Robust Adaptive Control for Uncertain Systems with Time Delay

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***Abstract*—In this article, we present at the first place the concept of the direct model reference adaptive control (DMRAC) algorithm which will be applied to the systems with time delay and with unmodeled dynamics. These unmodeled dynamics are represented as either additive or multiplicative incertitude in the transfer function; this algorithm will be applied to the linear variable systems such as SISO (Single-Input Single-Output). The Simulations show that the developed algorithm leads to an asymptotically stable error and the validity of the algorithm is illustrated by means of examples.**

I. Introduction

The DMRAC algorithm proposed by Sobel, Kaufman, and Barkana [1] provides an attractive adaptive control approach. Its control structure adopts the use of linear combination of feedforward model states, command inputs and the error feedback between the plant outputs and the model reference outputs. One of the properties that make the algorithm relatively easy to be implemented is that it only requires the plant and reference model outputs and reference model states to be available for measurement. Other related works such as Landau [2], termed the approach as an adaptive model following control. Another attractive characteristic of this algorithm that provides design convenience is that the order of the reference model can be made lower than that of the order of the plant to be controlled. This complements its ability of not needing the identification of process parameter.[3]

The simple adaptive control approach originated by Kaufman et al [4] which is an output feedback method that requires neither full state feedback nor adaptive observers. Asymptotic stability is guaranteed if the plant is almost strictly positive real (ASPR), that means there must exist a feedback gain K such that the resulting closed-loop transfer function is strictly positive real (SPR). This gain need not be physically realized during implementation.

We present the design of feedforward compensator for plants with plant uncertainty and time delay, as developed by Ozcelic,[5]. Development of design procedures for feedforward compensators are performed using the transfer function representation of a plant. Plant uncertainty is represented as variations in the coefficients of the numerator and denominator polynomials of the plant transfer function. Design conditions for a feedforward compensator are developed utilizing an optimization procedure for robust stability.

We develop in this paper the DMRAC algorithm for a system with time delay in the presence of structural additive or multiplicative incertitude.

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II. Formulation of DMRAC algorithm

The problem of DMRAC will be solved for the following equations of the process

$$\begin{aligned} \dot{x}_p(t) &= A_p x_p(t) + B_p u_p(t) \\ y_p(t) &= C_p x_p(t) \end{aligned} \tag{1}$$

Where $x_p(t)$ is the (n*1) state vector, $u_p(t)$ is the (m*1) control vector, $y_p(t)$ is the (q*1) plant output vector and A_p, B_p and C_p are matrices with appropriate dimensions. The range of the plant parameters is assumed to be bounded as defined by

$$\underline{a}_{ij} \le a_p(i,j) \le \overline{a}_{ij}, i = 1...n, j = 1...n$$

$$\underline{b}_{ij} \le b_p(i,j) \le \overline{b}_{ij}, i = 1...n, j = 1...m$$

Where $a_p(i,j)$ is the $(i,j)^{th}$ element of A_p, and $b_p(i,j)$ is the $(i,j)^{th}$ element of B_p .

The objective is to find without explicit knowledge of A_p and B_p a control $u_p(t)$ such that the outputs of the system $y_p(t)$ follow the output of reference model $y_m(t)$, this last is described by

$$\begin{aligned} \dot{x}_m(t) &= A_m x_m(t) + B_m u_m(t) \\ y_m(t) &= C_m x_m(t) \end{aligned} \tag{2}$$

The model incorporates desired plant behaviour and in many cases $\dim[x_p(t)] >> \dim[x_m(t)]$.

The adaptive control algorithm being presented is based upon the command generator tracker concept (CGT) developed by O'Brien and Broussard [6]. In the CGT method, it is assumed that there exists an ideal plant with ideal state and control trajectories, $x^*{}_p(t)$ and $u^*{}_p(t)$ respectively, which corresponds to perfect output tracking. It means $y_p(t) = y_m(t)$ for $t \geq 0$. By definition, this ideal plant satisfies the same dynamics as the real plant, and the ideal plant output is identically equal to the model output. Hence, when perfect tracking occurs, the real plant trajectories become the ideal plant trajectories. The ideal control law $u^*{}_p(t)$, generating perfect output tracking and the ideal state trajectories is assumed to be a linear combination of the model states and model input:

$$\begin{bmatrix} x_p^*(t) \\ u_p^*(t) \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} x_m(t) \\ u_m(t) \end{bmatrix} \tag{3}$$

Where the S_{ij} sub matrices satisfy the following conditions

$$\begin{aligned} S_{11}A_m &= A_pS_{11} + B_pS_{21} \\ S_{11}B_m &= A_pS_{12} + B_pS_{22} \\ C_m &= C_pS_{11} \\ 0 &= C_pS_{12} \end{aligned} \tag{4}$$

The adaptive control law based on this CGT approach is given as [1]

$$u_p(t) = K_x(t)x_m(t) + K_u(t)u_m(t) + K_e(t)(y_m(t) - y_p(t)) \tag{5}$$

Where $K_e(t)$, $K_x(t)$ and $K_u(t)$ are adaptive gains and concatenated into matrix $K(t)$ as follows

$$K(t) = \begin{bmatrix} K_e(t) & K_x(t) & K_u(t) \end{bmatrix}$$

Defining the vector $r(t)$ as

$$r(t) = \begin{bmatrix} y_m(t) - y_p(t) \\ x_{m(t)} \\ u_{m(t)} \end{bmatrix}$$

The control $u_p(t)$ is written in a compact form as follows.

