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Adaptive Observer's Design for A Nonlinear Hybrid System

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Abstract— **In this paper, a design of an adaptive observer for a class of uniformly observable MIMO nonlinear hybrid system is handled. It deals with a simultaneous estimation of the whole state as well as the unknown parameters while guaranteeing the recovering of the systems states and the reducing of the error estimation convergence during the transient period. The effectiveness of such observers is highlighted in the simulation results applied to the physical model of a quadruple tank process.**

Index Terms— **Adaptive observer; nonlinear switched systems; parameter estimation; quadruple tank process.**

I. INTRODUCTION

A larger class of systems has adopted the structure of hybrid systems that combine continuous and discrete dynamic behaviour. Typical examples of such systems include vehicles and motors control, chemical process, robotic systems, transmission [1,2]. To succeed in the control of hybrid dynamic systems, the real evolution of different states of the process must be known. Therefore the observability is crucial especially for the switched systems. In literature, many important researchs for linear switched systems were found in the past decade [3,4]. Indeed, authors introduced a set of observer designs which allow the reconstruction of the discrete and continuous states from the knowledge of the continuous and discrete outputs [5]. In [6], a design of Luenbeger-like observer for a class of linear hybrid systems is considered. Other designs of observer for a class of linear hybrid systems are treated in [7,8].

More recently, it has appeared that some parameters of a big number of physical systems could be unknown which leads the researchers to spread the last works on a new algotithms which are more compatible with the new conditions [9]. Early developments on adaptive observation are discussed for linear systems [10] and then for state affine systems in continuous-time [11]. A solution to the observation problem for a class of MIMO state affine systems with constant unknown parameters and discrete time output measurments is adressed in [12] assuming some persistent excitation conditions.

In the nonlinear case and comparing to the linear class, the synthesis of observers for hybrid systems presented more difficulties and only few results are obtained [13]. In [14] an obsever based on a step by step sliding mode observation is treated to estimate continuous and discrete states. Another nonlinear obsever for autonomous switching systems with jumps is developped in [15]. An approach to continuous and discrete observer design for Lipshitz nonlinear systems is discussed in [16] with the defining of a convergence conditions in terms of matrix inequalities for hybrid observer. Despite the variety of methods of adaptive observer design for a class of MIMO nonlinear system dressed, as well as the various basic problems solved using different approaches in [17,18] the problem of how to combine these techniques for a classe of nonlinear hybrid dynamic system has not been treated yet.

The observer designed in this work is an extension of the nonlinear adaptive observer structure presented in [19,20] and an adoption of an adaptive nonlinear hybrid design system (ANHS). It consist in a developped approach to perform a continuous state and parameter estimation jointly for a class of uniformly observable hybrid systems. However, the discrete mode or location is supposed to be known to guarantee the observability of the global system.

An important contribution of this paper is in the adoption of a class of the entire nonlinear hybrid system to develop a state and a parameter estimation shema that guarantee the fast convergence and ensure the stability of the overall system. Generally, for the class of hybrid sytems, we have to reinitialize the estimated states when the mode changes to avoid the chattering phenomena during the transient period. This jump lead to the loss of information if the observer is not robust. To analyse the performances of the nonlinear hybrid observer presented in this paper, we can suppose that the initial condition values for estimated states and parameters are not fixed for all subsystems.

The remainder of this paper is organized as follows. Section II is devoted to introduce the problem formulation and so the class of the nonlinear hybrid systems highlighted in the this work. Then, in Section III, we present a state and a parameter estimation where the transition conditions are known. The description for the quadruple tank process is presented in Section IV and simulation results are shown in order to illustrate the performance of the designed hybrid observer. Finally, some comments and conclutions are given.

