Fuzzy Lyapunov Approach for Robust Stabilization of Fuzzy Systems with Time Varying Delay

Chedia Latrach, Mourad Kchaou, Ahmed El Hajjaji and Abdelhamid Rabhi

***Abstract*— This paper investigates the problem of robust control design for a class of uncertain nonlinear systems with time varying delay. By evaluating the derivative of a new fuzyy weighting-dependent Lyapunov-Krasovskii functional (FLKF), less conservative convex conditions derived in terms of Linear Matrix Inequalities (LMIs),are established.Two new results are presented. The first one is based on FLKF and the so-called the free weighting matrices and the second uses the bounding techniques. Finally, to compare the effectiveness of the proposed methods in term of the conservatism reduction, the simulation example is presented.**

Keywords: Takagi-Sugeno(TS), Stability, Stabilization, Linear Matrix Inequalities (LMIs), Fuzzy Lyapunov-Krasovskii functions(LKF).

I. Introduction

In the last two decades, the stability analysis and control design of nonlinear systems described by Takagi-Sugeno (T-S) fuzzy model with time delay have been extensively studied ([1], [5], [6], [7], [11], [12], [20]. Two main reasons are contributed to this fact : 1) Ability of T-S fuzzy models [19] to approximate large class of nonlinear systems; 2) The existence of delays in many industrial processes such as transportation systems, communication systems, chemical processing systems, environmental systems and power systems ...) and its consequences on the stability and the performance degradation [18].

Stability analysis and control design for T-S systems have been widely and systematically handled in the literature via Linear Matrix Inequalities (LMIs) approach (see [4], [15]). In this framework, several strategies have been used to overcome mathematical and numerical difficulties, promoting less-conservative conditions. The standard quadratic Lyapunov approach is the most popular combing with the use of slack matrix variables into the LMI formulation [15], [16]. As an interesting alternative type of Lyapunov function, the fuzzy weighting-dependent Lyapunov function (FWLF) has been addressed for continuous-time fuzzy systems in [2], [9], [10], [17], and for discrete-time fuzzy system in [13], [14].

In the context of stabilization problem of delayed systems, some advanced techniques, aiming at reducing the conservatism, have been proposed. One approach is to take an appropriate and equivalent model transformation for the original systems. Under such transformations, the conservatism is mainly due to the bounding of the cross product terms which appear in the derivative of the Lyapunov-Krasovskii functional. To reduce the number of such terms and employ tighter bounds on them would certainly lead to better results. The free-weighting matrix methods, which are based on Leibniz-Newton formula, have been proposed to efficiently improve the delay-dependent results based on the bounding techniques on some cross product terms are no longer involved.

In this paper, new less-conservative conditions for stabilization of TS fuzzy systems subject to uncertainties and time delay using (FLKF) and the so-called free weighting matrix method, will be proposed. This paper is organized as follows: a system description and preliminaries are presented in Section 2. Section 3 contains the main results where the new delay dependent stabilization conditions formulated in LMI's terms are established. In Section 4, a numerical example is given to demonstrate the effectiveness and the benefits of the proposed method. A conclusion is drawn in Section 5.

Notations:$W + W^T$ is denoted as $Sym(W)$.

The symbol (*) within a matrix represents the symmetric entries.

II. System description and preliminaries

Consider a nonlinear system with state-delay which could be represented by a time-delay TS fuzzy model described by Plant Rule i(i=1,2,....,r):

IF $\theta_1(t)$ is N_{i1} and... $\theta_p(t)$ is N_{ip} THEN

$$\begin{aligned}\dot{x}(t) &= (A_i + \Delta A_i)x(t) + (A_{1i} + \Delta A_{1i})x(t - \sigma(t)) \\ &\quad + (B_i + \Delta B_i)u(t)\end{aligned} \tag{1}$$

Where $\theta_j(t)$ and $N_{ij}(i = 1, ..., r, j = 1, ..., p)$ are, respectively, the premise variables and the fuzzy sets ; $x(t) \in R^n$ is the state ; $u(t) \in R^m$ is the control input ; r is the number of IF-THEN rules; the time-delay, $\sigma(t)$, is a time-varying continuous function that satisfies,

$$0 \leq \sigma(t) \leq \bar{\sigma}, \qquad \dot{\sigma}(t) \leq \beta \tag{2}$$

The parametric uncertainties ΔA_i, ΔA_{1i}, ΔB_i are time-varying matrices that are defined as follows: $\Delta A_i = D_A F_i(t) E_{Ai}$, $\Delta A_{1i} = D_{A1} F_i(t) E_{A1i}$, $\Delta B_i = D_B F_i(t) E_{Bi}$.

