	-3,372	-3,409
inst010	0	0,000	-2,880	-2,996
inst011	0	0,000	-2,171	-3,010
inst012	0	0,000	-1,482	-1,487
inst013	0	0,000	-1,745	-2,393
inst014	0	0,000	-0,691	-0,825
inst015	0	0,000	-1,360	-1,547
inst016	0	0,000	-1,948	-2,190
inst017	0	0,000	-32,264	-32,539
inst018	0	0,000	-3,426	-3,818
inst019	0	-0,004	-0,663	-0,935
inst020	0	-0,015	-0,995	-1,555

constraints used in *0-1 ILP* does not give good results to our problem. But, it represents a new idea to the VRP formulation and to the sub tours elimination constraints which can be tested for other problems.

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