Decentralized observers for a class of large scale singular systems via LMI

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***Abstract*— This paper deals with the problem of observers design for large scale interconnected singular systems. A novel decentralized observer is proposed based on a new parameterization of the generalized Sylvester equations solution. The proposed approach unifies the large scale system observers design for the full-order, the reduced-order and the minimal order observers. Conditions for the existence and the stability of these observers are provided in terms of linear matrix inequalities (LMIs).**

I. INTRODUCTION

It is widely known, that large-scale systems are essential in our modern life. In general, a large scale system can be distinguished by a large number of variables and a complex structure where we can face with problems with many equations and/or unknowns (see e.g. [13]).To solve these problems, the use of some special techniques is required, where distributed solutions are adopted for technical or practical reasons. Thanks to decomposition techniques, several problems can be solved in a decentralized form, where large scale systems are considered as a set of interconnected subsystems. The problem of decentralized observer based control, has received many attention and several approaches have been developed to design decentralized local state estimator or feedback controllers (see for example, [1], [2]). The facility offered by this distributed form is to overcome complexity which allows the feasibility of observer based on a control implementation. However, the main difficulty of this representation is in handling of the interconnected terms between subsystems. For this reason, most of studies developed about decentralized observer based on a control assume that interconnections between subsystems are available [9], [10], This is not usually the case in practice. In this paper, we consider the problem of a decentralized state estimation, when the interconnections between subsystems are not available. A new decentralized observer is proposed based on the parameterization of the generalized Sylvester equations solution. The organization of the paper is as follows. The second section presents preliminary results, the problem is stated in section 3. The decentralized observer is developed in section 4, with discussion of the observers stability. A numerical example is proposed in section 5 to show the application of our results.

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II. PRELIMINARY RESULTS

In this section, we provide some basic results and notations that will be used throughout this paper.

1) Let $\mathcal{E}$ be an $(n \times p)$-dimensional matrix, the symbol $Ker^T(\mathcal{E})$ represents the set of all matrices Φ of full row such that $\Phi\mathcal{E} = 0$.
2) The symbol Σ^+ will be used to denote any generalized inverse of the matrix Σ, satisfying $\Sigma\Sigma^+\Sigma = \Sigma$.

Let us consider the standard system described by

$$\sigma x(t) = Ax(t) + Bv(t) \quad (1)$$
$$z(t) = Hx(t) + \mathcal{D}v(t) \quad (2)$$

where A, B, H and $\mathcal{D}$ are constant matrices of appropriate dimensions and where $\sigma x(t) = \dot{x}(t)$ for the continuous-time system, $t \in \mathbb{R}$, and $\sigma x(t) = x(t+1)$ for the discreet-time system, $t \in \mathbb{N}$.

The H_∞ performance index J_{zv} for the continuous-time case is given by

$$J_{zv}^c = \int_0^\infty (z^T(t)\, z(t) - \gamma_c^2 v^T(t)\, v(t))dt \quad (3)$$

for a given $\gamma_c > 0$ and for the discreet-time case, it is

$$J_{zv}^d = \sum_{t=0}^{\infty} (z^T(t)\, z(t) - \gamma_d^2 v^T(t)\, v(t)) \quad (4)$$

for a given $\gamma_d > 0$.

III. PROBLEM STATEMENT AND BASIC ASSUMPTIONS

We consider a linear descriptor large scale interconnected system composed of N subsystems described by the following equations:

$$E\sigma X(t) = AX(t) + Bu(t) \quad (5)$$
$$Z(t) = HX(t) \quad (6)$$

where vectors $X(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^q$ and $Z(t) \in \mathbb{R}^m$ denote the state, the known input and the known output respectively. Matrices A, B, H and E are constant real-valued such as : $E \in \mathbb{R}^{n\times n}$, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times q}$, $H \in \mathbb{R}^{m\times n}$, with E singular. Let the vectors $X(k)$ and $Z(k)$ be partitioned as:

$$X(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_N(t) \end{bmatrix} \text{ and } Z(t) = \begin{bmatrix} z_1(t) \\ z_2(t) \\ \vdots \\ z_N(t) \end{bmatrix},$$

and let matrices E, A, B and H be partitioned according to

the partition of $X(t)$ as follows:

$$E = \begin{bmatrix} E_1 & 0 & \dots & 0 \\ 0 & E_2 & \dots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & \dots & \dots & E_N \end{bmatrix}$$

$$A = \begin{bmatrix} A_{11} & A_{12} & \dots & A_{1N} \\ A_{21} & A_{22} & \dots & A_{2N} \\ \vdots & & \ddots & \vdots \\ A_{N1} & A_{N2} & \dots & A_{NN} \end{bmatrix},$$

$$B = \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_N \end{bmatrix}, \; H = \begin{bmatrix} H_1 & 0 & \dots & 0 \\ 0 & H_2 & \dots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & \dots & \dots & H_N \end{bmatrix}.$$