Let $\bar{A}_i = A_i + \Delta A_i$; $\bar{A}_{1i} = A_{1i} + \Delta A_{1i}$; $\bar{B}_i = B_i + \Delta B_i$.

where D_A, D_{A1} , D_B, E_{Ai}, E_{A1i}, and E_{Bi}, are constant matrices, and $F_i(t)$ is an unknown real time-varying matrix

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with Lebesgue measured elements bounded by $F_i(t)^T F_i(t) \leq I$. By using the commonly used center-average defuzzifier, product inference and singleton fuzzifier, the T-S fuzzy systems can be inferred as

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(\theta(t))\Big\{\bar{A}_i x(t) + \bar{A}_{1i} x(t-\sigma(t)) + \bar{B}_i u(t)\Big\} \tag{3}$$

where $\theta(t) = [\theta_1(t),...., \theta_p(t)]$ and $\upsilon_i(\theta(t) = \prod_{j=1}^{p} N_{ij}(\theta_j(t))$ are the membership functions of the system with respect to ith plan rule. Denote

$$h_i(\theta(t)) = \upsilon_i(\theta(t))/\sum_{i=1}^{r} \upsilon_i(\theta(t)). \tag{4}$$

$h_i(\theta(t))$ is the averaging weight for each rule, representing the normalized membership degrees, and satisfies

$$h_i(\theta(t)) \geq 0, \quad \text{and} \quad \sum_{i=1}^{r} h_i(\theta(t)) = 1, (i = 1, 2, ..., r) \tag{5}$$

In order to obtain the main results in this paper, the following lemma are needed:

Lemma 2.1: (Wang et al.[3]) Given matrices $D, E, F(t)$ with compatible dimensions and F(t) satisfying $F(t)^T F(t) \leq I$. Then,the following inequalities hold for any $\epsilon_1 > 0$: $DF(t)E + E^T F(t)^T D^T \leq \epsilon_1 DD^T + \epsilon_1^{-1} E^T E$

In this paper, for simplicity, we set $x_\sigma = x(t - \sigma(t))$, $x_{\bar{\sigma}} = x(t - \bar{\sigma})$, and $\zeta(t) = \begin{bmatrix} x^T(t) & x_\sigma^T & x_{\bar{\sigma}}^T \end{bmatrix}^T$.

III. Main Results

We consider the problem of delay-dependent stabilization of system (3). Based on the Parallel Distributed Compensation (PDC), the following fuzzy control law is employed to deal with the problem of stabilization via state feedback.

Controller Rule i:

IF $\theta_1(t)$ is N_{i1} and... $\theta_p(t)$ is N_{ip} THEN

$$u(t) = K_i x(t), i = 1, 2, ..., r. \tag{6}$$

Hence, the overall fuzzy control law is represented by

$$u(t) = \sum_{i=1}^{r} h_i(\theta(t)) K_i x(t), \tag{7}$$

where $K_i (i = 1, 2, ..., r)$ are the local control gain matrices. In the sequel, for brevity we use h_i to denote $h_i(\theta(t))$. Consider system (3) associated with the control law (7), then the resulting closed-loop system can be expressed as follows:

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \Big\{\hat{A}_{ij} x(t) + \bar{A}_{1i} x(t - \sigma(t))\Big\} \tag{8}$$

Where $\hat{A}_{ij} = \bar{A}_i + \bar{B}_i K_j$. In this part we consider that uncertainties ΔA_i , ΔB_i and ΔA_{1i} are zeros.

Theorem 3.1: System (8) is asymptotically stable, if there exist some symmetric positive-definite matrices $\bar{P}_i > 0$, $\bar{H} > 0$, and $\bar{S} > 0$, any matrices Θ_i, Y_i, $i = 1, \cdots, r$, U, $\bar{X}_{pq}$, $1 \leq q \leq p \leq 3$,$\bar{Q}_p$, $\bar{W}_p$, $p = 1, \cdots, 3$ satisfying the following set of LMIs:

$$P_l + \Theta_i > 0 \quad (i, l = 1, 2, ..., r) \tag{9}$$

$$\Lambda_{ii} < 0 \tag{10}$$

$$\Lambda_{ij} + \Lambda_{ji} < 0 \quad (i < j = 1, 2..., r) \tag{11}$$

$$\Xi_1 = \begin{bmatrix} X_{11} & * & * & * \\ X_{21} & X_{22} & * & * \\ X_{31} & X_{32} & X_{33} & * \\ Q_1^T & Q_2^T & Q_3^T & H \end{bmatrix} \geq 0 \tag{12}$$

$$\Xi_2 = \begin{bmatrix} X_{11} & * & * & * \\ X_{21} & X_{22} & * & * \\ X_{31} & X_{32} & X_{33} & * \\ W_1^T & W_2^T & W_3^T & H \end{bmatrix} \geq 0 \tag{13}$$

where

$$\Lambda_{ij} = \begin{bmatrix} \Gamma_{1ij} & * & * & * \\ \Gamma_{2ij} & \Gamma_{3i} & * & * \\ \Gamma_{4ij} & \Gamma_{5ij} & \Gamma_{6ij} & * \\ \Gamma_{7ij} & \mu_2(A_{1i}U) - \mu_1 U^T & 0 & \Gamma_8 \end{bmatrix}$$

