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An active diagnosis approach based on mixed fault detection filter and GLR test

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Abstract— **This paper presents a new approach for active diagnosis bsaed on adaptive Generalized Likelihood Ratio (GLR) test for multiple Faults Detection and Isolation (FDI) in dynamic stochastic linear systems. For sequential fault detection, the adaptive GLR test confronts the fault hypothesis $H_k h_j$ to the reference hypothesis H_k updating on line after each detection and isolation of one fault. The proposed scheme is based on the design of the Fault Detection Filter (FDF) and on the GLR test applied on the innovation sequence of the FDF.**

I. INTRODUCTION

Due to an increasing complexity of modern engineering systems, as well as the need for reliability, safety and efficient operation of complex systems. To improve the decision operation, we propose in this paper a solution to the problem of multiple FDI in discrete-time stochastic linear systems. In such systems, the FDI problem can be splitted into two steps: generation of residuals, which are ideally close to zero under no-fault conditions and minimally sensitive to noises, and residual evaluation, namely the design of decision rules based on these residuals. Frank (1990, 1991), Gertler (1991), and Patton and Chen (1991) have presented surveys of fault detection theory based on analytical redundancy.

There exists two classes of methods for model based FDI in dynamic linear systems :
In the first general structure, a filter is designed based on the assumption that no faults has occurred or will occur. The filter produces a prediction of the output signal based on this assumption and the past history of the output, and this prediction is substracted from the actual output to produce the innovation sequence close to zero for fault-free system. Consequently derivations from this behaviour are indicative of faults. According to each fault hypothesis h_i, the possible derivations from this behavior are linked to fault signatures recursively computed on-line. Depending to the fault occurrence time and to the fault magnitude evolution, the decision mechanism is based on the correlations existing between the innovation sequence and the signature of faults. Its the case of the GLR test using the Kalman filter and first presented in its most general form by Willsky and Jones (1976). More recent developments of this approach can be found in Basseville (1994).

In the second general structure, a filter is designed based on the assumption that all the possible faults are present on the system. The choice of the gain matrix is then made in order to guarantee the decoupling of the fault effects on the innovation sequence. Its the case of the Fault Detection Filter (FDF) first developed by Beard (1971) and Jones (1973). The problem of FDF

design was later re-visited by Massoumnia (1986) in the geometric framework and by White and Speyer (1987) in the context of eigenstructure assignment. Further improvements were suggested by Park and Rizzoni (1994) and Liu and Si (1997). This approach is very efficient for the treatment of multiple faults since the FDF generates fixed fault signatures independent to the fault occurrence time and to the fault magnitude evolution. This is the main motivation of Jamouli (2003) for extending of the FDF for multiple FDI in stochastic discrete-time systems. To solve the problem of fault detection and isolation of sequential faults, this paper proposes a mixed GLR/FDF approaches where the GLR test is applied on the innovation sequence of the FDF designed under the assumption H_k that a subset of possible faults are already detected and isolated. This will lead to an adaptive GLR test allowing the confrontation of faults hypothesis $H_k h_i$ relatively to H_k and the recursive treatment of multiple faults by updating on-line the reference assumption H_k and the FDF after each detection and isolation of one fault.
The paper is organized as follows : In section 2, we present the FDF concepts. The section 3 studies the adaptive GLR test for multiple faults appearing sequentially. The section 4 gives the conclusions.

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II. THE FAULT DETECTION FILTER

Under hypothesis h_0 , a fault-free discrete-time dynamic linear system is described by

$$x_{t+1} = Ax_t + Bu_t + w_t \tag{1}$$
$$y_t = Cx_t + v_t \tag{2}$$

where $x_t \in \Re^n$ is the state vector, $y_t \in \Re^m$ the measurement vector, $u_t \in \Re^r$ the input vector. The matrices A, B and C have appropriate dimensions and the zero mean white gaussian noises w_t and v_t

$$E\left(\begin{bmatrix} w_t \\ v_t \end{bmatrix}\begin{bmatrix} w_{t1}^T & v_{t1}^T \end{bmatrix}\right) = \begin{bmatrix} W & 0 \\ 0 & V \end{bmatrix}\delta_{tt_1} \tag{3}$$