The above partition provides the following model given by system equations $(7-8)$ with a global input $u(t)$, it can be always obtained by using similarity transformations (see for example [3], [5], [8] and [12]), However, there exist other decentralization methods that provide models with local inputs u_i like in hierarchical decentralization (see for example [2] and [4]).
Now, the large scale system $(5-6)$ can be rewritten as a set of N interconnected subsystems in the following form:

$$E_i\sigma x_i(t) = A_{ii}x_i(t) + B_iu(t) + \sum_{j=1,j\neq i}^{N} A_{ij}x_j(t) \quad (7)$$

$$z_i(t) = H_ix_i(t), \; i \in \{1 \dots N\} \quad (8)$$

where vectors $x_i(t) \in \mathbb{R}^{n_i}$ and $z_i(t) \in \mathbb{R}^{m_i}$ are the state and output measurement respectively of the i-th subsystem and $n = n_1 + \ldots + n_N$ and $m = m_1 + \ldots + m_N$. Matrices $E_i \in \mathbb{R}^{n_i \times n_i}$, $A_{ii} \in \mathbb{R}^{n_i \times n_i}$, $A_{ij} \in \mathbb{R}^{n_i \times n_j}$, $B_i \in \mathbb{R}^{n_i \times q}$, $H_i \in \mathbb{R}^{m_i \times n_i}$.

In the following, we will design a decentralized observer for the interconnected system $(7-8)$, where the interconnections of $i-th$ subsystem with other subsystems are assumed to be unknown. Let a matrix D_i be defined as

$$D_i = \begin{bmatrix} A_{i1} & \dots & A_{i(i-1)} & A_{i(i+1)} & \dots & A_{iN} \end{bmatrix}$$

and $v_i(t)$ is the unknown interconnection given by:

$$v_i(t) = \begin{bmatrix} x_1(t) \dots & x_{i-1}(t) & x_{i+1}(t) & \dots & x_N(t) \end{bmatrix}.$$

In this case system $(7-8)$ can be written as

$$E_i\sigma x_i(t) = A_{ii}x_i(t) + B_iu(t) + D_iv_i(t) \quad (9)$$

$$z_i(t) = H_ix_i(t), \; i \in \{1 \dots N\} \quad (10)$$

Now, let a partition of the matrix D_i and the vector v_i as: $D_iv_i = \begin{bmatrix} D_{i1} & D_{i2} \end{bmatrix} \begin{bmatrix} v_{i1} \\ v_{i2} \end{bmatrix}$ such that D_{i1} be of full column rank, then there exist, a full row rank matrix $\Phi_i \in \mathbb{R}^{r\varphi_i \times n_i}$ of $Ker^T(E_i) \cap Ker^T(D_{i1})$ such that $\Phi_i \begin{bmatrix} E_i & D_{i1} \end{bmatrix} = 0$.

Remark 1: The partition of the matrix D_i and the vector v_i is made so that we can extract full rank column part D_{i1} of the hole matrix D_i, because for unknown interconnection observer, the condition of existence is that necessary that the matrix D_{i1} representing the unknown interconnection is full rank column. We can always do this partition, since the matrix D_i is different to zero. If the matrix $D_i = 0$, we have a standard observer with a known inputs measurement $u(t)$.

Now, for the sequel of the paper, we make the following assumption which corresponds to the condition of the existence of the unknown interconnections observers.

Assumption 1:

$$rank \begin{bmatrix} E_i & D_{i1} \\ \Phi_iA_{ii} & 0 \\ H_i & 0 \end{bmatrix} = n_i + rank\ D_{i1}.$$

Remark 2: Assumption 1 is a generalization of the existence condition of unknown inputs observers for singular system, for non singular case, i.e. $E_i = I_{n_i}$, we have $\Phi_i = 0$ and Assumption 1 becomes:

$$rank \begin{bmatrix} I_{n_i} & D_{i1} \\ H_i & 0 \end{bmatrix} = n_i + rank\ D_{i1}.$$

or equivalently $rank\,(H_iD_{i1}) = rank\,(D_{i1})$ (see [6]).

The objective is to determine a completely decentralized state estimation for each subsystem given by $(9-10)$ satisfying the global performances obtained by the centralized observers for system $(5-6)$.

IV. Decentralized observer for large scale interconnected system

In this section, we consider the problem of the state estimation of the i-th subsystem given by $(9-10)$.

