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Integration of nonlinear damping with instantaneous harmonic control (IHC) for robust suppression of tonal disturbances

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Abstract—**This paper proposes a novel approach based on integrating the instantaneous harmonic control (IHC) with the cubic nonlinear damping to resolve the problem with the IHC robustness which relies on accurate knowledge of disturbance frequency. To assess the performance of this novel approach, numerical simulation studies are conducted on a single degree of freedom (sdof) vibration isolation system. The results clearly demonstrate the effectiveness of this novel approach in improving the IHC robustness. This study is of great significance as it provides a basis for the design and practical application of the cubic nonlinear damping integrated with the IHC to address vibration control problems in a wide range of engineering systems.**

I. INTRODUCTION

Working machineries such as propulsion engines, diesel generators and air compressors often generate vibration of periodic nature to supporting structures which can cause problems in a wide range of applications. In the marine sector, for example, this can lead to the excitation of ship hulls resulting in passenger discomfort in commercial vessels and producing acoustic noise which is a detection hazard for naval vessels [1]. As a result of such problems, the harmonic control algorithm is often employed in active vibration isolation devices to minimize the adverse effect of vibration [2]. The basic idea of the harmonic control is to make the magnitude the control signal match that of the disturbance whilst the phase of the control signal oppose the that of the disturbance signal. Consequently, the summation of the control and disturbance signals is driven to zero. In early days, the harmonic control was mainly applied in helicopter industry to control the periodic vibration generated by helicopter rotor blades induced by the aerodynamics excitation [3-4]. However, over the last decade, researchers started to deploy this control algorithm for the attenuation of periodic vibration caused by working machineries [5-6]. In its standard form, the harmonic control operates in the frequency domain using a steady state approach. Following each corrective control action, the algorithm waits for the transients associated with system dynamics to die out before executing the next update [7]. Obviously, this leads to a long convergence delay and increase the length of the algorithm which is unacceptable in practical situations. To avoid such problems, it is common in industrial application not to wait for reaching the steady state and update at the same rate as the fundamental controller sampling frequency. This is known as instantaneous harmonic control (IHC). The IHC have found widespread applications showing a remarkable vibration isolation and faster convergent rate [8-10]. However, the problem with the IHC is that it relies on the assumption that the disturbance frequency is known and time invariant. In practice, this assumption is unlikely to be valid, since it is well known that the frequency of the disturbance constantly varies with time [11-12]. In addition, it is well known that the IHC amplifies the vibration near the frequency where the IHC is designed to tackle. Therefore, one can conclude that if the IHC, for example, is designed to tackle a disturbance at the system's resonant frequency but the frequency of the disturbance suddenly has a slight change, then a large undesirable level of vibration can be the outcome. This result in our opinion means that the IHC is not robust enough because it cannot cope with uncertainties in disturbance frequencies.

To illustrate clearly the problem with the IHC robustness, consider a two degrees of freedom (2dof) system with resonance frequencies Ω_1 and Ω_2 subjected to two tone disturbance input frequencies ω_1 and ω_2, where $\omega_1 = \Omega_1$ and $\omega_2 = \Omega_2$. The IHC is then designed to tackle the two frequencies ω_1 and ω_2. The typical force transmissibility of this system under the IHC is shown in figure 1. As it can be seen from figure 1 that the IHC is effective in reducing the force transmissibility when $\omega_1 = \Omega_1$ and $\omega_2 = \Omega_2$, but this was achieved at the expense of increasing the force transmissibility at frequencies near the two resonant frequencies Ω_1 and Ω_2. As mentioned earlier, disturbance frequencies are likely to change with time. Hence, if the frequency of disturbance becomes Ω_3 or Ω_4, then, as shown in figure 1, the system will experience a large undesirable level of vibration.

To improve the IHC robustness, researchers often integrate the linear viscous damping with the IHC and apply this combined controller into vibration isolation systems [13]. However, it is well known that the linear viscous damping is not effective over all frequency regions of a vibrating system. That is, the addition of the linear viscous damping to the IHC reduces the force transmission over the resonant

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frequency region but increases the force transmission over higher frequency regions [14-15].