II. PROBLEM FORMULATION

In this section, we shall state the main problem under investigation which is an estimation of both continuous states and nonmeasured parameters for a class of nonlinear hybrid systems. Based on the work reported in [19] for MIMO nonlinear systems, the class of the system is described by

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the following subsystems:

$$\sum_{g} \left\{ \begin{array}{l} \dot{x} = A_g x + \text{Ł}_g(u_g, \wp_g, x) \\ y = C_g x. \end{array} \right. \tag{1}$$

where:

- The mode location is $g \in \nabla = \{1, 2, ..., N\}$.
- The states: $x \in \Re^n$; $x = \begin{pmatrix} x^1 \\ x^2 \\ \vdots \\ x^q \end{pmatrix}$; $x^k = \begin{pmatrix} x_1^k \\ x_2^k \\ \vdots \\ x_{\ell_k}^k \end{pmatrix} \in \Re^{n_k}$

  $x_i^k = \begin{pmatrix} x_{i,1}^k \\ x_{i,2}^k \\ \vdots \\ x_{p,k}^k \end{pmatrix}$; $x_i^k \in \Re^{p_k}$; $x_{i,j}^k \in \Re$.
- The output: $y = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_q \end{pmatrix} \in \Re^p$; $y_k = \begin{pmatrix} y_{k,1} \\ y_{k,2} \\ \vdots \\ y_{k,p_k} \end{pmatrix}$;
  $y_k \in \Re^{p_k}$; $y_{k,j} \in \Re$.
- The vector of the unknown constant parameters:
  $\wp_g = \begin{pmatrix} \wp_{g,1} \\ \wp_{g,2} \\ \vdots \\ \wp_{g,m} \end{pmatrix}$; $\wp_g \in \Re^m$; $\wp_{g,i} \in \Re$.
- The matrices A_g, C_g, and Ł_g are as follows:
  $$A_g = diag(A_{g,1}, \ldots, A_{g,q}) \tag{2}$$
  with $A_{g,k} = \begin{bmatrix} 0 & I_{g,p_k} & & 0 \\ \vdots & & \ddots & 0 \\ 0 & \ldots & 0 & I_{g,p_k} \\ 0 & \ldots & 0 & 0 \end{bmatrix}$;

  $$C_g = diag(C_{g,1}, \ldots, C_{g,q}) \tag{3}$$
  with $C_{g,k} = (C_{g,p_k}, 0, \ldots, 0)$, where $k = 1, \ldots, q$; $i = 1, \ldots, \ell_k$; $j = 1, \ldots, p_k$; $\sum_{k=1}^{p} n_k = n$ and $\sum_{k=1}^{q} p_k = p$.

  and

  $$\text{Ł}_g(u_g, \wp_g, x) = \begin{pmatrix} \text{Ł}_g^1(u_g, \wp_g, x) \\ \text{Ł}_g^2(u_g, \wp_g, x) \\ \vdots \\ \text{Ł}_g^q(u_g, \wp_g, x) \end{pmatrix} \in \Re^n. \tag{4}$$

  with $\text{Ł}_g^k(u_g, \wp_g, x) = \begin{pmatrix} \text{Ł}_{g,1}^1(u_g, \wp_g, x) \\ \text{Ł}_{g,2}^2(u_g, \wp_g, x) \\ \vdots \\ \text{Ł}_{g,\ell_k}^q(u_g, \wp_g, x) \end{pmatrix} \in \Re^{n_k}$;

  where $\text{Ł}_{g,i}^k \in \Re^{p_k}$.

We can suppose that the non linearities Ł_g depend on the state as shown:
For $1 \leq i \leq \ell_k - 1$:

$$\text{Ł}_{g,i}^k(u_g, \text{Ł}_g, x) = \text{Ł}_{g,i}^k(u_g, \wp_g, x^1, x^2, x^{k-1}, x_1^k, x_2^k, \tag{5}$$
$$x_i^k, \ldots, x_1^{k+1}, \ldots, x_1^q)$$

For $i = \ell_k$:

$$\text{Ł}_{g,i}^k(u_g, \text{Ł}_g, x) = \text{Ł}_{g,i}^k(u_g, \wp_g, x, x^2, \ldots, x^q). \tag{6}$$

Based on (1), the idea is to select the adequate subsystem which can describe the dynamic behavior of the hybrid system. Infact, to specify the active subsystem at every instant of time and to have acceptable switching strategies, the mode location g is supposed to be known. If the subsystem is specified and every subsystem is considered to be uniformly observable one can conclude the whole observability of the studied system.