A. Observer design

Let the decentralized observer for system $(9-10)$ be of the following form:

$$\sigma\xi_i(t) = N_i\xi_i(t) + M_iu(t) + J_iz_i(t) \quad (11)$$

$$\hat{x}_i(t) = P_i\xi_i(t) - Q_i\Phi_iB_iu(t) + F_iz_i(t) \quad (12)$$

where $\hat{x}_i$ is the estimate of x_i, ξ_i is the state of the observer, matrices N_i, M_i, J_i, P_i, Q_i and F_i are the appropriate dimensions and must be determined such that $e_i(t) = \hat{x}_i(t) - x_i(t)$ is unbiased, i.e. independent of $u(t)$ and $x_i(t)$, and converges to zero as $t \to \infty$.
The following proposition gives the conditions for the existence and the stability of the observer given by $(11-12)$.

Proposition 1: For $v_{i2}(t) = 0$, the system $(11-12)$ is an asymptotic observer (i.e.: $\lim_{t\to\infty} \widehat{x}_i(t) - x_i(t) = 0$) if :

i) $\sigma\epsilon_i(t) = N_i\epsilon_i(t)$ is asymptotically stable.
ii) $N_iT_iE_i - T_iA_i + J_iH_i = 0$
iii) $T_iD_{i1} = 0$
iv) $M_i = T_iB_i$
v) $\begin{bmatrix} P_i & Q_i & F_i \end{bmatrix} \begin{bmatrix} T_iE_i \\ \Phi_iA_i \\ H_i \end{bmatrix} = I_{n_i}$

Proof 1: Let $\epsilon_i(t) = \xi_i(t) - T_iE_ix_i(t)$ be the error between $\xi_i(t)$ and $T_iE_ix_i(t)$, then its dynamic is given by:

$$\begin{aligned} \sigma\epsilon_i(t) &= \sigma\xi_i(t) - T_iE_i\sigma x_i(t) \quad (13) \\ &= N_i\epsilon_i(t) + (N_iT_iE_i + J_iH_i - T_iA_{ii})\,x_i(t) \\ &+ (M_i - T_iB_i)\,u(t) - T_iD_{i1}v_{i1}(t) - T_iD_{i2}v_{i2}(t) \end{aligned}$$

The error $\epsilon_i(t)$ is independent of the state $x_i(t)$. The input $u(t)$, the unknown interconnections $v_{i1}(t)$, and the observer is asymptotically stable for $v_{i2}(t) = 0$ if

$$\sigma\epsilon_i(t) = N_i\epsilon_i(t)$$

is asymptotically stable and

$$\begin{aligned} N_iT_iE_i - T_iA_{ii} + J_iH_i &= 0 \\ T_iD_{i1} &= 0 \\ M_i &= T_iB_i \end{aligned}$$

what corresponds in condition $ii)$, $iii)$ and $iv)$ of Proposition 1.
On the other hand, the equation (12) can be written as:

$$\hat{x}_i(t) = P_i\epsilon_i(t) - Q_i\Phi_iB_iu(t) + F_iH_ix_i(t) \tag{14}$$

from equation (9), since $\Phi_iE_i = 0$ we can deduce that:

$$-\Phi_iB_iu(t) = \Phi_iA_{ii}x(t) + \Phi_iD_{i2}v_{i2}(t) \tag{15}$$

Then using (15), we can write the equation (14) as:

$$\begin{aligned} \hat{x}_i(t) &= P_i\epsilon_i(t) + Q_i\Phi_iD_{i2}v_{i2}(t) \\ &\quad + \begin{bmatrix} P_i & Q_i & F_i \end{bmatrix} \begin{bmatrix} T_iE_i \\ \Phi_iA_{ii} \\ H_i \end{bmatrix} x_i(t) \end{aligned} \tag{16}$$

consequently, if $v)$ is satisfied and $v_{i2}(t) = 0$ we obtain $e_i(t) = \hat{x}_i(t) - x_i(t) = P_i\epsilon_i(t)$ and $\lim_{t\to\infty} e_i(t) = 0$ since $\lim_{t\to\infty} \epsilon_i(t) = 0$
This completes the proof.

Remark 3: For $v_{i2}(t) \neq 0$, and if the equations $ii)$, $iii)$, $iv)$ and $v)$ of Proposition 1 are satisfied, we obtain:

$$\begin{aligned} \sigma\epsilon_i(t) &= N_i\epsilon_i(t) - T_iD_{i2}v_{i2}(t) &(17)\\ e_i(t) &= P_i\epsilon_i(t) + Q_i\Phi_iD_{i2}v_{i2}(t) &(18) \end{aligned}$$

B. Observer parameters determination

Now, in order to solve the equations $ii)$, $iii)$ and $iv)$ of Proposition 1, we define a matrix $T_i^{'}$ such that

$$T_i^{'} = T_i + \Psi_i\Phi_i \tag{19}$$

where Ψ_i is an arbitrary matrix. Under Assumption 1, and by using the fact that $\Phi_iE_i = 0$, we can rewrite the equations $ii)$ and $v)$ of Proposition 1 as:

$$\begin{bmatrix} N_i & \Psi_i & J_i \\ P_i & Q_i & F_i \end{bmatrix} \begin{bmatrix} T_i'E_i \\ \Phi_iA_{ii} \\ H_i \end{bmatrix} = \begin{bmatrix} T_i'A_{ii} \\ I_{n_i} \end{bmatrix} \tag{20}$$

