Improved LMI approach to Fuzzy H_∞ Filter Designs

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***Abstract*— The H_∞ filtering problem for a class of discrete-time Takagi-Sugeno fuzzy systems is studied. Attention is focused on the design of an H_∞ filter such that the filter error system is asymptotically stable and preserves a guaranteed H_∞ performance. By using the fuzzy Lyapunov function approach and adding slack matrix variables, the coupling between the Lyapunov matrix and the system matrices is eliminated. Then, a linear matrix inequality (LMI)-based approach is developed for designing the H_∞ fuzzy filter. Finally, an illustrative example is provided to show the effectiveness of the proposed approach and less conservatism.**

Key words−Discrete-time systems, fuzzy Lyapunov function, H_∞ filtering, linear matrix inequality (LMI), Takagi-Sugeno (T-S) fuzzy model.

I. INTRODUCTION

In the last few years, many researchers have studied the H_∞ filter design for a general class of linear systems due to a great practical importance. The filtering problem can be stated as follows: given a dynamic system with exogenous input and measured output, design a filter to estimate an unmeasured output such that the mapping from the exogenous input to the filter error is minimized or no larger than some prescribed level in terms of the H_∞ norm. In [1] and [2] , it has been shown that the existence of solution to H_∞ filtering problem is in fact related to the solvability of an appropriate algebraic Riccati equation. This result is then extended in [3] to a class of linear systems which are subject to parametric uncertainty. A sufficient condition for the existence of a solution is derived also via algebraic Riccati equations.

The advantage of the H_∞ filtering lies in that no statical assumption on the noise signals is needed, thus, it is more general than classical Kalman filtering[4]. Moreover, the H_∞ filter has been shown to be much more robust against unmodeled dynamics. For linear systems, there have been fruitful results of filter designs. Nevertheless, for complex nonlinear systems, it generally lacks common techniques in filter designs.

Recently, there has been a growing interest in the Takagi-Sugeno (T-S) fuzzy model since it is a powerful solution that bridges the gap between linear control and complex nonlinear systems [5]. The important advantage of the T-S fuzzy model is its universal approximation of any smooth nonlinear function by a "blending" of some local linear system models. Based on the local linearity, many complex nonlinear problems can be simplified by employing the Lyapunov function approach [6].

In this article, we investigate the H_∞ filtering problem for a class of discrete-time nonlinear systems, The nonlinear plant is described by the T-S fuzzy dynamic model. The fuzzy Lyapunov function, which is defined by fuzzily blending some quadratic Lyapunov functions, is used to derive our main results. By using the fuzzy Lyapunov function approach and adding slack matrix variable, a new condition for H_∞ performance analysis is proposed. The theoretical results are in the form of LMIs, which can be solved by standard numerical software. An example shows the effectiveness of the proposed approach.

II. Problem Description

Consider a class of discrete-time systems. Using the Takagi-Sugeno (TS) fuzzy dynamic model is possible to represent the system dynamics by a set of fuzzy implications that characterize local relations in the state space. This set is described by fuzzy **IF-THEN** rules which represents local linear input-output relations of the system.

For the class of nonlinear discrete-time systems considered, the ith rule is described by :

Plant Rule *i*: IF $\theta_1(k)$ is M_1^i, $\theta_2(k)$ is M_2^i and ... and $\theta_p(k)$ is M_p^i THEN

$$\begin{aligned} x(k+1) &= A_i x(k) + B_i w(k) \\ y(k) &= C_i x(k) + D_i w(k) \\ z(k) &= L_i x(k) \end{aligned} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state vector; $y(k) \in \mathbb{R}^m$ is the measured output vector; $z(k) \in \mathbb{R}^p$ is the signal to be estimated; and $w(k) \in \mathbb{R}^p$ is the external noise signal that is assumed to be the arbitrary signal in $l_2[0,\infty)$, A_i, B_i, C_i, D_i, and L_i are constant real matrices and of compatible dimensions; $\theta_1(k)$, $\theta_2(k)$,..., $\theta_p(k)$ represent the premise variables, and M_j^i, j=1,2,...,r, i=1,2,...,p, are fuzzy sets, r is the number of **IF-THEN** rules. For the sake of notational convenience, we denote $\mathcal{S}$:=$\{1, 2, ..., r\}$.