III. OBSERVER SYNTHESIS

In this section, we will be interested to design an adaptive observer class for nonlinear switched systems. The objective of the designed estimation observer is to allow the state and the paramter estimation for a class of a nonlinear hybrid system. It consist on using an appropriate adaptive observer for the global nonlinear system. Infact, for every subsytem and on switching between different values of the gain to achieve better parameters and states estimation and to garantree smooth error convergence. An observer for system (1) can be written in the following form:

$$\left\{ \begin{array}{l} \dot{\hat{x}}(t) = A_{g,k}\hat{x}^k + \text{Ł}_g^k(u_g, \hat{\wp}_g, \underline{\hat{x}}) - (O_g^{\aleph_k} \triangle_{g,k}^{-1} S_k^{-1} C_k^T K_k(C_k \tilde{x}^k) \\ \quad +O_g^{(-\aleph_k/2)+1} \bigwedge_{g,k}^{-1} Y_{g,k}(t) P_g(t) \\ \quad \sum_{l=1}^{q} O_g^{-\pounds_1^l + (\aleph_l/2)} Y_{g,l}^T(t) C_l^T K_l(C_l \tilde{x}^l)) \\ \dot{\hat{\wp}}(t) = -P_g(t) \sum_{l=1}^{q} O_g^{-\pounds_1^l + (\aleph_l/2) + (1/2)} Y_{g,l}^T(t) C_l^T K_l(C_l \tilde{x}^l) \end{array} \right. \tag{7}$$

where:

- $\hat{x} = \begin{pmatrix} \hat{x}^1 \\ \hat{x}^2 \\ \vdots \\ \hat{x}^q \end{pmatrix} \in \Re^n$; $\hat{x}^k = \begin{pmatrix} \hat{x}_1^k \\ \hat{x}_2^k \\ \vdots \\ \hat{x}_{\ell_k}^k \end{pmatrix} \in \Re^{n_k}$;
- $\underline{\hat{x}} = \begin{pmatrix} \underline{\hat{x}}^1 \\ \underline{\hat{x}}^2 \\ \vdots \\ \underline{\hat{x}}^q \end{pmatrix} \in \Re^n$; $\underline{\hat{x}}^k = \begin{pmatrix} \underline{\hat{x}}_1^k \\ \underline{\hat{x}}_2^k \\ \vdots \\ \underline{\hat{x}}_{\ell_k}^k \end{pmatrix} = \begin{pmatrix} \hat{x}_1^k \\ \hat{x}_2^k \\ \vdots \\ \hat{x}_{\ell_k}^k \end{pmatrix}$;
  $\hat{x}_i^k \in \Re^{p_k}$; $\underline{\hat{x}}^k \in \Re^{n(k)}$; $\underline{\hat{x}}_i^k \in \Re^{p_k}$; $k = 1 \ldots q$ with $\underline{\hat{x}}_1^k = \hat{x}_1^k$ and $\underline{\hat{x}}_i^k = \hat{x}_i^k$ for $i = 2 \ldots \ell_k$. $\sum_{k=1}^{p} n_k = n$.
  u_k and y_k are the inputs and the outputs of the system (1).
  The matrices A_k and C_k are defined respectively by (2) and (3).
- $\triangle_{g,k}$ and $\bigwedge_{g,k}$ is the bloc diagonal matrixes given by:
  $$\triangle_{g,k}(O_g) = diag(I_{p_k}, \tfrac{1}{O_g^{\aleph_k}} I_{p_k}, \ldots, \tfrac{1}{O_g^{(\aleph_k)(\ell_k - 1)}} I_{p_k}) \tag{8}$$

$$\bigwedge_{g,k}(O_g) = diag(\frac{1}{O_g^{\pounds_1^k}} I_{p_k}, \frac{1}{O_g^{\pounds_2^k}} I_{p_k}, \ldots, \frac{1}{O_g^{(\pounds_{\ell_k}^k)}} I_{p_k}) \quad (9)$$

with $O_g > 0$ a real number.

- S is the unique solution of the algebraic Lyapunov equation:
$$S_k + A_k^T S_k + S_k A_k - C_k^T C_k = 0$$
Furthermore, it was demonstrated in [1] that:
$$S_k^{-1} C_k^T = (C_{\ell_k}^1 I_{p_k}, \ldots, C_{\ell_k}^{\ell_k} I_{p_k})^T \quad (10)$$
where: $C_{\ell_k}^{\ell_k} = \frac{\ell_k}{i!(\ell_k - 1)!}$ for $i = 1 \ldots \ell_k$.
From the S_k , one defines the bloc diagonal matrix S as follows:
$$S = diag(S_1, S_2, \ldots, S_q)$$
- The real's $\aleph_k$ and $\pounds_i^k$ are as shown:

$$\begin{cases} \aleph_k = 2^{q-k}(\prod_{i=k+1}^q (\ell_k - \frac{3}{2})), \\ \aleph_k = 1 \end{cases} \quad (11)$$

for $k = 1, \ldots, q-1$.