On the other hand, since $\Phi_iD_{i1} = 0$, the equation $iii)$ of Proposition 1 is equivalent to $T_i^{'}D_{i1} = 0$, which have the following general solution

$$T_i^{'} = \bar{T}_i(I_{n_i} - D_{i1}D_{i1}^{+}) \tag{21}$$

where $\bar{T}_i$ is an arbitrary matrix to be determined. Then, the equation (20) becomes:

$$\begin{bmatrix} N_i & \Psi_i & J_i \\ P_i & Q_i & F_i \end{bmatrix} \begin{bmatrix} \bar{T}_i\bar{E}_i \\ \Phi_iA_{ii} \\ H_i \end{bmatrix} = \begin{bmatrix} \bar{T}_i\bar{A}_{ii} \\ I_{n_i} \end{bmatrix} \tag{22}$$

where

$$\bar{E}_i = (I_{n_i} - D_{i1}D_{i1}^{+})E_i \text{ and } \bar{A}_{ii} = (I_{n_i} - D_{i1}D_{i1}^{+})A_{ii}.$$

Consequently, the design of the decentralized observer given by $(11-12)$ is reduced to finding the matrices $\bar{T}_i$, N_i, Ψ_i, J_i, P_i, Q_i, F_i and M_i satisfying condition (22).

In the sequel, we give a solution for the generalized Sylvester equation (22), but before, we can give the following lemma which relates Assumption 1 to the rank of the matrix $\begin{bmatrix} \bar{E}_i \\ \Phi_iA_{ii} \\ H_i \end{bmatrix}$.

Lemma 1: The following assertions are equivalent:

1) $rank \begin{bmatrix} E_i & D_{i1} \\ \Phi_iA_{ii} & 0 \\ H_i & 0 \end{bmatrix} = n_i + rankD_{i1}$

2) $rank \begin{bmatrix} \bar{E}_i \\ \Phi_iA_{ii} \\ H_i \end{bmatrix} = n_i.$

Proof 2: Let $U_i = \begin{bmatrix} I - D_{i1}D_{i1}^{+} & 0 \\ D_{i1}D_{i1}^{+} & 0 \\ 0 & I \end{bmatrix}$ be a full column rank matrix and $V_i = \begin{bmatrix} I & 0 \\ -D_{i1}^{+}E_i & I \end{bmatrix}$ be a non singular matrix, then we have the following equality

$$\begin{aligned} rank \begin{bmatrix} E_i & D_{i1} \\ \Phi_iA_{ii} & 0 \\ H_i & 0 \end{bmatrix} &= rank \left(U_i \begin{bmatrix} E_i & D_{i1} \\ \Phi_iA_{ii} & 0 \\ H_i & 0 \end{bmatrix} V_i \right) \\ &= rank \begin{bmatrix} \bar{E}_i & 0 \\ 0 & D_{i1} \\ \Phi_iA_{ii} & 0 \\ H_i & 0 \end{bmatrix} \\ &= rank \begin{bmatrix} \bar{E}_i \\ \Phi_iA_{ii} \\ H_i \end{bmatrix} + rankD_{i1} \\ &= n_i + rankD_{i1} \end{aligned}$$

what proves the lemma.

Now, let R_i be a full row rank matrix such that $\Sigma_i = \begin{bmatrix} R_i \\ \Phi_iA_{ii} \\ H_i \end{bmatrix}$ is a full column rank matrix, i.e. $rank\Sigma_i = n_{ii}$.

Then, under Assumption 1, there always exist parameter matrices $\bar{T}_i$ and K_i, such that (see [7]):

$$\begin{bmatrix} \bar{T}_i & K_i \end{bmatrix} \begin{bmatrix} \bar{E}_i \\ \Phi_iA_{ii} \\ H_i \end{bmatrix} = R_i \tag{23}$$

The equation (23) has a solution if and only if

$$rank \begin{bmatrix} \bar{E}_i \\ \Phi_iA_{ii} \\ H_i \\ R_i \end{bmatrix} = rank \begin{bmatrix} \bar{E}_i \\ \Phi_iA_{ii} \\ H_i \end{bmatrix} = n_i. \tag{24}$$

under Assumption 1 and since $rank\Sigma_i = n_i$, this condition is satisfied.