Under hypothesis $H_k = h_1..h_j..h_k$ (k faults are assumed to be present on the system), the faulty system is modelized by

$$x_{t+1} = Ax_t + Bu_t + Fd_t + w_t \quad (4)$$
$$y_t = Cx_t + v_t \quad (5)$$

where $F = \begin{bmatrix} f_1 & .. & f_j & .. & f_k \end{bmatrix} \in \Re^{n,k}$ is the fault distribution matrix and $d_t = \begin{bmatrix} d_t^1 & .. & d_t^j & .. & d_t^k \end{bmatrix}^T \in \Re^k$ the vector of fault magnitudes.
Let

$$D = \begin{bmatrix} A^{\rho_1-1}f_1 & .. & A^{\rho_j-1}f_j & .. & A^{\rho_k-1}f_k \end{bmatrix} \quad (6)$$

where

$$\rho_j = mins : CA^{s-1}f_j \neq 0, s = 1, 2, .. \quad (7)$$

is the fault detectability index of fault h_j.
Consider the following observer

$$\hat{x}_{t+1} = A\hat{x}_t + Bu_t + K_t\gamma_t \quad (8)$$
$$\gamma_t = y_t - C\hat{x}_t \quad (9)$$

where $\hat{x}_t$ denotes the state estimation vector and $\gamma_t \in \Re^m$ the output residuals. From the fault model (2), the estimation errors $e_t = (x_t - \hat{x}_t) \in \Re^n$ and the innovation sequence γ_t propagates

$$e_{t+1} = (A - K_tC)e_t + Fd_t + w_t - Kv_t \quad (10)$$
$$\gamma_t = Ce_t + v_t \quad (11)$$

In the noise free-case, the FDF design problem is to find the observer's gain K_t such that the detection spaces Ω_j of minimal dimensions associated to each fault hypothesis h_j satisfy

$$(A - K_tC)\Omega_j \subseteq \Omega_j \qquad \forall\ \Omega_j \quad (12)$$
$$f_j \subseteq \Omega_j \qquad \forall\ \Omega_j \quad (13)$$
$$C\Omega_j \bigcap (\textstyle\sum_{s\neq j} C\Omega_s) = \emptyset \quad (14)$$

Consider the following algebraic constraints

$$(A - K_tC)^{\rho_j} f_j = 0 \quad for \quad j = 1, ..., k \quad (15)$$

represented in a matricial form as

$$K_t\Psi = \omega \quad (16)$$

with

$$\omega = AD \quad (17)$$
$$\Psi = CD \quad (18)$$

The solutions of (15) can be parameterized from the free parameters $\bar{K}_t \in \Re^{n,m}$ and $\alpha \in \Re^{m,n}$ as

$$K_t = \bar{K}_t + \eta_t\Pi_t \quad (19)$$
$$\Pi_t = (\Psi^T\alpha_t\Psi)^{-1}\Psi^T\alpha_t \quad (20)$$
$$\eta_t = \omega - \bar{K}_t\Psi \quad (21)$$

Inserting (19) into (10) and (11), we obtain

$$e_t = \bar{e}_t + \sum_{j=1}^{k} \begin{bmatrix} f_j & Af_j & .. & A^{\rho_j-1}f_j \end{bmatrix} \begin{bmatrix} d_{t-1}^j \\ d_{t-2}^j \\ \vdots \\ d_{t-\rho_j}^j \end{bmatrix} \quad (22)$$

and

$$\gamma_t = C\bar{e}_t + v_t + \Psi \begin{bmatrix} d_{t-\rho_1}^1 \\ \vdots \\ d_{t-\rho_k}^k \end{bmatrix} \quad (23)$$

where the independent columns of $\begin{bmatrix} f_j & Af_j & .. & A^{\rho_j-1}f_j \end{bmatrix}$ describe the detection space Ω_j satisfying (12,13,14) and where $\bar{e}_t$ is the estimation error under hypothesis h_0 given by

$$\bar{e}_{t+1} = (A - K_tC)\bar{e}_t + w_t - K_tv_t \quad (24)$$
$$\bar{P}_{t+1} = (A - K_tC)\bar{P}_t(A - K_tC)^T + W + K_tVK_t^T \quad (25)$$