- We can also defines the following sequence of scalar numbers for $k = 1, \ldots, q$ and $i = 1, \ldots, \ell_k$ as in [1]:
$$\pounds_i^k = \pounds_1^k + (i-1)\aleph_k \quad (12)$$
with:
$$\pounds_l^k = -(\ell_k - 1)\aleph_k + (\ell_1 - 1)\aleph_1 + (1 - \frac{1}{2^{k-i}})$$
We can deduce that:
$k = 1, \ldots, q$ and $i = 1, \ldots, \ell_k$: $\pounds_i^k \geq 0$
  - For $k = 1$, one has:
  $\pounds_i^k = (i-1)\aleph_1 \geq 0$ for $i \geq 1$ since $\aleph_1 > 0$ (13)
  - For $k \geq 2$, one has:
  $$\pounds_i^k = (\ell_1 - 1)\aleph_1 \geq 0 \quad (14)$$

- $Y_{g,k}$ is the solution of the following Ordinary Differential Equation (ODE):
$$\dot{Y}_{g,k}(t) = O_g^{\aleph_k}(A_k - S_k^{-1} C_k^T C_k) y_{g,k}(t) \quad (15)$$
$$+ O_g^{\frac{\aleph_k}{2} - \frac{1}{2}} \bigwedge_{g,k} \frac{\partial \pounds_g^k}{\partial \wp}(u_g, \hat{\wp}_g, \hat{\underline{x}})$$

  with $Y_k(0) = 0$; $k = 1, \ldots, q$.

- $P(t)$ is the solution of Ordinary Differential Equation (ODE).
$$\dot{P}_g(t) = -O_g P_g(t)(\sum_{k=1}^q Y_{g,k}^T(t) C_k^T C_k Y_{g,k}(t)) P_g(t) \quad (16)$$
$$+ O_g P_g(t)$$
with the choice of initial value, $P(t_0) \in \Re^m \times \Re^m$, symmetric definite and positive.
For $K_k : \Re^{P_k} \to \Re^{P_k}$, one has:
$$\forall \tilde{y}_k \in \Re^{p_k} : \tilde{y}_k^T K_k(\tilde{y}_k) \geq \frac{1}{2} \tilde{y}_k^T \tilde{y}_k \quad (17)$$

The observer synthesis necessites the following assumptions:

Assumption 3.1: [19] The state x, the input u_g and the unknown parameters $\wp_{g,i}$, $i = 1 \ldots m$ are bounded. Infact, there exist subsets $X \subset \Re^n$, $U \subset \Re^s$ and $\nabla \in \Re^m$such that for all $t \geq 0$, $x(t) \in X$, $u_g(t) \in U$ and $\wp_g \in \nabla$.

Assumption 3.2: The function $\pounds_g(u_g, \wp_g, x)$ is Lipschitz with respect to $\wp_g$ and x uniformly in u_g.

Assumption 3.3: The inputs $u_g(t)$ are that for any trajectory $\hat{x}(t)$ of the system (7), with $(\hat{x}(0), \hat{\wp}(0)) \in \Re^n \times \Re^m$, the matrix $(\sum_{k=1}^q Y_{g,k}^T(t) C_k^T C_k Y_{g,k}(t))$ is persistently exciting: $\exists \aleph_1, \aleph_2 > 0$; $T > 0$,$\forall t \geq 0$;

$$\aleph_1 I_m \leq (\sum_{k=1}^q \int_t^{t+T} Y_{g,k}^T(t) C_k^T C_k Y_{g,k}(t)) \leq \aleph_2 I_m \quad (18)$$

The state x and the parameters vector $\wp_g$ are confined to the bounded (A1) and $\pounds_g$ is considered as Lipshitz on the space $\Re^n \times \Re^m$ (A2). x and $\wp_g$ are defined to be smooth bounded saturation variables. Using the Lyapunov function, we can demonstrate that adopting (A1), (A2) and supposing that the inputs $u_g(t)$ are bounded, we can deduce the exponential convergence of the nonlinear system. Indeed, we state the following Theorem:

Theorem 3.1: If the nonlinear hybrid system described by (1) satisfy Assumptions (A1)-(A3), then, the system (7) is an adaptive observer with an exponential convergence for relatively high values of the design parameter for all O_g to a specified subsystem.
The proof of the Theorem (3.1) is in [19].