By fuzzy blending, the overall fuzzy model is inferred

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as follows:

$$\begin{aligned} x(k+1) &= A(h)x(k) + B(h)w(k) \\ y(k) &= C(h)x(k) + D(h)w(k) \\ z(k) &= L(h)x(k) \end{aligned} \quad (2)$$

where $A(h) = \sum_{i=1}^{r} h(\theta(k))A_i$, $B(h) = \sum_{i=1}^{r} h(\theta(k))B_i$, $C(h) = \sum_{i=1}^{r} h(\theta(k))C_i$, $D(h) = \sum_{i=1}^{r} h(\theta(k))D_i$, $L(h) = \sum_{i=1}^{r} h(\theta(k))L_i$, in which $\theta(k)$=[$\theta_1(k)$, $\theta_2(k)$,...,$\theta_p(k)$], $h_i(\theta(k))$=($\mu_i(\theta(k))/\sum_{i=1}^{r}\mu_i(\theta(k))$) and $\mu_i(\theta(k))$=$\Pi_{l=1}^{p}\zeta_{il}(\theta_l(k))$, $i \in \mathcal{S}$, in which $\zeta_{il}(\theta_l(k))$ is the membership degree of $\theta_i(k)$ in M_l^i. It is assumed that $\mu_i(\theta(k)) \geq 0$, $i \in \mathcal{S}$. Then, it can be seen that $\sum_{i=1}^{r}\mu_i(\theta(k)) > 0$ for all k. Therefore, for all k, $h_i(\theta(k)) \geq 0$, $i \in \mathcal{S}$, $\sum_{i=1}^{r} h(\theta(k)) = 1$.

The robust filtering problem addressed in this paper consists of obtaining an estimate $z_F(k)$ of the signal $z(k)$ based on the measurement set y(t), and provides the signal estimation error $e(k) \triangleq z(k) - z_F(k)$ for all $w(k) \in l_2[0,\infty)$. The main aim is to design a nonlinear filter, globally asymptotically stable of full order n and represented by local TS models. Using the same technique applied to (1) it follows that:

Filter Rule *i*: IF $\theta_1(k)$ is M_1^i, $\theta_2(k)$ is M_2^i and ... and $\theta_p(k)$ is M_p^i THEN

$$\begin{aligned} x_F(k+1) &= A_{Fi}x_F(k) + B_{Fi}y(k) \\ z_F(k) &= C_{Fi}x_F(k) + D_{Fi}y(k) \end{aligned} \quad (3)$$

where $x_F(k) \in \mathbb{R}^n$ is the filter state variable, and $z_F(k) \in \mathbb{R}^q$ is the output of the filter. A_{Fi}, B_{Fi}, C_{Fi} and D_{Fi} are the real matrices to be determined with appropriate dimensions.

The defuzzified output of (3) is inferred by:

$$\begin{aligned} x_F(k+1) &= A_F(h)x_F(k) + B_F(h)y(k) \\ z_F(k) &= C_F(h)x_F(k) + D_F(h)y(k) \end{aligned} \quad (4)$$

where $A_F(h) = \sum_{i=1}^{r} h(\theta(k))A_{Fi}$, $B_F(h) = \sum_{i=1}^{r} h(\theta(k))B_{Fi}$, $C_F(h) = \sum_{i=1}^{r} h(\theta(k))C_{Fi}$, $D_F(h) = \sum_{i=1}^{r} h(\theta(k))D_{Fi}$.

Defining $\xi(k) = [x(k)^T \;\; x_F(k)^T]^T$, the filtering error system can be written as

$$\begin{aligned} \xi(k+1) &= \tilde{A}(h)\xi(k) + \tilde{B}(h)w(k) \\ e(k) &= \tilde{C}(h)\xi(k) + \tilde{D}(h)w(k) \end{aligned} \quad (5)$$

where the filtering error output signal is denoted by $e(k) = z(k) - z_F(k)$ and
$\tilde{A}(h) = \begin{bmatrix} A(h) & 0 \\ B_F(h)C(h) & A_F(h) \end{bmatrix}$, $\tilde{D}(h) = -D_F(h)D(h)$,
$\tilde{B}(h) = \begin{bmatrix} B(h) \\ B_F(h)D(h) \end{bmatrix}$,
$\tilde{C}(h) = \begin{bmatrix} L(h) - D_F(h)C(h) & -C_F(h) \end{bmatrix}$.