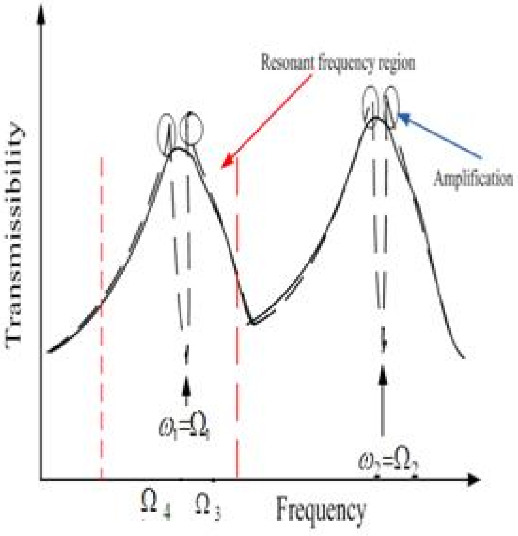

Figure 1: The force transmissibility of a 2dof system
Solid: Open loop control; **Dashed:** IHC

Recently, the authors introduced a cubic nonlinear damping into both sdof (single degree of freedom) and mdof (mutli degrees of freedom) vibrating systems and theoretically proved that the cubic nonlinear damping can reduce the force transmissibility over the system resonant frequency regions, whilst keeping the transmissibility over other frequency regions almost unaffected [16-17]. Given the beneficial effects of the cubic nonlinear viscous damping revealed in [16-17] and an intuitive approach to the IHC robustness problem is to exploit this beneficial effect by integrating the cubic nonlinear viscous damping with the IHC. This is the basic idea of the studies in this paper. To assess the performance of this novel approach, numerical simulation studies are conducted on sdof vibration isolation system.

This paper is organised as follows. Section II briefly introduces the basic principal of the IHC. Section III discusses the limitation of the widely used solution (linear viscous damping integrated with IHC) to the robustness problem of the IHC in vibration control applications. Section IV describes the idea of integrating cubic nonlinear damping with IHC to solve the IHC robustness problem. Numerical simulation results are also presented to demonstrate the effectiveness of this novel idea. Finally, Section V concludes the paper

II. INSTANTANEOUS HARMONIC CONTROL (IHC)

The standard harmonic control is based on the following equation:

$$F_{OUT}(j\omega) = G_c(j\omega)U(j\omega) + G_d(j\omega)F_{IN}(j\omega) \quad (1)$$

where $G_c(j\omega)$ and $G_d(j\omega)$ are the frequency response functions of control and the disturbance channels of the system respectively. $F_{IN}(j\omega)$ and $F_{OUT}(j\omega)$ are spectra of input and output forces $f_{IN}(t)$ and $f_{OUT}(t)$ respectively. $U(j\omega)$ is the spectrum of an active control force $u(t)$. If a system is subject to a harmonic disturbance force with frequency Ω, then the aim of the harmonic control algorithm is to determine the control signal spectrum $U(j\Omega)$ such that $F_{OUT}(j\Omega) = 0$. The solution to the problem can easily be obtained using the following simple feed-forward control law:

$$U(j\Omega) = G_c^{-1}(j\Omega)G_d(j\Omega)F_{IN}(j\Omega) \quad (2)$$

Unfortunately, in many practical situations, it is not always possible to implement this algorithm since $G_d(j\Omega)$ and $F_{IN}(j\Omega)$ are usually unknown and $G_c(j\Omega)$ cannot be estimated accurately. As a result, the following alternative iterative solution is often employed [12]:

$$u_p(j\Omega) = \alpha u_{p-1}(j\Omega) - \beta \tilde{G}_c^{-1}(j\Omega) f_{OUT,p}(j\Omega) \quad (3)$$

to obtain to approximate the magnitude and phase of the control signal. In equation (3), p represents an iteration index, $\beta>0$ is a learning gain, $0 < \alpha \le 1$ is the relaxation gain and $\tilde{G}_c^{-1}(j\Omega)$ is the inverse of the estimated control channel $\tilde{G}_c(j\Omega)$ evaluated at frequency Ω, and $f_{OUT,p}(j\Omega)$ is the spectra of the system output force evaluated at frequency Ω in the p^{th} iteration.