Assumption 3.4: There exists a positive real number $\Im_{min}$ defined by: $\Im_{i+1} - \Im_i \geq \Im_{min}$; $i = 1, \ldots, N$.
With $(\Im_{i+1} - \Im_i)$ define the duration between two switches.

Satisfying the Theorem and a good choice of $\Im_{min}$ can guarantee the exponential convergence of the hybrid observer. Assumption (A4) allow then the convergence of the observer before a new switch.

Theorem 3.2: If the nonlinear hybrid system described by (1) satisfy the Theorem (3.1) and the assumption (A4), then, the system (7) is an adaptive observer with an exponential convergence for relatively high values of the design parameter for all O_g to a global system (1).

The proof of the Theorem (3.2) will be demonstrated in a future works.

IV. APPLICATION TO A QUADRUPLE TANK PROCESS

A. Description of The Example

In this section, the effectiveness of an adaptive hybrid observer for nonlinear switched system previously described is highlighted. We defines a real example of quadruple tank process presented in [22]. The system presented in this paper is a multivariable process which consists of four interconnected water tanks as shown in fig. 1. This process has a great importance especially in chemical engineering laboratories. The quadruple tank system is considered as a suitable model to study the performances of the observers.

The flows of the independent pumps (P_1, P_2) are the two inputs (Q_1, Q_2) of the quadruple tank process and the levels in the four tanks (h_1, h_2, h_3, h_4) are the outputs. The two pumps are used to convey liquid from a basin into tank 3 and tank 4, respectively. They drain freely into tank 1 and tank 2, respectively. The upper tanks collect the liquid from the two tanks 1 and 2. The liquid levels in the bottom two tanks are measured using two ultrasonic sensors.

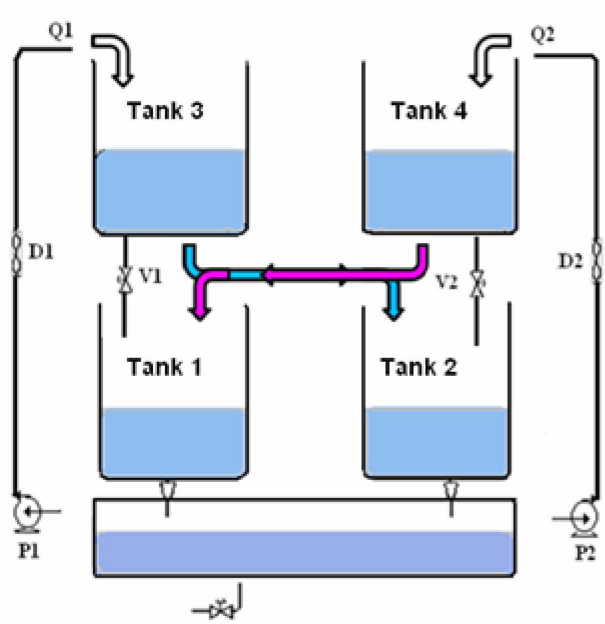

Fig. 1. The quadruple tank process.

The differential equations that describe the system dynamycs are given by:

- Model 1: $$\begin{pmatrix} \dot{x}_1 = -C_1\sqrt{x_1}+C_2\sqrt{x_3}+C_3\sqrt{x_4} \\ \dot{x}_2 = -C_4\sqrt{x_2}+C_5\sqrt{x_3}+C_6\sqrt{x_4} \\ \dot{x}_3 = -C_7\sqrt{x_3}+C_8u_1 \\ \dot{x}_4 = -C_9\sqrt{x_4}+C_{10}u_2 \end{pmatrix}; \quad (19)$$