The H_∞ norm of the transfer function matrix $G(z)$ of filtering error system (5), defined by: $\|G(z)\|_\infty = sup_{\omega\in[-\pi,\pi]}\bar{\sigma}[G(e^{j\omega})]$, where $\bar{\sigma}[.]$ denotes the maximum singular value of [.] and $G(z) = \tilde{C}(zI - \tilde{A})^{-1}\tilde{B} + \tilde{D}$.

Our objective in this paper is to develop new conditions for the existence of fuzzy H_∞ filters. Specifically, we are concerned with finding an asymptotically stable H_∞ in form of (4) such that two conditions are satisfied.

1) The filtering error system (5) is asymptotically stable when $w(k) \equiv 0$.
2) The filtering error system (5) has a prescribed level γ of H_∞ noise attenuation, i.e., under the zero initial condition $\xi(0) = 0$, $\|e\|_2 < \gamma\|w\|_2$ is satisfied for any nonzero $w(k) \in l_2[0,\infty)$.

In the following it is presented the solution to H_∞ filtering problem.

III. H_∞ Filtering Analysis

In this section, the filtering analysis problem is considered. More specifically, we assume that the filter matrices in (4) are known, and we will study the condition under which the filter error system (5) is asymptotically stable with H_∞-norm bounded γ. The following proposition shows that the H_∞ performance of the filtering error system can be guaranteed if there exist some matrix variables satisfying a certain matrix inequality. this proposition will play an instrumental role in the filter design problems.

Proposition 1 *Suppose system (2) and filter (4) are given, the filtering error system (5) is asymptotically stable with H_∞-norm bounded γ if there exist the symmetric positive definite matrix $0 < P(h) \in \mathbb{R}^{2n\times 2n}$, nonsingular matrix $M(h) \in \mathbb{R}^{2n\times 2n}$, and general matrices $S(h) \in \mathbb{R}^{2n\times 2n}$, $G(h) \in \mathbb{R}^{2n\times p}$ and $F(h) \in \mathbb{R}^{2n\times m}$ such that the following matrix inequality holds:*

$$\Theta(h) = \begin{bmatrix} \Gamma_1(h) & \Gamma_2(h) & \Gamma_4(h) & \Gamma_5(h) \\ \Gamma_2^T(h) & \Gamma_3(h) & \Gamma_6(h) & -F(h) \\ \Gamma_4^T(h) & \Gamma_6^T(h) & \Gamma_7(h) & \Gamma_8(h) \\ \Gamma_5^T(h) & -F^T(h) & \Gamma_8^T(h) & -I \end{bmatrix} < 0 \quad (6)$$

where $h^+ \triangleq h(\theta(k+1))$, and

$$\begin{aligned} \Gamma_1(h) &= P(h^+) - P(h) + S^T(h)(\tilde{A}(h) - I) \\ &\quad +(\tilde{A}(h) - I)^T S(h) \\ \Gamma_2(h) &= P(h^+) - S^T(h) + (\tilde{A}(h) - I)^T M(h) \\ \Gamma_3(h) &= P(h^+) - M^T(h) - M(h) \\ \Gamma_4(h) &= S^T(h)\tilde{B}(h) + (\tilde{A}(h) - I)^T G(h) \\ \Gamma_5(h) &= (\tilde{A}(h) - I)^T F(h) + \tilde{C}^T(h) \\ \Gamma_6(h) &= M^T(h)\tilde{B}(h) - G(h) \\ \Gamma_7(h) &= \tilde{B}^T(h)G(h) + G^T(h)\tilde{B}(h) - \gamma^2 I \\ \Gamma_8(h) &= \tilde{B}^T(h)F(h) + \tilde{D}^T(h) \end{aligned}$$