From (23), an unbiased estimator of the delayed fault magnitudes $\delta_t^k = \begin{bmatrix} d_{t-\rho_1}^1 & .. & d_{k-\rho_1}^k \end{bmatrix}^T$ is given by

$$\hat{\delta}_t^k = \Pi_t\gamma_t \quad (26)$$
$$P_t^d = E(e_t^d e_t^{dT}) = \Pi_t\bar{H}_t\Pi_t^T \quad (27)$$

where $e_t^d = (\delta_t^k - \hat{\delta}_t^k) \in \Re^k$ and $\bar{H}_t = E(\bar{\gamma}\bar{\gamma}_t^T) = C\bar{P}_tC^T + V$ with $\bar{\gamma}_t = C\bar{e}_t + v_t$. In the unbiased minimum variance sense, $\bar{\gamma}_t$ and $\hat{\delta}_t^k$ can be optimized by minimizing the trace of Ω_{t+1} defined as

$$\Omega_{t+1} = E\left\{ \begin{bmatrix} \bar{e}_{t+1} \\ e_t^d \end{bmatrix} \begin{bmatrix} \bar{e}_{t+1}^T & e_t^{dT} \end{bmatrix} \right\}$$

(28)

$$= \begin{bmatrix} \bar{P}_{t+1} & P_t^{ed} \\ P_t^{de} & P_t^{d} \end{bmatrix}$$

where

$$\bar{P}_t = \bar{P}_{t+1}^{Kal} - \eta_t P_t^d \eta_t^T - P_t^{ed}\eta_t^T - \eta_t P_t^{de} \quad (29)$$

$$P_t^{ed} = (P_t^{de})^T = (A\bar{P}_t C^T - \bar{K}_t \bar{H}_t - \eta_t \Pi_t \bar{H}_t)\Pi_t^T \quad (30)$$

$$\bar{P}_{t+1}^{Kal} = (A - \bar{K}_t C)\bar{P}_t (A - \bar{K}_t C)^T + W + \bar{K}_t V \bar{K}_t^T \quad (31)$$

with respect to the free parameters $\bar{K}_t$ and α_t or equivalently the trace of $\bar{\Omega}_{t+1}$ given by

$$\bar{\Omega}_{t+1} = \begin{bmatrix} I & \eta_t \\ 0 & I \end{bmatrix} \Omega_{t+1} \begin{bmatrix} I & 0 \\ \eta_t^T & I \end{bmatrix} \quad (32)$$

$$= \begin{bmatrix} \bar{P}_{t+1}^{Kal} & (A\bar{P}_t C^T - \bar{K}\bar{H}_t)\Pi_t^T \\ \Pi_t (A\bar{P}_t C^T - \bar{K}\bar{H}_t)^T & P_t^d \end{bmatrix}$$

From the standard Kalman filter's gain $\bar{K}_t = A\bar{P}_t C^T \bar{H}_t^{-1}$ minimizing the trace of $\bar{P}_{t+1}^{Kal}$, we obtain

$$\tilde{\Omega}_{t+1} = \begin{bmatrix} A\bar{P}_t A^T - A\bar{P}_t C^T \bar{H}_t^{-1} C\bar{P}_t A^T + W & 0 \\ 0 & P_t^d \end{bmatrix} \quad (33)$$

and the trace of $\tilde{\Omega}_{t+1}$ is minimal with respect to α_t if and only if

$$\bar{\alpha}_t = \bar{H}_t^{-1} \quad (34)$$

since $P_t^d = (\Psi^T \bar{H}_t^{-1}\Psi)^{-1}$ and $P_t^d > (\Psi^T \bar{H}_t^{-1}\Psi)^{-1}$ for any choice of α_t. So, with $\bar{K}_t = A\bar{P}_t C^T \bar{H}_t^{-1}$ and $\bar{\alpha}_t = \bar{H}_t^{-1}$ we obtain the following FDF

$$\hat{x}_{t+1} = (A - \bar{K}_t C)(\hat{x}_t + D\hat{\delta}_t^k) + Bu_t + \bar{K}_t \gamma_t \quad (35)$$