In this case, the matrices $\bar{T}_i$ and K_i are given by

$$\bar{T}_i = R_i \begin{bmatrix} \bar{E}_i \\ \Phi_iA_{ii} \\ H_i \end{bmatrix}^{\dagger} \begin{bmatrix} I \\ 0 \end{bmatrix} \tag{25}$$

and

$$K_i = R_i \begin{bmatrix} \bar{E}_i \\ \Phi_i A_{ii} \\ H_i \end{bmatrix}^{\dagger} \begin{bmatrix} 0 \\ I \end{bmatrix}. \quad (26)$$

Also, under Assumption 1, the general solution to (22) is given by

$$\begin{bmatrix} N_i & \Psi_i & J_i \\ P_i & Q_i & F_i \end{bmatrix} = \begin{bmatrix} \bar{T}_i \bar{A}_{ii} \\ I_{n_i} \end{bmatrix} \Omega_i^{\dagger} - \begin{bmatrix} Y_i \\ Z_i \end{bmatrix} \left(I - \Omega_i \Omega_i^{\dagger} \right) \quad (27)$$

Where $\Omega_i = \begin{bmatrix} \bar{T}_i \bar{E}_i \\ \Phi_i A_{ii} \\ H_i \end{bmatrix} \in \mathbb{R}^{(n_i + r_{\varphi_i} + m_i) \times n_i}$, and $\begin{bmatrix} Y_i \\ Z_i \end{bmatrix}$ is an arbitrary matrix of appropriate dimension to be determined.

Now, define the following matrices

$$\begin{aligned}
\Lambda_{P_i} &= \Omega_i^{\dagger} \begin{bmatrix} I \\ 0 \\ 0 \end{bmatrix}, \Lambda_{Q_i} = \Omega_i^{\dagger} \begin{bmatrix} 0 \\ I \\ 0 \end{bmatrix}, \\
\Lambda_{F_i} &= \Omega_i^{\dagger} \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix}, \Delta_{P_i} = (I - \Omega_i \Omega_i^{\dagger}) \begin{bmatrix} I \\ 0 \\ 0 \end{bmatrix}, \\
\Delta_{Q_i} &= (I - \Omega_i \Omega_i^{\dagger}) \begin{bmatrix} 0 \\ I \\ 0 \end{bmatrix}, \Delta_{F_i} = (I - \Omega_i \Omega_i^{\dagger}) \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix}, \\
\Lambda_{N_i} &= \bar{T}_i \bar{A}_{ii} \Omega_i^{\dagger} \begin{bmatrix} I \\ 0 \\ 0 \end{bmatrix}, \Lambda_{\Psi_i} = \bar{T}_i \bar{A}_{ii} \Omega_i^{\dagger} \begin{bmatrix} 0 \\ I \\ 0 \end{bmatrix}, \\
\Lambda_{J_i} &= \bar{T}_i \bar{A}_{ii} \Omega_i^{\dagger} \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix}, \Delta_{N_i} = (I - \Omega_i \Omega_i^{\dagger}) \begin{bmatrix} I \\ 0 \\ 0 \end{bmatrix}, \\
\Delta_{\Psi_i} &= (I - \Omega \Omega^{\dagger}) \begin{bmatrix} 0 \\ I \\ 0 \end{bmatrix}, \Delta_{Ji} = (I - \Omega_i \Omega_i^{\dagger}) \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix}.
\end{aligned}$$

Then, we obtain

$$\begin{aligned}
N_i &= \Lambda_{N_i} - Y_i \Delta_{N i} && (28) \\
\Psi_i &= \Lambda_{\Psi_i} - Y_i \Delta_{\Psi_i} && (29) \\
J_i &= \Lambda_{J_i} - Y_i \Delta_{J i} && (30) \\
P_i &= \Lambda_{P_i} - Z_i \Delta_{P i} && (31) \\
Q_i &= \Lambda_{Q_i} - Z_i \Delta_{Q_i} && (32) \\
F_i &= \Lambda_{F_i} - Z_i \Delta_{F_i} && (33)
\end{aligned}$$

Now, we give the conditions for the existence of the parameter matrices Y_i and Z_i such that for $v_{i2}(t) = 0$, the system $(17-18)$ is asymptotically stable, and for $v_{i2}(t) \neq 0$, it satisfies the H_∞ performance index $J_{zv}^c < \gamma_c$ for the continuous-time case and $J_{zv}^d < \gamma_d$ for the discreet-time case.