Proof: We first show that the filtering error system (5) with $w(k) \equiv 0$ is asymptotically stable and then prove that, under the zeros initial condition, $\|z\|_2 < \gamma\|w\|_2$ holds for all nonzero $w(k) \in l_2[0,\infty)$. To prove the first part, we choose the following fuzzy Lyapunov function:

$$V(k) = \xi^T(k)P(h)\xi(k).$$

When $w(k) \equiv 0$, the first equation in (5) becomes

$$\xi(k+1) = \tilde{A}(h)\xi(k). \tag{7}$$

Defining $\zeta(k) = \xi(k+1) - \xi(k)$ and taking the forward difference of V(k) along the solution trajectories of system (5) given in [7] as:

$$\begin{aligned} \Delta V(k) &= V(k+1) - V(k) \\ &= \xi^T(k+1)P(h^+)\xi(k+1) - \xi^T(k)P(h)\xi(k) \\ &= \lambda^T(k)\Phi(h)\lambda(k) \end{aligned}$$

where $\lambda(k) = [\xi^T(k) \ \ \zeta^T(k)]^T$, and

$$\Phi(h) = \begin{bmatrix} \Gamma_1(h) & \Gamma_2(h) \\ \Gamma_2^T(h) & \Gamma_3(h) \end{bmatrix}$$

where $\Gamma_1(h)$, $\Gamma_2(h)$, and $\Gamma_3(h)$ are given in (6).
By [7], $\Delta V(k) < 0$ for nonzero $\lambda(k)$, That implies the filtering error system (5) with $w_k \equiv 0$ is asymptotically stable.
To establish the H_∞ performance of system (5) under initial condition, for any $N \in \{1, 2, ...\}$, we introduce the following index:

$$J_N \triangleq \sum_{k=0}^{N-1} \{e^T(k)e(k) - \gamma^2 w^T(k)w(k)\}$$

Under the zero initial condition, $V(k)|_{k=0} = 0$, and we have

$$\begin{aligned} J_N \ \leq\ & \sum_{k=0}^{N-1} \{-e^T(k)e(k) - \gamma^2 w^T(k)w(k) + \Delta V(k) \\ & +2[\xi^T(k)S^T(h) + \zeta^T(k)M^T(h) + w^T(k)G^T(h) \\ & +e^T(k)F^T(h)] \times [(\tilde{A}(h) - I)\xi(k) - \zeta(k) \\ & +\tilde{B}(h)w(k)] + e^T(k)[\tilde{C}(h)\xi(k) + \tilde{D}(h)w(k)] \\ & +[\xi^T(k)\tilde{C}^T(h) + w^T(k)\tilde{D}^T(h)]e(k)\} \\ =\ & \sum_{k=0}^{N-1} \eta^T(k)\Theta(h)\eta(k) \end{aligned} \tag{8}$$

where $\eta(k) = [\xi^T(k) \ \ \zeta^T(k) \ \ w^T(k) \ \ e^T(k)]^T$, and $\Theta(h)$ given in (6).
(6) and (8) implies $J_N < 0$ for any N, i.e., $\|e\|_2 < \gamma\|w\|_2$ is satisfied for all nonzero $w(k) \in l_2[0,\infty)$. This completes the proof. ■

Remark 1 *In the derivation of proposition 1, four slack variables M(h), S(h), G(h) and F(h) are introduced. By setting G(h)=0 and F(h)=0, Proposition 1 coincides with the results of Lemma 1 in [7]. Thus, Proposition 1 would generally render a less conservative evaluation the upper bound of the H_∞ norm, which can be seen from the numerical example.*

IV. Fuzzy H_∞ Filter Design

In this section, a new liberalization will be established for designing a fuzzy H_∞ filter in (4), that is, to determine the filter matrices in (4) such that the filter error system (5) is asymptotically stable with H_∞-norm bounded γ.