where

$$\begin{aligned}
\Gamma_{1ij} &= Sym(A_i U + B_i Y_j) + \bar{P}_{\Phi,i} + \bar{S} + Sym(\bar{Q}_1) + \bar{\sigma}\bar{X}_{11} \\
\Gamma_{2ij} &= \mu_1(A_i U + B_i Y_j) + U^T A_{1i}^T + \bar{\sigma}\bar{X}_{21} + \bar{Q}_2 - \bar{Q}_1^T \\
&\quad + \bar{W}_1^T \\
\Gamma_{3i} &= \mu_1(A_{1i}U + U^T A_{1i}^T) - (1-\beta)\bar{S} + \bar{\sigma}\bar{X}_{22} \\
&\quad - Sym(\bar{Q}_2) + Sym(\bar{W}_2) \\
\Gamma_{4ij} &= -\bar{W}_1^T + \bar{Q}_3 + \bar{\sigma}\bar{X}_{31} \\
\Gamma_{5ij} &= -\bar{W}_2^T - \bar{Q}_3 + \bar{W}_3 + \bar{\sigma}\bar{X}_{32} \\
\Gamma_{6ij} &= -Sym(\bar{W}_3) + \bar{\sigma}\bar{X}_{33} \\
\Gamma_{7ij} &= \mu_2(A_i U + B_i Y_j) - U^T + \bar{P}_i^T \\
\Gamma_8 &= -\mu_2 Sym(U) + \bar{\sigma}\bar{H} \\
P_{\Phi,i} &= \sum_{l=1}^{r} \Phi_l(P_l + \Theta_i)
\end{aligned}$$

Proof: Choose Lyapunov function as follows:

$$\begin{aligned}
V(x(t)) &= V_1(x(t)) + V_2(x(t)) + V_3(x(t)) \\
V_1(x(t)) &= x^T(t)P(t)x(t) \\
V_2(x(t)) &= \int_{t-\sigma(t)}^{t} x^T(s)Sx(s)\,\mathrm{d}s \\
V_3(x(t)) &= \int_{-\bar{\sigma}}^{0}\int_{t+\alpha}^{t} \dot{x}^T(s)H\dot{x}(s)\,\mathrm{d}s\mathrm{d}\alpha
\end{aligned} \tag{14}$$

where $P(t) = \sum_{i=1}^{r} h_i P_i, \quad i = 1, 2, \cdots, r$, is a fuzzy weighting-dependent Lyapunov function. Taking the time-derivative of $V(x(t))$ along the trajectories of (8) yields

$$\dot{V}_1(x(t)) = 2\dot{x}^T(t)P(t)x(t) + x^T(t)\dot{P}(t)x(t) \tag{15}$$

$$\begin{aligned}\dot{V}_2(x(t)) &= x^T(t)Sx(t) - (1-\dot{\sigma}(t))x_\sigma^T S x_\sigma \\ &\le x^T(t)Sx(t) - (1-\beta)x_\sigma^T S x_\sigma\end{aligned} \tag{16}$$

$$\begin{aligned}\dot{V}_3(x(t)) &= \bar{\sigma}\dot{x}^T(t)H\dot{x}(t) - \int_{t-\bar{\sigma}}^{t} \dot{x}^T(\alpha)H\dot{x}(\alpha)\,\mathrm{d}\alpha \\ &= \bar{\sigma}\dot{x}^T(t)H\dot{x}(t) - \int_{t-\bar{\sigma}}^{t-\sigma(t)} \dot{x}^T(\alpha)H\dot{x}(\alpha)\,\mathrm{d}\alpha \\ &- \int_{t-\sigma(t)}^{t} \dot{x}^T(\alpha)H\dot{x}(\alpha)\,\mathrm{d}\alpha\end{aligned} \tag{17}$$

(18)

The derivative of $P(t)$ provides information about time derivative of membership functions. Then, we assume that

$$|\dot{h}_l(\theta(t))| \le \Phi_l \tag{19}$$

where $\Phi_l \ge 0 (l = 1, 2...r)$.
From the properties of the membership functions in (5) it follows that $\sum_{l=1}^{r} \dot{h}_l(\theta(t)) = 0, \quad \forall\theta(t)$ and thus the following equation holds:

$$\sum_{i=1}^{r}\sum_{l=1}^{r} h_i\dot{h}_l x^T(t)\Theta_i x(t) = 0 \tag{20}$$

where Θ_i are slack matrices with appropriate dimensions and $\Theta_i = \Theta_i^T$.
Assuming that the assumption (9) and (19) hold, it is easy to have

$$\begin{aligned}\dot{P}(t) &= \sum_{l=1}^{r}\dot{h}_l(t)P_l = \sum_{i=1}^{r}(h_i(t))\sum_{l=1}^{r}\dot{h}_l(t)P_l \\ &\le \sum_{i=1}^{r}\sum_{l=1}^{r} h_i(t)\Phi_l(P_l+\Theta_i)\end{aligned} \tag{21}$$

Finally, we get

$$\dot{V}_1(x(t)) \le \sum_{i=1}^{r} h_i\Big\{2x^T(t)P_i\dot{x}(t) + \sum_{l=1}^{r}\Phi_l x^T(t)(P_l+\Theta_i)x(t)\Big\} \tag{22}$$

From (8) we may construct for any appropriately dimensional matrices $M_i, i = 1, 2, 3$ the following equation:

$$\begin{aligned}2\Big[x^T(t)M_1 + x_\sigma^T M_2 + \dot{x}(t)^T M_3\Big]\Big[&-\dot{x}(t) \\ &+ \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j(\hat{A}_{ij}x(t) + \bar{A}_{1i}x_\sigma)\Big] = 0\end{aligned} \tag{23}$$

Let us define $\xi^T(t) = \begin{bmatrix} x^T(t) & x_\sigma^T & \dot{x}^T(t) & x_{\bar{\sigma}}^T \end{bmatrix}$. Then, the following equations hold for any matrices Q and W with appropriate dimensions:

$$0 = 2\zeta^T(t)Q\Big[x(t) - x_\sigma - \int_{t-\sigma(t)}^{t}\dot{x}(\alpha)\,\mathrm{d}\alpha\Big] \tag{24}$$

$$0 = 2\zeta^T(t)W\Big[x_\sigma - x_{\bar{\sigma}} - \int_{t-\bar{\sigma}}^{t-\sigma(t)}\dot{x}(\alpha)\,\mathrm{d}\alpha\Big] \tag{25}$$