- Model 2: $$\begin{pmatrix} \dot{x}_1 = -C_1\sqrt{x_1}+C_2\sqrt{x_3} \\ \dot{x}_2 = -C_4\sqrt{x_2}+C_6\sqrt{x_4} \\ \dot{x}_3 = -C_7\sqrt{x_3}+C_8u_1 \\ \dot{x}_4 = -C_9\sqrt{x_4}+C_{10}u_2 \end{pmatrix}; \quad (20)$$

- Model 3: $$\begin{pmatrix} \dot{x}_1 = -C_1\sqrt{x_1}+C_3\sqrt{x_4} \\ \dot{x}_2 = -C_4\sqrt{x_2}+C_5\sqrt{x_3} \\ \dot{x}_3 = -C_7\sqrt{x_3}+C_8u_1 \\ \dot{x}_4 = -C_9\sqrt{x_4}+C_{10}u_2 \end{pmatrix} \quad (21)$$

$y = (x_1 \quad x_2)^T$ for all subsystems.

Where the state vector $x(t) \in R^4$ represent the liquid levels in different tanks, $u(t) = (u_1u_2)^T \in R^2$, denote the flow of every pump, the output vector $y(t) = (h_1h_2)^T \in R^2$, and the constants of the system $C_i, i = 1, \ldots, 10$.
The purpose of this work is to estimate the two upper tanks (x_3, x_4) and some unknown parameters of the process. (x_1, x_2) are supposed to be measured. To put the models of the quadruple tank process (19)-(20)-(21) under the nonlinear system class (1), one shall apply the following changes of coordinates: (a) for the model 1, (b) for the model 2 and (c) for the model 3.

$$a : \begin{pmatrix} z_1 = x_1 \\ z_2 = x_2 \\ z_3 = C_2\sqrt{x_3}+C_3\sqrt{x_4} \\ z_4 = C_5\sqrt{x_3}+C_6\sqrt{x_4} \end{pmatrix} ; b : \begin{pmatrix} z_1 = x_1 \\ z_2 = x_2 \\ z_4 = C_2\sqrt{x_3} \\ z_4 = C_6\sqrt{x_4} \end{pmatrix}$$

and $c : \begin{pmatrix} z_1 = x_1 \\ z_2 = x_2 \\ z_3 = C_3\sqrt{x_4} \\ z_4 = C_5\sqrt{x_3} \end{pmatrix}$.

We can deduce the models of the quadruple tank process in the new coordinates. The new representation coincide with the class of the system (1).

- Model 1: $$\begin{pmatrix} \dot{z}_1 = z_3 - C_1\sqrt{z_1} \\ \dot{z}_2 = z_4 - C_4\sqrt{z_2} \\ \dot{z}_3 = -\frac{C_2C_7+C_3C_9}{2} \\ +\frac{C_2C_6-C_3C_5}{2}[\frac{C_2C_8}{C_6z_3-C_3z_4}u_1 - \frac{C_3C_{10}}{C_5z_3-C_2z_4}u_2] \\ \dot{z}_4 = -\frac{C_5C_7+C_6C_9}{2} \\ +\frac{C_2C_6-C_3C_5}{2}[\frac{C_5C_8}{C_6z_3-C_3z_4}u_1 - \frac{C_6C_{10}}{C_5z_3-C_2z_4}u_2] \end{pmatrix};$$

- Model 2: $$\begin{pmatrix} \dot{z}_1 = z_3 - C_1\sqrt{z_1} \\ \dot{z}_2 = z_4 - C_4\sqrt{z_2} \\ \dot{z}_3 = -\frac{C_2^2}{2} + \frac{C_2^2C_8u_1}{2z_3} \\ \dot{z}_4 = -\frac{C_6^2}{2} + \frac{C_6^2C_{10}u_2}{2z_4} \end{pmatrix};$$

- Model 3: $$\begin{pmatrix} \dot{z}_1 = z_3 - C_1\sqrt{z_1} \\ \dot{z}_2 = z_4 - C_4\sqrt{z_2} \\ \dot{z}_3 = -\frac{C_3^2}{2} + \frac{C_3^2C_{10}u_2}{2z_3} \\ \dot{z}_4 = -\frac{C_5^2}{2} + \frac{C_5^2C_8u_1}{2z_4} \end{pmatrix};$$

The system output is $y = (z_1 \quad z_2)^T$.