Based on Proposition 1, and we choose the four slack variables as following forms:

$$\begin{aligned} S &= \begin{bmatrix} S_1 & S_2 \\ \hat{S} & \hat{S} \end{bmatrix}, \quad M = \begin{bmatrix} M_1 & M_2 \\ \hat{S} & \hat{S} \end{bmatrix}, \\ G &= \begin{bmatrix} G_1 \\ 0 \end{bmatrix}, \quad F = \begin{bmatrix} F_1 \\ 0 \end{bmatrix} \end{aligned} \tag{9}$$

Proposition 2 *Suppose system (1) is given, an admissible full-order fuzzy H_∞ filter in the form of (4) exits if there exit matrices $0 < \tilde{P} \in \mathbb{R}^{2n\times 2n}$, $S_1 \in \mathbb{R}^{n\times n}$, $S_2 \in \mathbb{R}^{n\times n}$, $M_1 \in \mathbb{R}^{n\times n}$, $M_2 \in \mathbb{R}^{n\times n}$, $F_1 \in \mathbb{R}^{n\times m}$, $G_1 \in \mathbb{R}^{n\times p}$, $\hat{S} \in \mathbb{R}^{n\times n}$, $\hat{A}_F \in \mathbb{R}^{n\times n}$, $\hat{B}_F \in \mathbb{R}^{n\times m}$, $\hat{C}_F \in \mathbb{R}^{q\times n}$, and $\hat{D}_F \in \mathbb{R}^{q\times m}$ satisfying*

$$\begin{bmatrix} \Phi_1(h) & \Phi_2(h) & \Phi_3(h) & \Phi_4(h) \\ * & \Phi_5(h) & \Phi_6(h) & -F(h) \\ * & * & \Phi_7(h) - \gamma^2 I & \Phi_8(h) \\ * & * & * & -I \end{bmatrix} < 0 \tag{10}$$

where
$\Phi_1(h) = \tilde{P}(h^+) - \tilde{P}(h) + \Sigma_1^T(h) + \Sigma_1(h)$
$- S^T(h) - S(h),$
$\Phi_2(h) = \tilde{P}(h^+) + \Sigma_5(h) - S^T(h) - M(h),$
$\Phi_3(h) = \Sigma_2(h) - G(h),$
$\Phi_4(h) = \Sigma_3(h) - F(h),$
$\Phi_5(h) = \tilde{P}(h^+) - M^T(h) - M(h),$
$\Phi_6(h) = \Sigma_4(h) - G(h),$
$\Phi_7(h) = B^T(h)G_1(h) + G_1^T(h)B(h),$
$\Phi_8(h) = B^T(h)F_1(h) - D^T(h)\hat{D}_F^T(h)$
with

$$\Sigma_1(h) = \begin{bmatrix} S_1^T(h)A(h) + \hat{B}_F(h)C(h) & \hat{A}_F(h \\ S_2^T(h)A(h) + \hat{B}_F(h)C(h) & \hat{A}_F(h \end{bmatrix},$$

$$\Sigma_2(h) = \begin{bmatrix} S_1^T(h) + \hat{B}_F h)D(h) + A^T(h)G_1(h) \\ S_2^T(h) + \hat{B}_F h)D(h) \end{bmatrix},$$

$$\Sigma_3(h) = \begin{bmatrix} L^T(h) - C^T(h)\hat{D}_F^T(h) + A^T(h)F_1(h) \\ -\hat{C}_F^T(h) \end{bmatrix},$$

$$\Sigma_4(h) = \begin{bmatrix} S_1^T(h) + \hat{B}_F h)D(h) \\ S_2^T(h) + \hat{B}_F h)D(h) \end{bmatrix},$$

$$\Sigma_5(h) = \begin{bmatrix} M_1^T(h)A(h) + \hat{B}_F(h)C(h) & \hat{A}_F(h \\ M_2^T(h)A(h) + \hat{B}_F(h)C(h) & \hat{A}_F(h \end{bmatrix}.$$

and S(h), M(h), G(h), and F(h) defined in (9).

The matrices for an admissible full-order fuzzy H_∞ filter given by

$$A_F = \hat{S}^{-T}\hat{A}_F, B_F = \hat{S}^{-T}\hat{B}_F, C_F = \hat{C}_F, D_F = \hat{D}_F \tag{11}$$

Now, we are in the position to present our main result.