Moreover, for any matrix $X = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ * & X_{22} & X_{23} \\ * & * & X_{33} \end{bmatrix} > 0$, one has

$$\begin{aligned}0 = \bar{\sigma}\zeta^T(t)X\zeta(t) &- \int_{t-\bar{\sigma}}^{t-\sigma(t)}\zeta^T(t)X\zeta(t)\mathrm{d}\alpha \\ &- \int_{t-\sigma(t)}^{t}\zeta^T(t)X\zeta(t)\mathrm{d}\alpha\end{aligned} \tag{26}$$

Adding (23)-(26) to the sum of $V_l(x(t)), \; l = 1; 2; 3$, yields

$$\begin{aligned}\dot{V}(x(t)) &\le \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \Psi^T(t)\Phi_{ij}\Psi(t) - \\ &\int_{t-\sigma(t)}^{t}\zeta_1^T(t,\alpha)\Xi_1\zeta_1(t,\alpha)\,\mathrm{d}\alpha - \\ &\int_{t-\bar{\sigma}}^{t-\sigma(t)}\zeta_1^T(t,\alpha)\Xi_2\zeta_1(t,\alpha)\,\mathrm{d}\alpha \\ = \sum_{i=1}^{r} h_i^2\Psi(t)^T\Phi_{ii}\Psi(t) &+ 2\sum_{i=1}^{r}\sum_{j>i}^{r} h_i h_j \Psi(t)^T(\Phi_{ij} \\ &+\Phi_{ji})\Psi(t) - \int_{t-\sigma(t)}^{t}\zeta_1^T(t,\alpha)\Xi_1\zeta_1(t,\alpha)\,\mathrm{d}\alpha \\ &- \int_{t-\bar{\sigma}}^{t-\sigma(t)}\zeta_1^T(t,\alpha)\Xi_2\zeta_1(t,\alpha)\,\mathrm{d}\alpha\end{aligned} \tag{27}$$

where

$$\Phi_{ij} = \begin{bmatrix} \delta_1 & * & * & * \\ \delta_2 & \delta_3 & * & * \\ -W_1^T + Q_3 + \bar{\sigma}\bar{X}_{31} & \delta_4 & -Sym(W_3) + \bar{\sigma}X_{33} & * \\ M_3A_{ij} - M_1^T + P_i^T & M_3A_{1i} - M_2^T & 0 & -Sym(M_3) + \bar{\sigma}H \end{bmatrix}$$

$\delta_1 = Sym(M_1A_{ij}) + P_{\Phi,i} + S + Sym(Q_1) + \bar{\sigma}X_{11}$
$\delta_2 = M_2A_{ij} + A_{1i}^T M_1^T + Q_2 - Q_1^T + W_1^T + \bar{\sigma}X_{21}$
$\delta_3 = Sym(M_2A_{1i}) - S(1-\beta) - Sym(Q_2) + Sym(W_2) + \bar{\sigma}X_{22}$
$\delta_4 = -W_2^T - Q_3 + W_3 + \bar{\sigma}X_{32}$
$\zeta_1^T(t,\alpha) = \begin{bmatrix}\zeta^T(t) & \dot{x}^T(\alpha)\end{bmatrix}$
$\Psi^T(t) = \begin{bmatrix}x^T(t) & x_\sigma^T & x_{\bar{\sigma}}^T & \dot{x}^T(t)\end{bmatrix}$

From (11) we have $\Gamma_8 < 0$, this implies $0 < \bar{\sigma}\bar{H} < \mu_2 Sym(U)$, this indicates that U is non singular when $\mu_2 > 0$. Define $G = U^{-1}$ and $\Upsilon = diag(G, G, G, G)$. Let $\bar{S} = U^T SU$, $\bar{P}_{\Phi,i} = U^T P_{\Phi,i}U$, $\bar{H} = U^T HU$, $\bar{P}_i = U^T P_i U$ and $Y_i = K_i U$. Pre and post multiplying (11) by Υ^T and Υ, respectively, and then, we change $M_1 = G^T$, $M_2 = \mu_1 G^T$ $M_3 = \mu_2 G^T$, we obtain $\bar{\Phi}_{ij} + \bar{\Phi}_{ji} < 0$. By following the similar way it is easy to obtain from (10) that $\bar{\Phi}_{ii} < 0$ with the consideration of (12) and (13), we have $\dot{V}(x(t)) < 0$ when $h_i \ge 0$, therefore, the system (8) is asymptotically stable. ■

Remark 3.1: In the proof of Theorem 3.1, we introduce the equality (26) to estimate the upper bound of the time derivative of $V(t)$. However, the inequality $-2a^Tb \leq a^TSa + b^TS^{-1}b$, is sometimes used to bound the terms $-2\zeta^T(t)Q\int_{t-\sigma(t)}^{t}\dot{x}(\alpha)\,\mathrm{d}\alpha$ and $-2\zeta^T(t)W\int_{t-\bar{\sigma}}^{t-\sigma(t)}\dot{x}(\alpha)\,\mathrm{d}\alpha$ in (24). This treatment may give conservative results. The following Theorem summarizes this case.