C. Decentralized observer: continuous-time case

Theorem 1: Under Assumption 1 and for a given $\gamma_c > 0$, there exists an observer of the form $(11-12)$ such that the error $e_i(t)$ given by $(17-18)$ is asymptotically stable for $v_{i2}(t) = 0$ and $\|e_i(t)\|_2 < \gamma_c \|v_{i2}(t)\|_2$ for $v_{i2}(t) \neq 0$, if there exist a matrix $X_{c_i} = X_{c_i}^T > 0$ and a matrices Ω_{Y_i} and Z_i such that the following LMI is satisfied

$$\begin{bmatrix} (1,1) & (1,2) & \left(\Lambda_{P_i} - Z_i \Delta_{P i} \right)^T \\ (1,2)^T & \gamma_{c_i} I & (2,4)^T \\ \Lambda_{P_i} - Z_i \Delta_{P i} & (2,4) & \gamma_{c_i} I \end{bmatrix} > 0 \quad (34)$$

where:

$$\begin{aligned}
(1,1) &= -X_{c_i} N_i - N_i^T X_{c_i} \\
&= -X_{c_i} \Lambda_{N_i} + \Omega_{Y_i} \Delta_{N i} - \Lambda_{N_i}^T X_{c_i} + \Delta_{N i}^T \Omega_{Y_i}^T \\
(1,2) &= -X_{c_i} T_i D_{i2} = -X_{c_i} \left(T_i^{'} - \Psi_i \Phi_i \right) D_{i2} \\
&= -X_{c_i} \bar{T}_i (I_{n_i} - D_{i1} D_{i1}^{+}) D_{i2} \\
&+ X_{c_i} \Lambda_{\Psi_i} \Phi_i D_{i2} - \Omega_{Y_i} \Delta_{\Psi_i} \Phi_i D_{i2} \\
(2,4) &= -Q_i \Phi_i D_{i2} = -\Lambda_{Q_i} \Phi_i D_{i2} + Z_i \Delta_{Q_i} \Phi_i D_{i2}
\end{aligned}$$

where $\Omega_{Y_i} = X_{c_i} Y_i$.

Proof 3: From $i)$ of Proposition 1 and using the fact that $T_i = \bar{T}_i (I - D_{i1} D_{i1}^{+}) - \Psi_i \Phi_i$, we can write

$$\begin{aligned}
\dot{\epsilon}_i(t) &= N_i \epsilon_i(t) - T_i D_{i2} v_{i2}(t) \\
&= N_i \epsilon_i(t) - (\bar{T}_i \bar{D_{i2}} - \Psi_i \Phi_i D_{i2}) v_{i2}(t) \quad (35)
\end{aligned}$$

where $\bar{D}_{i2} = (I - D_{i1} D_{i1}^{+}) D_{i2}$.
and by using the equation (16), we obtain

$$e_i(t) = P_i \epsilon_i(t) + Q_i \Phi_i D_{i2} v_{i2}(t) \quad (36)$$

Therefore, applying the result of [11] to the continuous-time system given by (35) and (36) and by substituting $\bar{T}_i$, N_i, Ψ_i, P_i and Q_i by there values given respectively in $(25)-(28)-(29)-(31)$ and (32), and by taking $\Omega_{Y_i} = X_{c_i} Y_i$, we obtain the LMI of the Theorem 1.

D. Decentralized observer: discret-time case

Theorem 2: Under Assumption 1 and for a given $\gamma_d > 0$, there exists an observer of the form $(11-12)$ such that the error $e(t)$ given by $(17-18)$ is asymptotically stable for $v_{i2}(t) = 0$ and $\|e_i(t)\|_2 < \gamma_d \|v_{i2}(t)\|_2$ for $v_{i2}(t) \neq 0$, if there exist a matrix $X_{d_i} = X_{d_i}^T > 0$ and a matrix Ω_{Y_i} such that the following LMI is satisfied

$$\begin{bmatrix} X_{d_i} & 0 & (1,3) & (1,4) \\ 0 & \gamma_{d_i} I & \Lambda_{P_i} - Z_i \Delta_{P i} & (2,4) \\ (1,3)^T & \left(\Lambda_{P_i} - Z_i \Delta_{P i} \right)^T & X_{d_i} & 0 \\ (1,4)^T & (2,4)^T & 0 & \gamma_{d_i} I \end{bmatrix} > 0 \quad (37)$$

where:

$$\begin{aligned}
(1,3) &= X_{d_i} N_i = X_{d_i} \Lambda_{N_i} - X_{d_i} Y_i \Delta_{N_i} \\
&= X_{d_i} \Lambda_{N_i} - \Omega_{Y_i} \Delta_{N_i} \\
(1,4) &= -X_{d_i} T_i D_{i2} \\
&= -X_{d_i} \bar{T}_i (I_{n_i} - D_{i1} D_{i1}^{+}) D_{i2} + X_{d_i} \Lambda_{\Psi_i} \Phi_i D_{i2} \\
&- \Omega_{Y_i i} \Delta_{\Psi_i} \Phi_i D_{i2} \\
(2,4) &= Q_i \Phi_i D_{i2} = \Lambda_{Q_i} \Phi_i D_{i2} - Z_i \Delta_{Q_i} \Phi_i D_{i2}
\end{aligned}$$

where $\Omega_{Y_i} = X_{d_i} Y_i$.