$$\bar{P}_{t+1} = (A - \bar{K}_t C)(\bar{P}_t + DP_t^d D^T)(A - \bar{K}_t C)^T \quad (36)$$

$$+ W + \bar{K}_t V \bar{K}_t^T$$

$$\bar{K}_t = A\bar{P}_t C^T \bar{H}_t^{-1} \quad (37)$$

$$\hat{\delta}_t^k = P_t^d \Psi^T \bar{H}_t^{-1} \gamma_t \quad (38)$$

$$P_t^d = (\Psi^T \bar{H}_t^{-1} \Psi)^{-1} \quad (39)$$

$$\gamma_t = y_t - C\hat{x}_t \quad (40)$$

$$\bar{H}_t = C\bar{P}_t C^T + V \quad (41)$$

III. FDI FOR SEQUENTIAL FAULTS

Under hypothesis $H_k h_i$, where h_i belong to the set $\bar{H}_k$ of all possible faults remaining to detect and isolate, the faulty system modelized by

$$x_{t+1} = Ax_t + Bu_t + Fd_t + f_i(t, r_i)\nu_i(t, r_i) + w_t \quad (42)$$

$$y_t = Cx_t + v_t \quad (43)$$

where $f_i(t, r_i)$ is the fault distribution vector, $\nu_i(t, r_i)$ the fault magnitude and r_i the unknown time of failure occurrence. Two different kinds of faults are considered: under: H_k, no any assumption is made concerning the fault magnitudes d_t. Under $H_k h_i$, the fault magnitude $\nu_i(t, r_i)$ is assumed to follow a constant bias model: $\nu_i(t, r_i) = \nu_i, \{t \geq r_i\}$. This will justified more later in the updating scheme.

In this section, the problem is to detect and isolate the possible faults not taken into account in the design of the FDF. The hypothesis $H_k h_i$ can be confronted from H_k as

$$H_k : E(\gamma_t) = \Psi\delta_t^k, t < r_i \quad (44)$$

$$H_k h_i : E(\gamma_t) = \Psi\delta_t^k + \rho_i(t, r_i)\nu_i \quad t \geq r_i \quad (45)$$

where $\rho_i(t, r_i)$ is the fault signature recursively computed for all possible faults h_i belonging to $\bar{H}_k$ as

$$\zeta_i(t+1, r_i) = (A - K_t C)\zeta_i(t, r_i) + f_i \quad (46)$$

$$\rho_i(t, r_i) = C\zeta_i(t, r_i) \quad (47)$$

with $\zeta_i(r_i, r_i) = 0$. The signature $\rho_i(t, r_i)$ depends upon both t and r_i during the transient behaviour of the FDF. When the steady-state behaviour of the FDF is reached, this signature depends only upon $t - r_i$. If $D = 0$, then $K_t = \bar{K}_t$ and the above signatures are those obtained by Willsky and Jones (1976) since $\zeta_i(t, r_i) = \alpha_i(t, r_i) - \beta_i(t-1, r_i)$ describe the additive effect of fault h_i on the state and on its estimate, respectively.

Let $D(r_i, t) = \{\delta_{ri}^k, .., \delta_{t-1}^k, \delta_t^k\}$. The generalized likelihood ratio between hypotheses $H_k h_i$ and H_k can be expressed as

$$\lambda(D(r_i, t), \nu_i) = \frac{P(\gamma/H_k h_i)}{P(\gamma/H_k)} \quad (48)$$

$$= \frac{exp - (\sum_{j=r_i}^{t} \|\gamma_j - \Psi\delta_j^k - \rho_i(j, r_i)\nu_i\|^2_{\bar{H}_j^{-1}})}{exp(-\sum_{j=r_i}^{t} \|\gamma_j - \Psi\delta_j^k\|^2_{\bar{H}_j^{-1}}} \quad (49)$$

Replacing $D^j(r_i, t)$ by their unbiased minimum variance estimate under H_k given by their (35,..), noted $\hat{D}(r_i, t) =$