Theorem 3.2: System (8) is asymptotically stable, if there exists some matrices $\bar{P}_i > 0$, $\bar{H} > 0$, $\bar{S} > 0$ and $\bar{T} > 0$, any matrices Θ_i, Y_i, U , $\bar{Q}$, and $\bar{W}$ satisfying the following set of LMIs : (9), (10), and (11). Where

$$\Lambda_{ij} = \begin{bmatrix} \Gamma_{1ij} & * & * & * & * & * \\ \Gamma_{2ij} & \Gamma_{3i} & * & * & * & * \\ \Gamma_{4ij} & \mu_2(A_{1i}U) - \mu_1 U^T & \Gamma_5 & * & * & * \\ -\bar{W}_1^T & \bar{W}_2^T & 0 & \Gamma_6 & * & * \\ \bar{Q}_1^T & \bar{Q}_2^T & 0 & 0 & -\bar{\sigma}^{-1}\bar{H} & 0 \\ \bar{W}_1^T & \bar{W}_2^T & 0 & 0 & 0 & -\bar{\sigma}^{-1}\bar{H} \end{bmatrix} \quad (28)$$

where $\Gamma_{1ij} = Sym(A_iU+B_iY_j)+\bar{P}_{\Phi,i}+\bar{S}+\bar{T}+Sym(\bar{Q}_1)$
$\Gamma_{2ij} = \mu_1(A_iU + B_iY_j) + U^TA_{1i}^T + \bar{Q}_2 - \bar{Q}_1^T + \bar{W}_1^T$
$\Gamma_{3i} = \mu_1(A_{1i}U+U^TA_{1i}^T)-Sym(\bar{Q}_2)+Sym(\bar{W}_2)-(1-\beta)\bar{S}$
$\Gamma_{4ij} = \mu_2(A_iU + B_iY_j) - U^T + \bar{P}_i^T$
$\Gamma_5 = -\mu_2(U + U^T) + \bar{\sigma}\bar{H}$
$\Gamma_6 = -\bar{T}$

Proof: With considering the same FLKF in Theorem 3.1 and using that

$$\int_{t-\bar{\sigma}}^{t}\dot{x}^T(\alpha)H\dot{x}(\alpha)\,\mathrm{d}\alpha = \int_{t-\bar{\sigma}}^{t-\sigma(t)}\dot{x}^T(\alpha)H\dot{x}(\alpha)\,\mathrm{d}\alpha + \int_{t-\sigma(t)}^{t}\dot{x}^T(\alpha)H\dot{x}(\alpha)\,\mathrm{d}\alpha$$

where the following equations hold for any matrices Q and W with appropriate dimensions:

$$\begin{aligned} -\int_{t-\sigma(t)}^{t}\dot{x}^T(\alpha)H\dot{x}(\alpha)\,\mathrm{d}\alpha &\leq \sigma(t)\xi^T(t)QH^{-1}Q^T\xi(t) \\ &\quad +2\xi^T(t)Q\int_{t-\sigma(t)}^{t}\dot{x}(\alpha)\,\mathrm{d}\alpha \\ &\leq \bar{\sigma}\xi^T(t)QH^{-1}Q^T\xi(t) + 2\xi^T(t)Q\int_{t-\sigma(t)}^{t}\dot{x}(\alpha)\,\mathrm{d}\alpha \end{aligned} \quad (29)$$

$$\begin{aligned} -\int_{t-\sigma(t)}^{t}\dot{x}^T(\alpha)H\dot{x}(\alpha)\,\mathrm{d}\alpha &\leq \sigma(t)\xi^T(t)WH^{-1}W^T\xi(t) \\ &\quad +2\xi^T(t)W\int_{t-\sigma(t)}^{t}\dot{x}(\alpha)\,\mathrm{d}\alpha \\ &\leq \bar{\sigma}\xi^T(t)WH^{-1}W^T\xi(t) + 2\xi^T(t)W\int_{t-\sigma(t)}^{t}\dot{x}(\alpha)\,\mathrm{d}\alpha \end{aligned} \quad (30)$$

, we can prove that $\dot{V}(x(t)) \leq 0$ if conditions (9), (10), and (11) are satisfied. ■

Remark 3.2: As in [8], if we consider that

$$-\int_{t-\bar{\sigma}}^{t}\dot{x}^T(\alpha)H\dot{x}(\alpha)\,\mathrm{d}\alpha \leq -\int_{t-\sigma(t)}^{t}\dot{x}^T(\alpha)H\dot{x}(\alpha)\,\mathrm{d}\alpha \quad (31)$$

we obtain the following result given in Corollary 3.1.