Proof 4: the proof is similar to the one given for theorem 1 and it's omitted due to the lack of place.

Remark 4: We can see that the observer $(11-12)$ is of dimension $\beta_i \leqslant n_i$. Then, the presented approach unifies the observers design for the full-order ($\beta_i = n_i$), the reduced-order ($\beta_i = n_i - m_i$) and the minimal order observers (obtained for $\beta_i = n_i - r_{\varphi_i} - m_i$).

The algorithm for the decentralized observer design under Assumption 1, is given as follows:

- **Step 1** : Find a matrix R_i such that $\Sigma_i = \begin{bmatrix} R_i \\ \Phi_i A_{ii} \\ H_i \end{bmatrix}$ is of full column $rank$.
- **Step 2** : Compute the matrix $\bar{T}_i$ by using (25), then deduce the matrix $T_i^{'}$ from $T_i^{'} = \bar{T}_i \left(I_{n_i} - D_{i1} D_{i1}^{+}\right)$.
- **Step 3** : Solve the LMI of Theorem 1 for the continuous-time case and the LMI of Theorem 2 for the discreet-time case, to compute the parameter matrices Y_i and Z_i.
- **Step 4** : Compute $N_i, \Psi_i, J_i, P_i, Q_i$ and F_i by using the equation (28), (29), (30), (31), (32) and (33) respectively.
- **Step 5** : Compute T_i by using the fact that $T_i = T_i^{'} - \Psi_i \Phi_i$, then deduce M_i from $M_i = T_i B_i$.

In the following section, we apply the results developed in this paper to an illustrative example

V. Numerical example

Consider the following interconnected descriptor system of the form $(5-6)$ with:

$$\begin{aligned} E &= \begin{bmatrix} E_1 & 0 \\ 0 & E_2 \end{bmatrix}, A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \\ B &= \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, H = \begin{bmatrix} H_1 & 0 \\ 0 & H_2 \end{bmatrix}. \end{aligned}$$

According to the considered decomposition in section III, we have two subsystems:

Subsystem 1 is given by**:**

$$\begin{aligned} E_1 &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, A_{11} = 10^{-3} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}, \\ A_{12} &= D_1 = \begin{bmatrix} 0.001 & 0 & 0 \\ 0 & -0.002 & 0 \\ 0 & 0 & -0.003 \end{bmatrix}, \\ D_{11} &= \begin{bmatrix} 0.001 \\ 0 \\ 0 \end{bmatrix}, D_{12} = \begin{bmatrix} 0 & 0 \\ -0.002 & 0 \\ 0 & -0.003 \end{bmatrix}, \\ B_1 &= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \text{ and } H_1 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} \end{aligned}$$

For this system, matrix $\Phi_1 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$. In this case, it is easy to see that Assumption 1 is verified. We shall design a reduced-order observer of dimension $\beta_1 = 1$, let

$$R_1 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}.$$

The solution of the LMI of Theorem 2 gives

$$\begin{aligned} Z_1 &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ -222.04 & -0.40 & -67.98 & 67.98 & 135.97 \\ 0 & -0.002 & 0 & 0 & 0 \end{bmatrix} \\ Y_1 &= \begin{bmatrix} 0.44 & 0.0008 & 0.13 & -0.13 & 0-0.27 \end{bmatrix} \text{ and} \\ X_{d1} &= 1.3705. \end{aligned}$$

The parameters of the observer are given by:

$$\begin{aligned} N_1 &= -0.0020, M_1 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}, \\ J_1 &= 10^{-12} \begin{bmatrix} -0.58 & 0.58 \end{bmatrix}, \\ P_1 &= \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, Q_1 \Phi_1 B_1 = 10^{-12} \begin{bmatrix} 111 & 111 & 0 \\ -5 & -5 & 0 \\ -222 & -222 & 0 \end{bmatrix}, \\ F_1 &= \begin{bmatrix} 0.66 & 0.33 \\ -0.33 & 0.33 \end{bmatrix}. \end{aligned}$$

Subsystem 2 is given by**:**

$$\begin{aligned} E_2 &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, A_{22} = 10^{-4} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -5 \end{bmatrix}, \\ A_{21} &= D_2 = \begin{bmatrix} -0.0005 & 0 & 0 \\ 0 & -0.0008 & 0 \\ 0 & 0 & -0.0002 \end{bmatrix}, \\ D_{21} &= \begin{bmatrix} -0.0005 \\ 0 \\ 0 \end{bmatrix}, D_{22} = \begin{bmatrix} 0 & 0 \\ -0.0008 & 0 \\ 0 & -0.0002 \end{bmatrix}, \\ B_2 &= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \text{ and } H_2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} \end{aligned}$$