Theorem 1 *Suppose system (1) is given, an admissible full-order fuzzy H_∞ filter in the form (4) exists if there exist* $0 < \tilde{P}_i \in \mathbb{R}^{2n\times 2n}$, $S_{1i} \in \mathbb{R}^{n\times n}$, $S_{2i} \in \mathbb{R}^{n\times n}$, $M_{1i} \in \mathbb{R}^{n\times n}$, $M_{2i} \in \mathbb{R}^{n\times n}$, $F_{1i} \in \mathbb{R}^{n\times m}$, $G_{1i} \in \mathbb{R}^{n\times p}$, $\hat{S}i \in \mathbb{R}^{n\times n}$, $\hat{A}_{Fi} \in \mathbb{R}^{n\times n}$, $\hat{B}_{Fi} \in \mathbb{R}^{n\times m}$, $\hat{C}_{Fi} \in \mathbb{R}^{q\times n}$, *and* $\hat{D}_{Fi} \in \mathbb{R}^{q\times m}$ $i \in \mathcal{S}$ *satisfying the following LMIs:*

$$\Upsilon_{iil} < 0, \quad i,l \in \mathcal{S} \tag{12}$$

$$\Upsilon_{ijl} + \Upsilon_{jil} < 0, \quad i < j; \quad i,j,l \in \mathcal{S} \tag{13}$$

where

$$\Upsilon_{ijl} = \begin{bmatrix} \Phi_{1ijl} & \Phi_{2ijl} & \Phi_{3ij} & \Phi_{4ij} \\ \Phi_{2ijl}^T & \Phi_{5il} & \Phi_{6ij} & -F_i \\ \Phi_{3ij}^T & \Phi_{6ij}^T & \Phi_{7ij} - \gamma^2 I & \Phi_{8ij} \\ \Phi_{4ij}^T & -F_i^T & \Phi_{8ij}^T & -I \end{bmatrix}$$

$$\begin{aligned}
\Phi_{1ijl} &= \tilde{P}_l - \tilde{P}_i + \Sigma_{1ij}^T + \Sigma_{1ij} - S_i^T - S_i, \\
\Phi_{2ijl} &= \tilde{P}_l + \Sigma_{5ij} - S_i^T - M_i, \\
\Phi_{3ij} &= \Sigma_{2ij} - G_i, \\
\Phi_{4ij} &= \Sigma_{3ij} - F_i, \\
\Phi_{5il} &= \tilde{P}_l - M_i^T - M_i, \\
\Phi_{6ij} &= \Sigma_{4ij} - G_i, \\
\Phi_{7ij} &= B_j^T G_{1i} + G_{1i}^T B_j, \\
\Phi_{8ij} &= B_j^T F_{1i} - D_j^T \hat{D}_{Fi}^T \\
\Sigma_{1ij} &= \begin{bmatrix} S_{1i}^T A_j + \hat{B}_{Fi} C_j & \hat{A}_{Fi} \\ S_{2i}^T A_j + \hat{B}_{Fi} C_j & \hat{A}_{Fi} \end{bmatrix}, \\
\Sigma_{2ij} &= \begin{bmatrix} S_{1i}^T + \hat{B}_{Fi} D_j + A_j^T G_{1i} \\ S_{2i}^T + \hat{B}_{Fi} D_j \end{bmatrix}, \\
\Sigma_{3ij} &= \begin{bmatrix} L_j^T - C_j^T \hat{D}_{Fi}^T + A_j^T F_{1i} \\ -\hat{C}_{Fi}^T \end{bmatrix}, \\
\Sigma_{4ij} &= \begin{bmatrix} S_{1i}^T + \hat{B}_{Fi} D_j \\ S_{2i}^T + \hat{B}_{Fi} D_j \end{bmatrix}, \\
\Sigma_{5ij} &= \begin{bmatrix} M_{1i}^T A_j + \hat{B}_{Fi} C_j & \hat{A}_{Fi} \\ M_{2i}^T A_j + \hat{B}_{Fi} C_j & \hat{A}_{Fi} \end{bmatrix}. \\
S_i &= \begin{bmatrix} S_{1i} & S_{2i} \\ \hat{S}_i & \hat{S}_i \end{bmatrix}, \quad M_i = \begin{bmatrix} M_{1i} & M_{2i} \\ \hat{S}_i & \hat{S}_i \end{bmatrix}, \\
G_i &= \begin{bmatrix} G_{1i} \\ 0 \end{bmatrix}, \quad F_i = \begin{bmatrix} F_{1i} \\ 0 \end{bmatrix}
\end{aligned}$$