Corollary 3.1: System (8) is asymptotically stable, if there exists some matrices $\bar{P}_i > 0$, $\bar{H} > 0$ and $\bar{S} > 0$, any matrices Θ_i, Y_i, U and $\bar{L}$ satisfying the following set of LMIs : (9), (10) , and (11). Where

$$\Lambda_{ij} = \begin{bmatrix} \Gamma'_{1ij} & * & * & * \\ \Gamma'_{2ij} & \Gamma'_{3i} & * & * \\ \Gamma'_{4ij} & \mu_2(A_{1i}U) - \mu_1 U^T & \Gamma'_5 & 0 \\ \bar{\sigma}\bar{L}_1^T & \bar{\sigma}\bar{L}_2^T & 0 & -\bar{H} \end{bmatrix}$$

where $\Gamma'_{1ij} = Sym(A_iU + B_iY_j) + \bar{P}_{\Phi,i} + \bar{S} + Sym(\bar{L}_1)$
$\Gamma'_{2ij} = \mu_1(A_iU + B_iY_j) + U^TA_{1i}^T + \bar{L}_2 - \bar{L}_1^T$
$\Gamma'_{3i} = \mu_1(A_{1i}U + U^TA_{1i}^T) - Sym(\bar{L}_2) - (1-\beta)\bar{S}$
$\Gamma'_{4ij} = \mu_2(A_iU + B_iY_j) - U^T + \bar{P}_i^T$
$\Gamma'_5 = -\mu_2(U + U^T) + \bar{\sigma}\bar{H}$.

In the next, we propose to extend the result in theorem 3.1 which gives the best result to robust case (ΔA_i , ΔB_i and $\Delta A_{1i} \neq 0$).

Theorem 3.3: System (8) is asymptotically stable, if there exists some matrices $\bar{P}_i > 0$, $\bar{H} > 0$, and $\bar{S} > 0$, any matrices Θ_i, Y_i, U ,$\bar{X}_{11}$, $\bar{X}_{21}$, $\bar{X}_{22}$, $\bar{X}_{31}$, $\bar{X}_{32}$, $\bar{X}_{33}$, $\bar{Q}_1$,$\bar{Q}_2$, $\bar{Q}_3$, $\bar{W}_1$, $\bar{W}_2$, $\bar{W}_3$, and ϵ_i. satisfying the following set of LMIs : (9), (10), (11), (12), and (13). where

$$\Lambda_{ij} = \begin{bmatrix} \Gamma_{1ij} & * & * & * & * & * & * \\ \Gamma_{2ij} & \Gamma_{3i} & * & * & * & * & * \\ \Gamma_{4ij} & \Gamma_{5ij} & \Gamma_{6ij} & * & * & * & * \\ \Gamma_{7ij} & \mu_2(A_{1i}U) - \mu_1 U^T & 0 & \Gamma_8 & * & * & * \\ E_{Ai}U & 0 & 0 & 0 & -\epsilon_i I & * & * \\ E_{Bi}Y_j & 0 & 0 & 0 & 0 & -\epsilon_i I & * \\ 0 & E_{A1i}U & 0 & 0 & 0 & 0 & -\epsilon_i I \end{bmatrix} \quad (32)$$

where $\Gamma_{1ij} = Sym(A_iU + B_iY_j) + \bar{P}_{\Phi,i} + \bar{S} + Sym(\bar{Q}_1) + \bar{\sigma}\bar{X}_{11} + \epsilon_iD_AD_A^T + \epsilon_iD_BD_B^T + \epsilon_iD_{A1}D_{A1}^T$
$\Gamma_{2ij} = \mu_1(A_iU + B_iY_j) + U^TA_{1i}^T + \bar{\sigma}\bar{X}_{21} + \bar{Q}_2 - \bar{Q}_1^T + \bar{W}_1^T$
$\Gamma_{3i} = \mu_1(A_{1i}U + U^TA_{1i}^T) - (1-\beta)\bar{S} + \bar{\sigma}\bar{X}_{22} - Sym(\bar{Q}_2) + Sym(\bar{W}_2) + \mu_1^2\epsilon_iD_AD_A^T + \mu_1^2\epsilon_iD_BD_B^T + \mu_1^2\epsilon_iD_{A1}D_{A1}^T$
$\Gamma_{4ij} = -\bar{W}_1^T + \bar{Q}_3 + \bar{\sigma}\bar{X}_{31}$

$\Gamma_{5ij} = -\bar{W}_2^T - \bar{Q}_3 + \bar{W}_3 + \bar{\sigma}\bar{X}_{32}$
$\Gamma_{6ij} = -Sym(\bar{W}_3) + \bar{\sigma}\bar{X}_{33} + \mu_2^2\epsilon_i D_A D_A^T + \mu_2^2\epsilon_i D_B D_B^T + \mu_2^2\epsilon_i D_{A1} D_{A1}^T$
$\Gamma_{7ij} = \mu_2(A_i U + B_i Y_j) - U^T + \bar{P}_i^T$
$\Gamma_8 = -\mu_2(U + U^T) + \bar{\sigma}\bar{H}$

Proof: With considering the same FLKF in Theorem 3.1 and by using Lemma 2.1 and the Schur complement, we obtain Theorem 3.3 .

Where the uncertain part is represented as :
$\Delta\bar{\Phi}_{ij}$=$Sym(\check{D}\check{F}(t)\check{E}) \leq \epsilon\check{D}\check{D}^T + \epsilon^{-1}\check{E}^T\check{E}$ With

$$\check{D} = \begin{bmatrix} D_A & D_B & D_{A1} & 0 \\ \mu_1 D_A & \mu_1 D_B & \mu_1 D_{A1} & 0 \\ \mu_2 D_A & \mu_2 D_B & \mu_2 D_{A1} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{33}$$

$$\check{F}(t) = \begin{bmatrix} F(t) & 0 & 0 & 0 \\ 0 & F(t) & 0 & 0 \\ 0 & 0 & F(t) & 0 \\ 0 & 0 & 0 & F(t) \end{bmatrix} \tag{34}$$

$$\check{E} = \begin{bmatrix} E_{Ai}U & 0 & 0 & 0 \\ E_{Bi}Y_j & 0 & 0 & 0 \\ 0 & E_{A1i}U & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{35}$$

■

IV. Numerical Example

To illustrate the designed controller law for the T-S fuzzy model with time-varying delay, we consider the following example.