As mentioned earlier, in its standard form the harmonic control approach described by equation (3) waits for the transients associated with system dynamics to die out before executing the next update which can lead to a long convergence delay and adds to the coding overhead [7]. To avoid such problems, the IHC which update at the same rate as the fundamental controller sampling frequency is often used. The starting point of this IHC is the assumption of discrete time representation of the plant:

$$f_{OUT}(t) = \tilde{G}_c(q)u(t) + \tilde{G}_d(q) f_{IN}(t) \tag{4}$$

where q^{-1} is the backward shift operator. The first step in the IHC is to approximate the Fourier coefficient with its instantaneous value using the equation [7]:

$$\tilde{f}_{OUT}(e^{j\Omega t}) = f_{OUT}(t) e^{-j\Omega t} \tag{5}$$

where $e^{-j\Omega t}$ is a complex reference signal having the same frequency as the disturbance force and $\tilde{f}_{OUT}(e^{j\Omega t})$ an estimate of the Fourier coefficient of $f_{OUT}(t)$ evaluated at frequency Ω . Based on this instantaneous estimate, the control signal is updated using the formula [9]:

$$\tilde{u}_p(e^{j\Omega t}) = \alpha \tilde{u}_{p-1}(e^{j\Omega t}) - \beta \tilde{G}_c(e^{j\Omega T_s})^{-1} \tilde{f}_{out,p}(e^{j\Omega t}) \tag{6}$$

where $\tilde{G}_c(e^{j\Omega T_s})^{-1}$ is the inverse of $\tilde{G}_c(q)$ evaluated at frequency Ω, $\tilde{u}(e^{j\Omega t})$ is an estimate of the Fourier coefficient of $u(t)$ evaluated at frequency Ω.

Equation (6) is equivalent to equation (3), but in equation (6) the control signal $\tilde{u}(e^{j\Omega t})$ is updated at every step of the sampling time interval T_s.

The final step of the algorithm is to transform the control signal back to the time domain using the instantaneous inverse Fourier formula [8]:

$$u_{IHC}(t) = \underbrace{\tilde{u}_p(e^{j\Omega t})e^{j\Omega t}}_{u_3(t)} + \underbrace{\tilde{u}_p(e^{-j\Omega t})e^{-j\Omega t}}_{u_4(t)} = 2\mathrm{Re}[u_3(t)] \tag{7}$$

where $u_{IHC}(t)$ is the IHC force.

III. THE PROBLEM WITH INTEGRATION OF LINEAR DAMPING WITH IHC

The aim of this section is to demonstrate the limitation of the widely used solution (integration of linear damping with IHC) to the robustness problem of the IHC in vibration control applications. Consider the sdof vibration isolation in figure 2.

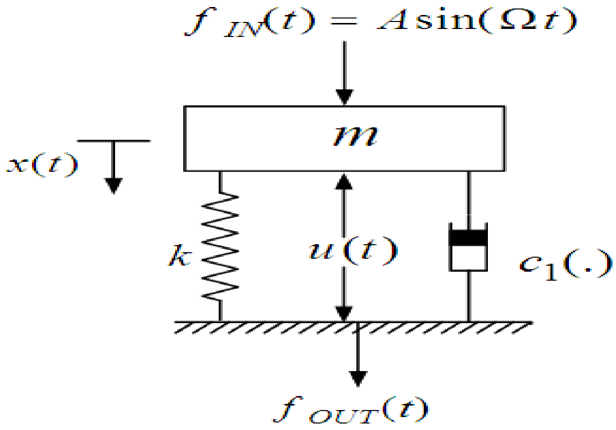

Figure 2 An sdof vibration isolation system

This system consists of mass m supported on a spring with stiffness k and a parallel damper which consists of linear viscous damping characteristic parameter c_1. $u(t)$ is an active control force . $f_{IN}(t)$ is a harmonic disturbance force with amplitude A and frequency Ω . When the system is subjected to $f_{IN}(t)$, this result in a displacement of the mass $x(t)$ and an output force $f_{OUT}(t)$ transmitted to the base.

The equation of motion of the system can readily be obtained as:

$$\begin{cases} m\ddot{x} = -kx(t) - u(t) - c_1\dot{x}(t) + f_{IN}(t) \\ f_{OUT}(t) = kx(t) + u(t) + c_1\dot{x}(t) \end{cases} \tag{8}$$

The IHC combined with linear viscous damping is normally implemented through the active control force as shown below:

$$u(t) = r_2\dot{x}(t) + u_{IHC}(t) \tag{9}$$

where $u_{IHC}(t)$ denotes the IHC force implemented using equations (5)-(7) and $r_2\dot{x}(t)$ is a linear velocity feedback with linear damping parameter r_2.