The matrices for an admissible full-order fuzzy H_∞ filter given by

$$\begin{aligned}
A_{Fi} &= \hat{S}_i^{-T} \hat{A}_{Fi}, \quad B_F = \hat{S}_i^{-T} \hat{B}_{Fi}, \\
C_{Fi} &= \hat{C}_{Fi}, \quad D_{Fi} = \hat{D}_{Fi}
\end{aligned} \tag{14}$$

Proof: From Proposition 2, an admissible full-order fuzzy H_∞ filter exits if there exit matrix functions $\tilde{P}(h) > 0$, $S_1(h)$, $S_2(h)$, $M_1(h)$, $M_2(h)$, $F_1(h)$, $G_1(h)$ and matrix $\hat{S}$ satisfying (10).

$$\tilde{P}(h) = \sum_{i=1}^{r} h(\theta(k))\tilde{P}_i = \sum_{i=1}^{r} h(\theta(k)) \begin{bmatrix} \tilde{P}_{1i} & \tilde{P}_{2i} \\ \tilde{P}_{2i}^T & \tilde{P}_{3i} \end{bmatrix}$$

$$\begin{aligned}
S_1(h) &= \sum_{i=1}^{r} h(\theta(k))S_{1i}, \quad S_2(h) = \sum_{i=1}^{r} h(\theta(k))S_{2i} \\
M_1(h) &= \sum_{i=1}^{r} h(\theta(k))M_{1i}, \quad M_2(h) = \sum_{i=1}^{r} h(\theta(k))M_{2i} \\
F_1(h) &= \sum_{i=1}^{r} h(\theta(k))F_{1i}, \quad G_1(h) = \sum_{i=1}^{r} h(\theta(k))G_{1i} \\
A_F(h) &= \sum_{i=1}^{r} h(\theta(k))A_{Fi}, \quad B_F(h) = \sum_{i=1}^{r} h(\theta(k))B_{Fi} \\
C_F(h) &= \sum_{i=1}^{r} h(\theta(k))C_{Fi}, \quad D_F(h) = \sum_{i=1}^{r} h(\theta(k))D_{Fi}
\end{aligned}$$

For simplicity, define the left-hand side of a relation as LH(.). Then, from (12) and (13), we have

$$\begin{aligned}
LH(12) &= \sum_{i=1}^{r}\sum_{l=1}^{r} h_i^2 h_l^+ \Upsilon_{iil} \\
&\quad + \sum_{i=1}^{r}\sum_{j=i+1}^{r}\sum_{l=1}^{r} h_i h_j h_l^+ (\Upsilon_{ijl} + \Upsilon_{jil}) < 0
\end{aligned}$$

which implies that all the conditions of Proposition 2 hold and that the result immediately follows. ■

V. Numerical Example

Consider the following discrete time fuzzy system:

$$\begin{aligned}
x(k+1) &= A(h)x(k) + B(h)w(k) \\
y(k) &= C(h)x(k) + D(h)w(k) \\
z(k) &= L(h)x(k)
\end{aligned} \tag{15}$$

where

$$\begin{aligned}
A(h) &= \sum_{i=1}^{2} h_i A_i, \quad B(h) = \sum_{i=1}^{2} h_i B_i, \quad C(h) = \sum_{i=1}^{2} h_i C_i, \\
D(h) &= \sum_{i=1}^{2} h_i D_i, \quad L(h) = \sum_{i=1}^{2} h_i L_i,
\end{aligned}$$