$\{\hat{\delta}^k_{ri}, .., \hat{\delta}^k_{t-1}, \hat{\delta}^k_t\}$, and by their unbiased minimum variance estimate under $H_k h_i$ given by $\hat{D}(r_i, t) = \{\hat{\delta}^k_{ri}, .., \hat{\delta}^k_{t-1}, \hat{\delta}^k_t\}$ with $\hat{\delta}^k_t = P^d_t \Psi^T \bar{H}^{-1}_t (\gamma_t - \rho_i(t, r_i)\nu_i)$, we obtain

$$\lambda(\hat{D}(r_i,t),\nu_i) \tag{50}$$

$$= \frac{exp(-\sum_{j=r_i}^t \|\gamma_j - \rho_i(j,r_i)\nu_i)\|^2_{Q^k_j}}{exp(-\sum_{j=r_i}^t \|\gamma_j)\|^2_{Q^k_j}} \tag{51}$$

where $Q^H_j = \bar{H}^{-1}_j - \bar{H}^{-1}_j \Psi P^d_t \Psi^T \bar{H}^{-1}_j$ is the statistical rejector. So, from the maximum likelihood estimate of ν_i (under hypothesis $H_k H_i$) given by

$$\hat{\nu}_i(t, r_i, k) \tag{52}$$

$$= [\sum_{j=r_i}^t \rho_i^T(j,r_i) Q^k_j \rho_i(j,r_i)]^{-1} \tag{53}$$

$$\times \sum_{j=r_i}^t \rho_i^T(j,r_i) Q^k_j \gamma_j$$

and from $T_i(t, r_i, k) = ln(\hat{D}(r_i, t), \nu_i)$, we obtain the following adaptive GLR test for multiple faults detection

$$max\{T_i(t,r_i,k)\} \begin{smallmatrix} >^{H_k h_i} \\ <^{H_k} \end{smallmatrix} \mu_i(t,r_i,k) \tag{54}$$

$h_i \in \bar{H}_k, t - M + 1 \le r_i \le t - \rho_i$

with

$$T_i(t,r_i,k) = b_i(t,r_i,k)^2 a_i(t,r_i,k)^{-1} \tag{55}$$

$$a_i(t,r_i,k) = \sum_{j=r_i}^t \rho_i^T(j,r_i) Q^k_j \rho_i(j,r_i) \tag{56}$$

$$b_i(t,r_i,k) = \sum_{j=r_i}^t \rho_i^T(j,r_i) Q^k_j \rho_i \gamma_j \tag{57}$$

In theory, the research of the maximum value of $T_i(t, r_i, k)$ should be made $M = t$ to use all past information. In order not to increase linearly the size of the window of this search and to avoid the case where $a_i(j, r_i, k) = 0$ for $r_i \in [t, t-(\rho_i)]$, r_i is estimated by looking for the maximum value of $T_i(t, r_i, k)$ inside the sliding window $[t-M, t-\rho_i]$.

The statistical variables $T_i(t, r_i, k)$ follow Chi2 distribution with one degree of freedom and the probability of good decisions of the hypothesis $H_k h_i$ increases with the Kullback divergence $a_i(t, r_i, k)\nu_i^2$. The choice of the threshold level $\mu_i(t, r_i, k)$ must depend of t, r_i and k because the noncentrality parameter of the Chi2 distribution is given by $a_i(t, r_i, k)\nu_i^2$. Even if $a_i(t, r_i, k)$ depends only up $t - r_i$ and k when the steady-state behaviour of the FDF is reached, this is the main motivation of Basseville and Benveniste (1983) for the derivation of a modified version of the GLR algorithm. The fault h_s is then detected and isolated at time $\hat{r}_i$ if

$$T_s(t,\hat{r}_i,k) \tag{58}$$

$$= max\{T_i(t,r_i,k)\} > \mu_i(t,r_i,k) \tag{59}$$

$h_i \in \bar{H}_k, t - M + 1 \le r_i \le t - \rho_i.$

For the updating strategy $H_{k+1} = H_k h_s$, the fault magnitude ν_s is then considered as an unknown input at time instant $\hat{r}_s$ and the FDF is recomputed with $F = [F f_s]$. The only variables affected by this updating strategy are the terms $D\hat{\delta}^k_t$ and $DP^d_t D^T$ appearing in relations ((35),(36)). They are similar to those added by Willsky and Jones (1976) in the estimation of the state variable and the covariance matrix of the state estimation error of the Kalman filter to improve its traking ability. However, our updating scheme forgets the information $\hat{\nu}_s(t, \hat{r}_s, k)$. This leads to maximum number of faults which can be treated equals to the number of available measurements. It's the main difference with the updating scheme of Willsky and Jones (1976) allowing the treatment of an infinite number of faults. In Jamouli (2012), we have proposed a GLR method based an augmented Kalman filter and the updating strategy is similar to the two stage Kalman filter (Friedland (1969)).