The objective of introducing controller (9) in this case is to reduce the force transmitted to the base $f_{OUT}(t)$ at the resonant frequency and, at the same time, to remove the amplification effect induced by the conventional IHC over the frequencies near the system resonant frequency as illustrated in figure 1.

Figure 3 shows the simulation result of force transmission of system (8) with k =16000N/m, m =240Kg, and $c_1 = 296$ N.s/m, and in the case where a linear damping with parameter $r_2 = 600$ N/m.s is integrated with IHC to tackle the disturbance at the system resonant frequency 8.1rad/sec.

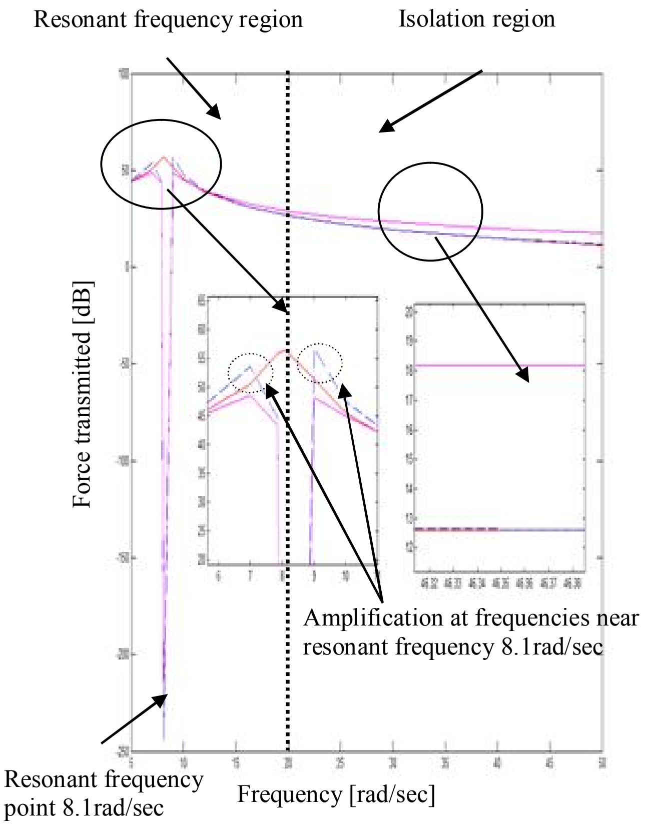

Figure 3: Force transmission to the base of system (8) where controller (9) is applied under different configurations. **Red Solid**: $r_3 = 0N.s/m$ and $u_{IHC}(t) = 0$; **Solid Pink:** $r_2 = 600N.s/m$ and $u_{IHC8.1}(t) \neq 0$; **Blue Dashed**: $r_2 = 0N.s/m$ and $u_{IHC8.1}(t) \neq 0$.

Note in figure 3, $u_{IHC,8.1}(t) \neq 0$ means the IHC force is employed to tackle the disturbance at resonant frequency 8.1rd/sc of system (8), whereas $u_{IHC}(t) = 0$ indicates that the IHC force is switched off. It can be observed from figure 3 that using the IHC alone and using the IHC integrated with the linear damping can achieve almost the same reduction of the force transmission at resonant frequency point 8.1rd/sc of the system. In addition, while the IHC on its own amplifies the force transmission at frequencies surrounding the resonant frequency 8.1rad/sec, the IHC integrated with the linear damping provides a significant reduction of the force over the whole resonant frequency region. However, the IHC integrated with the linear damping achieves this good result at the expense of increasing the force transmission over the isolation area. To resolve this problem motivates the studies in this paper.

IV. THE INTEGRATION OF CUBIC NONLINEAR DAMPING WITH IHC

Given the beneficial effects of cubic nonlinear damping revealed in [16-17], an intuitive approach to tacking the problem described in Section III is to integrate a cubic nonlinear damping with the IHC and implementing the resulting control for system (8). The controller which integrates a cubic nonlinear damping with the IHC is given below:

$$u(t) = r_3 \dot{x}^3(t) + u_{IHC}(t) \tag{10}$$

where r_3 is the cubic nonlinear damping characteristic parameter.

In order to demonstrate the effectiveness of this idea, three simulation studies were conducted for a specific model (8) with k=16000N/m, c_1 = 296N.s/m and m =240 Kg. In these simulation studies, the system was excited by a sinusoidal force with a 100N magnitude and over the following harmonic frequencies:

5, 7, 7.8, 8.1, 9, 10, 12, 16, 20, 25, 30, 40, 50rd/sc.

respectively.