with

$$\begin{aligned}
A_1 &= \begin{bmatrix} 1.05 & 0.35 \\ -0.42 & 0.07 \end{bmatrix} & A_2 &= \begin{bmatrix} 0.792 & -0.432 \\ -0.36 & 0 \end{bmatrix} \\
B_1 &= \begin{bmatrix} -1 \\ -0.5 \end{bmatrix} & B_2 &= \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\
C_1 &= \begin{bmatrix} 1.71 & 2.85 \end{bmatrix} & C_2 &= \begin{bmatrix} -1.9 & 2.28 \end{bmatrix} \\
D_1 &= \begin{bmatrix} 0.005 \end{bmatrix} & D_2 &= \begin{bmatrix} 0.005 \end{bmatrix} \\
L_1 &= \begin{bmatrix} 0.81 & 0.27 \end{bmatrix} & L_2 &= \begin{bmatrix} 0.4 & 1.2 \end{bmatrix}
\end{aligned}$$

The membership functions are given as follows: $M_1(x_1(k)) = \left\{ \begin{array}{ll} |\frac{sin(x_1(k))}{x_1(k)}|, & for\ x_1(k) \neq 0 \\ 1, & for\ x_1(k) = 0 \end{array} \right\}$ and $M_2(x_1(k)) = 1 - M_1(x_1(k))$.

By solving LMIs (12-13), the minimum H_∞ attenuation level γ_{min} are listed in table1:

	$Our\ Theorem$	$Theorem\ 1[7]$	$Theorem\ 3[9]$
γ_{min}	4.8855	5.8247	11.6585

Table 1. Calculation Results of Example

And the filter matrices are obtained:

$$A_{F1} = \begin{bmatrix} 1.0411 & 0.5402 \\ -0.4935 & -0.2135 \end{bmatrix},$$
$$A_{F2} = \begin{bmatrix} 0.7435 & -0.1810 \\ -0.2632 & 0.3088 \end{bmatrix},$$
$$B_{F1} = \begin{bmatrix} -0.1109 \\ -0.0542 \end{bmatrix}, \qquad B_{F2} = \begin{bmatrix} 0.0970 \\ 0.0115 \end{bmatrix},$$
$$C_{F1} = \begin{bmatrix} -0.5066 & 0.0828 \end{bmatrix},$$
$$C_{F2} = \begin{bmatrix} -0.7814 & -0.4945 \end{bmatrix},$$

$$D_{F1} = [0.2182], \qquad D_{F2} = [0.2400]$$

The simulation results of Transfer function and the squared roots of the ratio of estimation error's energy to disturbance's energy, which is defined by

$$\gamma(k) = \sqrt{\frac{\sum_{i=0}^{k} e^T(i)e(i)}{\sum_{i=0}^{k} w^T(i)w(i)}}$$

is shown in Fig.1, where the noise signal is chosen as

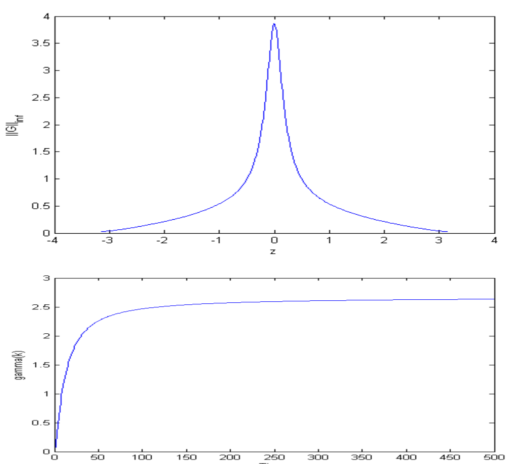

Fig. 1: Simulation results

$w(k) = (2 + k^{1.3})^{-1}$, k=1,2,.... It can be seen that the squared root $\gamma(k)$ is about 2.672, which reveals that the H_∞ performance level is less than the prescribed level, i.e.,4.8855.

VI. Conclusion

In this article, we investigated the H_∞ filtering problem for a class of discrete-time nonlinear systems described by the T-S fuzzy dynamic model. we used the fuzzy Lyapunov function approach and adding slack matrix variables, a new condition for H_∞ performance analysis is proposed with LMI technique. The numerical example is used to illustrate the effectiveness of the proposed method.

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