IV. The jumps detectability conditions

A satisfactory choice of M involved in the sliding windows $[t - M, t - \rho_i]$ depends to the time deal between the occurence of two jumps. The inferior bound of α of the sliding windows must be determined in relation with the detectability condition of jumps based on the Kullback distance (Kullback, 1959).

Theorem1: The jumps detectability condition is satisfied if and only if the transfer function μ to y given by $H(z) = C(Iz - A)^{-1}F$ with $F = [f_1..f_i..f_k]$ is left invertible,i.e $rank[H(z)] = k$. If $H(z)$ is left invertible then α is given by the degree of the polynomial matrix $\xi(z)$ (Wolowich and Falb, 1976) satisfying $rank\{lim_{z\to\infty} H(z)\xi(z)\} = k$.

Proof

Define the least favorable jumps scenario as the case where all the jump h_i for $i \in [1, .., k]$ appear simultaneously at the occurence time r here assumed to be known. In this case, the steady state jumps signatures are expressed as

$$\zeta_{t+1} = (A - \bar{K}C)\zeta_t + F, \zeta_r = 0 \tag{60}$$

$$\rho_t = C\zeta_t \tag{61}$$

The Kullback distence δ_t between this least favorable scenario and h_0 is then espressed

$$\delta_t = \mu^T a_t \mu \quad (62)$$

with $a_t = \sum_{l=r}^{t} \rho_l^T \bar{H}_l^{-1} \rho_l$

The Kullback distance (62) cannot satisfy $\delta_k = \mu^T a_t \mu = 0,\ \forall \mu \neq 0$ if and only if $a_t > 0$ and the problem now to determine the minimum value of t denoted α so that $\sum_{l=r}^{\alpha} \rho_l^T \rho_l > 0$. From the transfer function $C[Iz - (A - \bar{K}C)]^{-1}F$ of (60) this problem can solved by determining a minimal polynomial matrix $\xi(z)$ so that $rank\{lim_{z\to\infty} H(z)\xi(z)\} = k$ since $C[Iz-(A-\bar{K}C)]^{-1}F = [I + C(Iz-A)^{-1}\bar{K}]H(z)$. The condition $rank[H(z)] = k$ ensures the existence of an interactor matrix $\xi(z)$ such that $H(z)\xi(z) = [C(zI-A)^{-1}F_\alpha + G_\alpha]$ with $rank[G_\alpha] = k$ and $lim_{z\to\infty} H(z)\xi(z)\} = G_\alpha$

(Wolowich and Falb, 1976) closing the proof. To illustrate this theorem, the interactor matrix $\xi(z)$ of $H(z) = C(Iz - A)^{-1}F$ with

$A = \begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 1 & \lambda_2 & 0 & 0 \\ 0 & 1 & \lambda_3 & 0 \\ 0 & 0 & 1 & \lambda_4 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ and $F = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ is given by $\xi(z) = \begin{bmatrix} 0.5z & -\frac{\sqrt{3}}{2}z^2 \\ 0.5z & \frac{\sqrt{3}}{2}z^2 \end{bmatrix}$ and $H(z)\xi(z) = [C(Iz-A)^{-1}F_2 + G_2]$

holds with $F_2 = \begin{bmatrix} \lambda_1 & 0 \\ 1 + 0.5\lambda_2 & \frac{\sqrt{3}}{2}\lambda_2^2 \\ 0.5 & \frac{\sqrt{3}}{2}(\lambda_2 + \lambda_3) \\ 0 & \frac{\sqrt{3}}{2} \end{bmatrix}$ and $G_2 = \begin{bmatrix} 1 & 0 \\ 0 & \frac{\sqrt{3}}{2} \\ 0 & 0 \end{bmatrix}$. The degree $\alpha = 2$ of the polynomial matrix $\xi(z)$ also represents the input delay of $H(z)$. We are going to derive another detectability condition based on Tanaka (1990). Assume that the Ricatti difference equation (36) of the jump-free Kalman filter ($D = 0$) has a stabilizing solution $(eig(A - \bar{K}C) < 1)$, i.e.