For this system, the matrix $\Phi_2 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$. In this case it is easy to see that Assumption 1 is verified. We shall design a reduced-order observer of dimension $\beta_2 = 1$, let

$$R_2 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}.$$

The solution of the LMI of Theorem 2 gives

$$\begin{aligned} Z_2 &= \begin{bmatrix} 0 & 0.0005 & 0 & 0 & 0 \\ -222.04 & -0.15 & -157 & 157 & 0 \\ 0 & -0.001 & 0 & 0 & 0 \end{bmatrix} \\ Y_2 &= \begin{bmatrix} 0.0888 & 0.0001 & 0.0628 & -0.0628 & 0 \end{bmatrix} \text{ and} \\ X_2 &= 2.3705. \end{aligned}$$

The parameters of the observer are given by:

$$\begin{aligned} N_2 &= -0.0004, M_2 = 10^{-20} \begin{bmatrix} -2 & -2 & 0 \end{bmatrix}, \\ J_2 &= 10^{-18} \begin{bmatrix} -12.73 & -6.9 \end{bmatrix}, \\ P_2 &= \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, Q_2 \Phi_2 B_2 = 10^{-15} \begin{bmatrix} 12.5 & 12.5 & 0 \\ -0.03 & -0.03 & 0 \\ -25 & -25 & 0 \end{bmatrix}, \\ F_2 &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ -1 & 1 \end{bmatrix}. \end{aligned}$$

Figures 1, 2, 3 and 4 illustrate the results of the simulation obtained with the above sub-systems 1 and 2. They show the performances of our approach.

VI. Conclusion

In this paper, we have presented a new method for the decentralized observers design for large scale singular systems. The method gives an unified approach to full order, reduced order and minimal order observers design for each discreet-time and continuous-time large scale interconnected systems. The observers design procedure is based on a new parametrization of decentralized Sylvester equation under constraint. Conditions for the existence and stability of the obtained observer are given. The sufficient conditions for the stability are derived using an LMI formulation.

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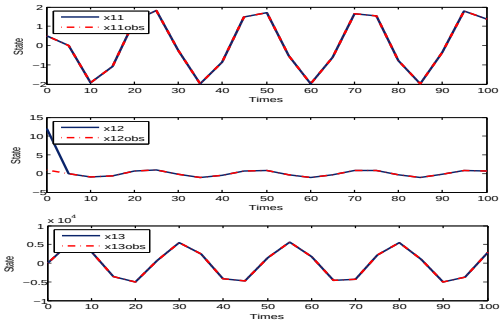

Fig. 1. $x_{1j}(t)$ (solid lines) and $\hat{x}_{1j}(t)$ (dashed lines) states of the sub-systems 1

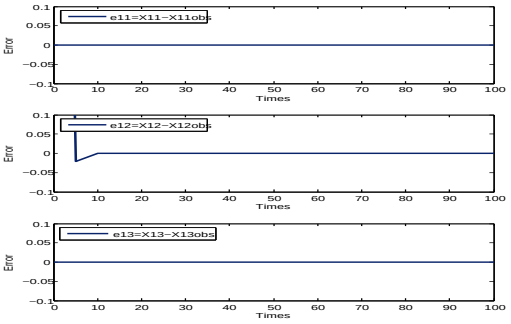

Fig. 2. Error between $x_{1j}(t)$ and $\hat{x}_{1j}(t)$ for sub-system 1

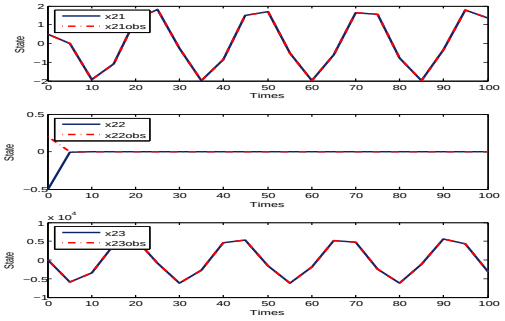

Fig. 3. $x_{2j}(t)$ (solid lines) and $\hat{x}_{2j}(t)$ (dashed lines) states of the sub-systems 2

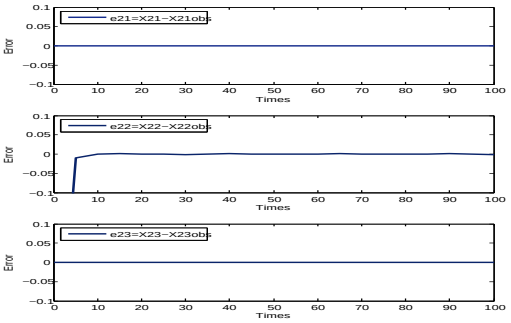

Fig. 4. Error between $x_{2j}(t)$ and $\hat{x}_{2j}(t)$ for sub-system 2