B. Simulation Results

We apply the nonlinear hybrid adaptive observer (7) to estimate both the parameters (C_1, C_8) and so (C_4, C_{10}) of the system and the unknown states. The vector of estimated parameters is defined as:

$$\wp = \begin{pmatrix} \wp_1 \\ \wp_2 \end{pmatrix} = \begin{pmatrix} C_1 \\ C_8 \end{pmatrix} \in \Re^2 \quad (22)$$

The initial conditions of the real and estimated states are $z(0) = [0.001 \quad 0.001 \quad 2.5 \quad 2]^T$ and $z(0) = [0.001 \quad 0.001 \quad 4.5 \quad 4]^T$, respectively. The initial conditions of the unknown parameters are $\hat{\wp}(0) = [0 \quad 0]^T$.
This section is dedicated to illustrate the performances of the observer proposed in the previous part. Infact, we can choise the different parameters for all subsystems as follows:

for $g = 1,2,3$; $q = 2$; $\ell_{g,1} = 2$; $\ell_{g,2} = 2$; $\aleph_{g,1} = 1$; $\aleph_{g,2} = 1$; $\pounds^1_{g,1} = 0$; $\pounds^2_{g,1} = \frac{1}{2}$; $\pounds^1_{g,2} = 1$; $\pounds^2_{g,2} = \frac{3}{2}$.

TABLE I

PRAMETERS VALUES OF THE SYSTEM

Prameters	Values
C_1, C_4	0.005
C_2, C_6	0.014
C_3, C_5	0.02
C_7, C_9	0.02
C_8, C_{10}	43

One can also adopt a design of K_k as $K_k(\tilde{y}_k) = \tilde{y}_k$ satisfying the property (17).

The expression of the inputs u_g is used in simulations that bounded by $u_{max} = 1ml/s$. The mode location g is fixed as shown in fig. 2 The optimal values of O_g chosen are equal to 15, 3 and 1.5 for $g = 1,2,3$ respectively. This choice insures fast convergence and good speed. So the results of real and estimated states are drawn in figure.2 for z_3 and in figure.3 for z_4. Besides, the obtained estimation of the nonmeasured parameters for all subsystems is depicted in figure 4 which demonstrate the convergence of different parameters to the true values. The Simulation of the hybrid adaptive observer (HNAO) shows the effectiveness of such observer to estimate both the continuous states and parameters of the system so as to give a satisfactory convergence when the transient value g is known without the choice of initial conditions for all states and parameters of subsystems. The estimation evolution gives a satisfactory convergence and demonstrates the robustness of such observer.

To demonstrate the robustness of such observer, one can change the values of the unknown parameters for every subsystem. The parameter values used in simulation are so $(C_1, C_8) = (0.001, 53)$ for subsystem 1, $(C_1, C_8) = (0.009, 43)$ for subsystem2 and $(C_1, C_8) = (0.005, 63)$ for subsystem 3. We have reported in fig. 6 and 7 the estimation of the new values. The results can be considered acceptable as they show fast convergence in all subsystems and a respectable behaviour as well.

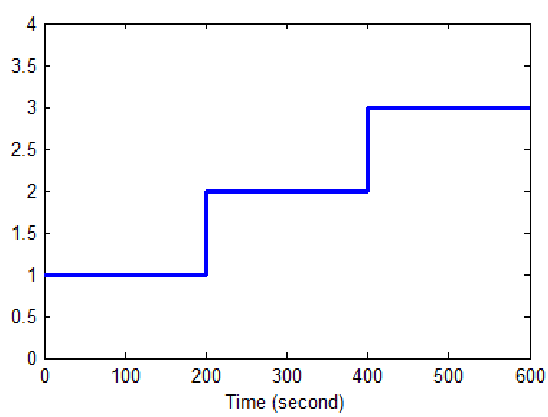

Fig. 2. Evolution of the mode location g.