The controller (10) with three different parameter settings was applied to control the force transmission to the system support base in order to investigate the system performance in the different cases. In the first case, $r_3 = 0N.s/m^3$ and $u_{IHC}(t) = 0$; in the second case, $r_3 = 0N.s/m^3$ and $u_{IHC,8.1}(t) \neq 0$; and in the third case, $u_{IHC,8.1}(t) \neq 0$ and $r_3 = 35000N.s/m^3$.

Figure 4 shows the simulation results. From these results, it can be observed that the integration of a cubic nonlinear damping with the IHC can not only reduce the amplification of the force transmission induced by the IHC around the resonant frequency of the system, but also keeps the force transmission almost unchanged over the isolation frequency range. This is exactly the ideal performance required for the control.

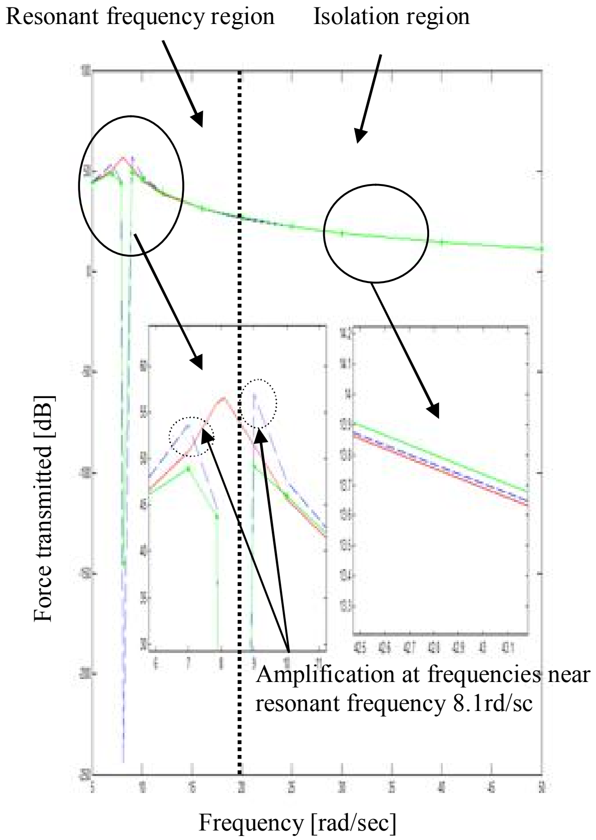

Figure 4 Force transmission to the base of system (8) under three different parameter settings of controller (10): **Red Solid**: $r_3=0N.s/m^3$ and $u_{IHC}(t)=0$; **Green Crossed:** $r_3=35000N.s/m^3$ and $u_{IHC,8.1}(t)\neq 0$; **Blue Dashed**: $r_3=0N.s/m^3$ and $u_{IHC,8.1}(t)\neq 0$.

In order to further verify the advantage of integrating a cubic nonlinear damping with the IHC, two additional simulation studies under the same disturbance condition were conducted. First, controller (9) with $r_2=500N.s/m$ and $u_{IHC,8.1}(t)\neq 0$ was applied to system (8). This is to compare the system performance under the control scheme of IHC integrated with a linear damping with the system performance under the control scheme of IHC integrated with an equivalent nonlinear damping. Secondly, a cubic nonlinear damping was solely used with $r_3=35000N.s/m^3$ and $u_{IHC}(t)=0$ in (10). This is to compare the system performance under the control scheme of IHC integrated with a nonlinear damping with the system performance under a cubic nonlinear damping only control scheme. The simulation results and corresponding comparisons are shown in figures 5 and 6, respectively