Example . Consider the nonlinear systems with time-delay:

$$\begin{cases} \dot{x}_1(t) = -x_2(t) - x_3(t) + ax_1(t - \sigma(t)) \\ \dot{x}_2(t) = x_1(t) + b_1 x_2(t) \\ \dot{x}_3(t) = (x_1(t) - c)x_3(t) + u(t) \end{cases} \tag{36}$$

The following fuzzy T-S model is presented:

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \Big\{[(A_i + \Delta A_i) + B_i K_j]x(t) + (A_{1i} + \Delta A_{1i})x(t - \sigma(t))\Big\} \tag{37}$$

where

$$A_1 = \begin{bmatrix} 0 & -1 & -1 \\ 1 & b_1 & 0 \\ 0 & 0 & -\gamma \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 0 & -1 & -1 \\ 1 & b_1 & 0 \\ 0 & 0 & \gamma \end{bmatrix},$$

$$A_{11} = A_{12} = \begin{bmatrix} a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad B_1 = B_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},$$

Where $\gamma = 6.3$, $a = 0.5$, $b_1 = b_2 = 0.2$, $c = 5.7$
The fuzzy rules are : h_1=$\frac{1}{2(1+(c-x_1(t)/\gamma))}$ and $h_2 = 1/2(1 - (c - /x_1(t)/\gamma)$.

The operating domain of the nonlinear plant are assumed to be $x_1(t) \in [-10, 12]$, $x_2(t) \in [-12, 8]$ and $x_3(t) \in [0, 25]$. Thus, based on the derivative of the membership functions, we choose Φ_l= {2.15,2.15}.

It appears from Table 1 that the free weighting approach gives the best result in the case of uncertain T-S fuzzy model with varying time-delay as we said in Remark 3.1.

In fact, by solving the LMIs in Theorem 3 of [9], in Theorem 3.1 , in Corollary 3.1 , and Theorem 3.2, the maximum value of the time delay $\bar{\sigma}$ is found in theorem 3.1. Table 1 summarizes the results. By using theorem 3.1 with the choice of $\beta = 0.15$ and $\sigma(t)$=3.05088+0.15 sin(t)($\bar{\sigma} = 3.2088, \mu_1 = \mu_2 = 1$) we can obtain the following state-feedback gain matrices:

$$K_1 = \begin{bmatrix} 11.2167 & 2.0460 & 0.1499 \end{bmatrix} \tag{38}$$
$$K_2 = \begin{bmatrix} 11.2167 & 2.0460 & -12.4501 \end{bmatrix} \tag{39}$$

The simulation results depicted in figures (1)-(3) show that:

- The evolution of the membership derivative shown in Figure 1 allows us to verify that the assumption (19) is verified.
- The closed-loop behavior of the total system with above fuzzy controller for initial conditions $x_0 = \begin{bmatrix} 1.4 & 0.3 & -2.4 \end{bmatrix}$ tends to zero, which is in accordance with the analysis in this paper.

Methods	Maximum allowed $\bar{\sigma}$
Theorem 3.1	3.2088 s
Theorem 3.2	2.5 s
Corollary 3.1	2.4225 s
Theorem 3 of [9]	0.4770 s

TABLE I
Comparison of time-varying delay-dependent stabilization methods without uncertainties

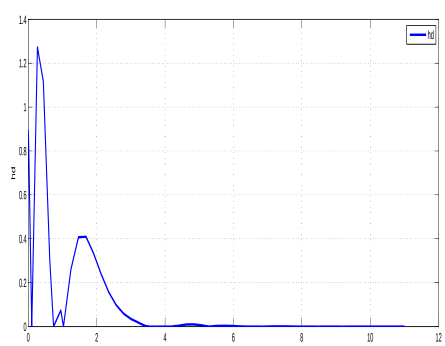

Fig. 1. The dynamic of the derivative of the membership function h_1

V. Conclusion

In this paper, we have presented two new conditions for robust stabilization for uncertain fuzzy systems with time varying delay. Based on the fuzzy weighting-dependent Lyapunov-Krasovskii functional method combined with the introduction of free weighting matrices, the less conservatism

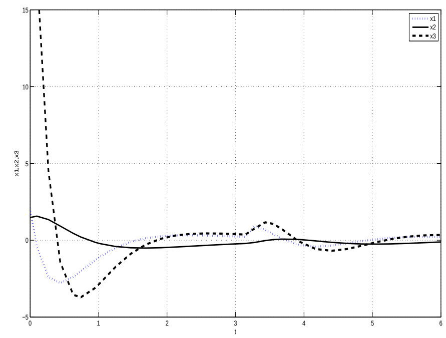

Fig. 2. Control results for system (37) without uncertainties $\bar{\sigma} = 3.2088$

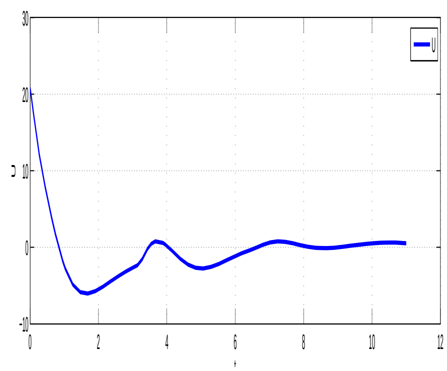

Fig. 3. Control input

sufficient conditions formulated in LMI terms are established. We find that the free weighting approach is less conservative than the bounding techniques. The numerical example has been given to illustrate the conservatism reduction of the proposed methods.