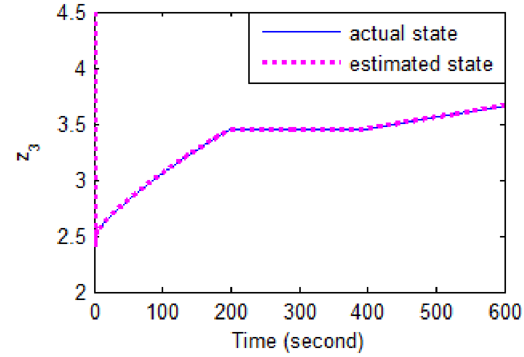

Fig. 3. State estimation 3 using HNAO.

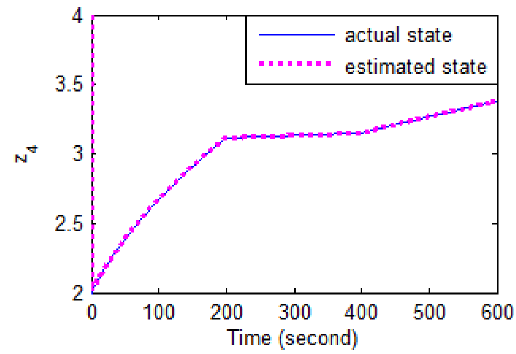

Fig. 4. State estimation 4 using HNAO.

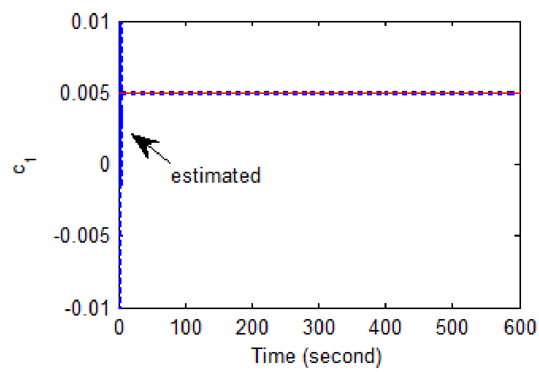

Fig. 5. Parameter estimation c_1 using HNAO.

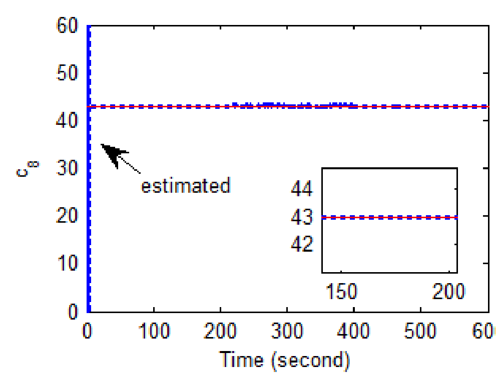

Fig. 6. Parameter estimation c_8 using HNAO.

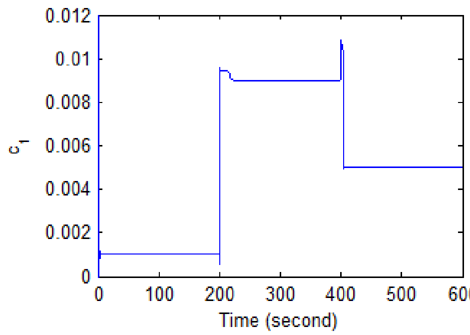

Fig. 7. Parameter estimation c_1 using HNAO.

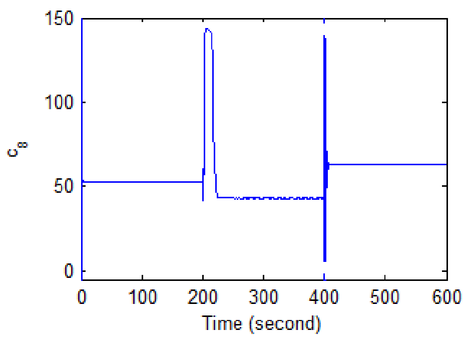

Fig. 8. Parameter estimation c_8 using HNAO.

V. Conclusions

In this paper, an adaptive observer for a large class of MIMO uniformly observable nonlinear hybrid systems is adressed. This class of systems is composed by cascade blocks with non trivial interconnections which facilitate the synthesis of the observer. Infact, we proposed a model-based adaptive observer for this large class of nonlinear hybrid systems. Simulation results carried out of a quadruple tank process under realistic conditions have been reported. These simulations showed the good capabilities of the designed adaptive observer in providing good estimates for the non-measurable states and the unknown parameters of the hybrid process.

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