$$\begin{bmatrix} Iz - A \\ C \end{bmatrix} = n, \quad \forall z \in C, |\, z \,| \geq 1 \quad (63)$$

$$rank[-e^{jw}I + A \quad W^{1/2}] = n, \quad \forall w \in [0, 2\pi] \quad (64)$$

Theorem2: Under (63,64), the Kullback distance (62) strictly increases with time k for $k > \alpha$ and any new observation bring a new information about a possible jump if and only if

$$rank \begin{bmatrix} Iz - A & F \\ C & 0 \end{bmatrix} = n + k \quad (65)$$

Proof

The sequence $a_{\alpha+1}^{-1}, a_{\alpha+2}^{-1}, ...a_t^{-1}$, belongs to the sequence $\Omega_{\alpha+1}, \Omega_{\alpha+2}, ..., \Omega_t$ defined as $\begin{bmatrix} \times & \times \\ \times & a_t^{-1} \end{bmatrix}$ and generated by the following augmented state Ricatti difference equation

$$\Omega_{k+1} = \bar{A}\Omega_k\bar{A}^T + \quad (66)$$

$$+\bar{\Gamma}W\bar{\Gamma}^T - \bar{A}\Omega_k\bar{C}^T(\bar{C}\Omega_k\bar{C}^T + V)^{-1}\bar{C}\Omega_k\bar{A}^T$$

with $\bar{A} = \begin{bmatrix} A & F \\ 0 & I \end{bmatrix}$, $\bar{C} = \begin{bmatrix} C & 0 \end{bmatrix}$ and $\bar{\Gamma} = \begin{bmatrix} I \\ 0 \end{bmatrix}$ if and only if $\Omega_\alpha = \begin{bmatrix} \bar{P}_\alpha + \zeta_\alpha a_\alpha^{-1}\zeta_\alpha^T & \zeta_\alpha a_\alpha^{-1} \\ a_\alpha^{-1}\zeta_\alpha^T & a_\alpha^{-1} \end{bmatrix}$ defined from the quantities involved in theorem 1. Under (63,64), the pair $(A, \bar{\Gamma}W^{1/2})$ has k unreachable modes on the unit circle and the detectability of the pair $(\bar{A}, \bar{C})$ ensures the existence of a solution Ω of (66) $(eig(\bar{A} - \bar{K}\bar{C}) \leq 1$ with $\bar{K} = \bar{A}\Omega\bar{C}^T(\bar{C}\Omega\bar{C}^T + V)^{-1}$. The detectability of $(\bar{A}, \bar{C})$ corresponds to the geometrical jump detectability condition obtained by Caglayan (1980).

Under (62), the detectability of $(\bar{A}, \bar{C})$ is equivalent to (63) and ensures the existence the asymptotical convergence of a_t^{-1} to zero. So, $a_{t+1}^{-1} < a_t^{-1}, \forall k > \alpha$ or $\tilde{a}_{t+1} > 0, \forall k > \alpha$ where $\tilde{a}_{t+1} = a_{t+1} - a_t$ is the Kullback increment. The Kullback distance (62) is maximized with respect to μ since the trace of Ω_t generated by (66) is minimum. This remark remains valid for the relative Kullback distance $\delta_i^{new}(t, r) = a_i^{new}(t, r)\mu^2$ between h_i^{new} and H_j.

V. Conclusion

We have proposed an adaptive GLR test for model-based FDI in dynamic stochastic linear systems allowing the recursive treatment of multiple additive faults. The GLR test ha been applied after the statistical rejection of faults already detected and isolated. The statistical and geometrical detectability conditions of multiples faults have been estabilished. We have studied the choice of the inferior bound of the sliding window and the jump detectability conditions in the sense of Kullback (1959), Caglayan (1980) and Tanaka (1990).

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