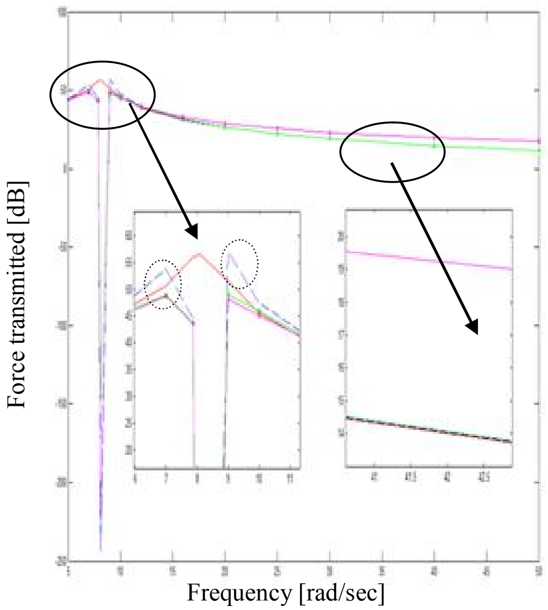

Figure 5 Force transmission to the base of system (8) under three different parameter settings of controller (10) and controller (9) respectively: **Red Solid**: $r_3=0N.s/m^3$ and $u_{IHC}(t)=0$; **Green Crossed** $r_3=35000N.s/m^3$ and $u_{IHC,8.1}(t)\neq 0$; **Blue Dashed**: $r_3=0N.s/m^3$ and $u_{IHC,8.1}(t)\neq 0$; **Pink Squared**: $r_2=500N.s/m$ and $u_{IHC,8.1}(t)\neq 0$.

It can be observed from figure 5 that although the integration of IHC with a nonlinear or a linear damping achieves the same performance over the resonant frequency region, a much better performance can be achieved by the IHC integrated with a nonlinear damping over the isolation frequency region. These results are consistent with the theoretical conclusions reached by the authors in [16-17]. On the other hand, figure 6 shows that apart from at the resonant frequency of 8.1rd/sc, the system force transmission under the IHC combined with a nonlinear damping is about the same as the force transmission when the system is under the control of a nonlinear damping only scheme. This indicates that the novel approach which is based on the integration of IHC with a cubic nonlinear damping effectively merges the advantages of cubic nonlinear damping and the IHC, and achieves an ideal system performance that has never been reached before.

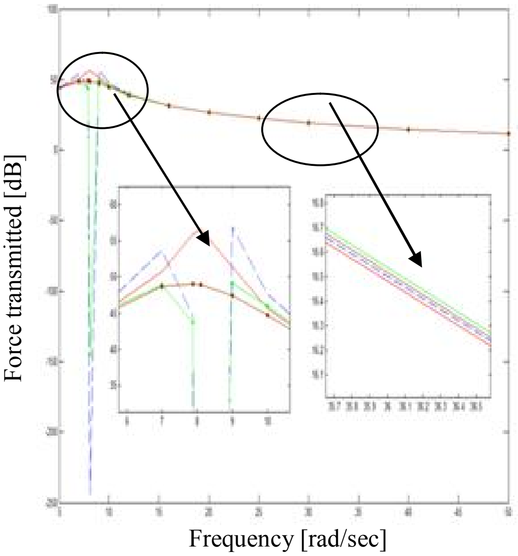

Figure 6 Force transmission to the base of system (3.1) under parameter settings of controller (4.2). **Red Solid**: $r_3=0N.s/m^3$ and $u_{IHC}(t)=0$; **Green Crossed**: $r_3=35000N.s/m^3$ and $u_{IHC,8.1}(t)\neq 0$; **Brown Dotted:** $r_3=35000N.s/m^3$ and $u_{IHC}(t)=0$.

It is important to emphasize that the IHC algorithm described by equations (5)-(7) was originally developed for linear systems. Although adding the nonlinear viscous damping to the IHC algorithm was a natural step to take given the beneficial effects of nonlinear damping revealed by the authors in [16-17] and this intuitive approach has lead to a much superior results compared with the traditional IHC technique, there is still no theoretical analysis and the beneficial effects of this novel approach was only demonstrated through simulation studies. A rigorous theory is yet to be developed in future studies based on these very promising results.

V. CONCLUSIONS

In this paper, a novel approach based on the integration of IHC with cubic nonlinear damping has been proposed to resolve the problem with the robustness of conventional IHC which relies on an accurate knowledge of disturbance frequencies. This idea of using a cubic nonlinear damping is drawn from the previous work carried out by the authors in which they theoretically demonstrated the beneficial effect of cubic nonlinear damping on vibration control systems. To assess the performance of this novel approach, numerical simulation studies have been conducted on sdof vibration isolation system. The results clearly demonstrate the effectiveness of this new approach. The results of this study is of great significance as it provides a basis for the design and practical application of the novel integration of IHC with cubic nonlinear damping to address vibration control problems in a wide range of engineering systems. It is also important to emphasis that the authors have experimentally demonstrated the effectiveness of combining cubic nonlinear damping with IHC on a vibration isolation rig. The results will be presented in a future paper.

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