REFERENCES

[1] Teixeira M.C.M Assuncao and E Avellar R.G. On relaxed lmibased designs for fuzzy regulators and fuzzy observers. *IEEE Trans Fuzzy Syst*, 11:613–623, 2003.

[2] El-Kebir Boukas. Free-weighting matrices delay-dependent stabilization for systems with time-varying delays. *ICIC Express Letters ICIC International c 2008 ISSN 1881-803X*, 2, 2008.

[3] Wang Y Xie L De Souza CE. Robust control of a class of uncertain nonlinear systems. *Systems and Control Letters*, 19:139–149, 1992.

[4] C.Lin, Q-G.Wang, T.H .Lee, and Y.He. Lmi approach to analysis and control of takagi-sugeno fuzzy systems with time delay. *Newyork/Berlin,Heidelberg,Germany:Springer-Verlag*, 15:482–493, 2007.

[5] C.Lin, Q-G.Wang, T.H.Lee, and Y.He. Design of observer -based H_∞ control for fuzzy time-delay systems. *IEEE Trans.Fuzzy Syst*, 16:534–543, 2008.

[6] Kim E and Lee H. New approaches to relaxed quadratic stability condition of fuzzy control systems. *IEEE Trans. Fuzzy Syst*, 8:523–534, 2000.

[7] H. Gassara, A. EL Hajjaji, and M. Chaabane. Delay-dependant H_∞ exponential stabilization of T-S fuzzy systems with interval time-varying delay. *49th IEEE Conference on Decision and Control, IEEE-CDC, December 15-17, Atlanta-Georgia, USA*, .:4281–4286, 2010.

[8] Wang H, Xue A, Lu R Wang, and J.. Delay-dependent robust stability and stabilization for uncertain singular system with time-varying delay. *American Control Conference, Westin Seattle Hotel, Seattle, Washington, USA,*, .:36263631, 2008.

[9] H.Gassara, A. El Hajjaji, and M. Chaabane. Robust stability and stabilization conditions for time delay fuzzy systems using weighting-dependent approach. *Proceeding of ECC*, .:422–429, 2009.

[10] Zhang Jinxian, Wang Wu, Cui Fujun, and Yang Fuwen. Free-weighting matrices approach to h?filtering for a class of uncertain linear systems with time-varying delay. *CCDC'09 Proceedings of the 21st annual international conference on Chinese Control and Decision Conference*, 2009.

[11] Tanaka k, Ikeda T, and Wang HO. Robust stabilization of a class of uncertain nonlinear systems via fuzzy control : quadratic stabilizability, control theory, and linear matrices inequalities. *IEEE Trans.Fuzzy Syst*, 4:1–13, 1996.

[12] Tanaka k, Ikeda T, and Wang HO. Fuzzy regulators and fuzzy observers : relaxed stability conditions and lmi-based designs. *IEEE Trans. Fuzzy Syst*, 6:250–265, 1998.

[13] Mourad Kchaou, Mansour Souissi, and Ahmed Toumi. A new approach to non-fragile H_∞ observer-based control for discretetime fuzzy systems. *International Journal of Systems Science*, .:1–12, 2010.

[14] Mourad Kchou, Mansour Souissi, and Ahmed Toumi. Robust H_∞guaranteed cost fuzzy control for uncertain discrete-time fuzzy systems via poly-quadratic lyapunov functions. *Asian Journal of Control*, 13:309–316, 2011.

[15] K.Tanaka and H.O.Wang. Fuzzy control systems design and analysis: A linear matrix inequality approach. *New York: Wiley*, .:324–332, 2001.

[16] L.A.Mozelli, C.D.Campos, R.M.Pahares, L.A.B.Torres, and E.M.A.M.Mendes. Chaotic synchronization and information transmission experiments:a fuzzy relaxed H∞ control approach. *Circ.Syst.Signal Process*, 26:427–449, 2007.

[17] L.A.Mozelli and R.M.Palhares. A systematic approach to improve multiple lyapunov function stability and stabilization conditions for fuzzy systems. *Informations Sciences*, 179:1149–1162, 2008.

[18] R.MPalhares, C.D.Capmos, P.Y Ekel M.C.R Lesles, and M.F.S. V .D'Angelo. Delay-dependent robust H_∞ control of uncertain linear systems with time-varying delays. *.Comp.Math.Appl*, 50:13–32, 2005.

[19] T.Takagi and M.Sugeno. Fuzzy identification of system and its applications to modeling and control. *IEEE Trans ,Sys.,Man,Cybern*, 15:116–132, 1985.

[20] Liu X and Zhang Q. New approaches to H_∞ controller designs based on fuzzy observers for T-S fuzzy systems via lmi. *Automatica*, 39:1571–1582, 2003.