$$u_p(t) = K(t)r(t) \tag{6}$$

$K(t)$ is defined according to the following adaptive rule [7]

$$K(t) = K_I(t) + K_p(t) \tag{7}$$

where

$$\begin{aligned} \dot{K}_I(t) &= C_p(y_m(t) - y_p(t))r^T T;\ K_I(0) = K_{I0} \\ K_p(t) &= C_p(y_m(t) - y_p(t))r^T \overline{T} \end{aligned} \tag{8}$$

Sufficiency for asymptotic tracking is achieved if:
1. There exists a solution to the CGT problem, equation (4).
2. The plant is ASPR, that is there exists a gain matrix $K_e(t)$, not needed for implementation, such that the closed-loop transfer function

$G_c(s) = \left[I + G_p(s)K_e\right]^{-1} G_p(s)$ is SPR

In general, the ASPR conditions are not satisfied by most real systems. Barkana and Kaufman [6] have remedied this situation by showing that a non-ASPR plant of the form $G_p(s) = C_p(SI - A_p)^{-1}B_p$ Can be augmented with a feedforward compensator $H(s)$ such that the augmented plant transfer matrix

$G_a(s) = G_p(s) + H(s)$ is ASPR.

It was shown in [1] that, in general the resulting adaptive controller result in a model following error that is bounded, but not zero, the modification incorporating the supplementary feedforward into the reference model output, has been developed [7] where the asymptotic model following was achieved using a strictly proper stable feedforward compensator. However it is also possible using a proper but not strictly proper stable feedforward compensator [8].

The DMRAC algorithm with the augmented plant and the augmented reference model is summarized and gives as follows:

The plant and the model are given by the equation (1) and (2) respectively.

The compensator is given by:
A. the compensator proposed for the plant is:

$$\begin{aligned} \dot{s}_p(t) &= A_s s_p(t) + B_s u_p(t) \\ r_p(t) &= C_s s_p(t) \end{aligned}$$

B. the compensator proposed for the reference model is:

$$\begin{aligned} \dot{s}_m(t) &= A_s s_m(t) + B_s\left[u_p(t) - K_e(t)(z_m(t) - z_p(t))\right] \\ r_m(t) &= C_S s_m(t) \end{aligned}$$

The augmented outputs of the plant and the model are:

$$\begin{aligned} z_p(t) &= y_p(t) + r_p(t) \\ z_m(t) &= y_m(t) + r_m(t) \end{aligned}$$

Such that the output of the system is $Z_p(t)$ and for the model is $Z_m(t)$.

The augmented error is given by:

$$e_{ya}(t) = z_m(t) - z_p(t) = y_m(t) - y_p(t) + r_m(t) - r_p(t)$$

The control law is:

$$u_p(t) = K(t)r(t)$$

The convergence of the adaption gains as well as recent studies on the DMRAC can be found in [9].

The vector $r(t)$ is written in a form as follows.

$$r^T(t) = \left[(z_m(t) - z_p(t))^T, x_m{}^T, u_m{}^T \right]$$

In this paper we use the ASPR lemma [10], which is needed for the development of the design procedure for a feedforward compensator.

Lemma [10]

Let *G(s)* be a strictly minimum phase *m*x*m* transfer matrix of relative McMillan degree *m* or zero. Let *G(s)* have the minimal realization (*A*, *B*, *C*) with the high frequency gain *CB*>0 (positive definite), then *G(s)* is ASPR.

III. Analyse of stability

The stability will be analyzed by using an approach of Lyapunov illustrated by:

$$V(t) = e^T(t)P(t)e(t) + Tr[S(K_I(t) - \tilde{K}(t))T^{-1}(K_I(t) - \tilde{K}(t)^T S^T \qquad (10)$$

Tr: Trace of matrix.

With *T* and *P (t)* are respectively symmetrical matrixes positive definite, *S* is a nonsingular matrix and $\tilde{K}$ is written the same manner as *K*, it means

$$\tilde{K}(t) = [\tilde{K}_e(t), \tilde{K}_x(t), \tilde{K}_u(t)]$$

And it is supposed that

$$\tilde{K} = C_p e(t) r^T(t) T_I(t)$$

With $T_I(t)$ is a time matrix. After some algebraic manipulations, the derivative of *V(t)* is given by

$$\dot{V} = e^T(t)[\dot{P}(t) + P(t)(A_p(t) - B_p(t)\tilde{K}'_e(t)C_p) + (A_p(t) - B_p(t)\tilde{K}'_e(t)C_p)^T P(t) - 2e^T(t)P(t)B_p(t)(S^T S)^{-1}B_p{}^T(t)P(t)e(t)r^T(t)\bar{T}r(t)] \qquad (11)$$

This expression was established by supposing that

$$C_p = (S^T S)^{-1} B^T(t) P(t)$$

And if $\tilde{K}'_u(t)$ and $\tilde{K}'_x(t)$ are selected so that

$$\tilde{K}'_x(t) = S_{21}(t) \text{ And } \tilde{K}'_u(t) = S_{22}(t)$$

Then the derivative of *V (t)* is negative if

1. *T* is positive definite.
2. T_I is semi positive definite
3. $C_p = (S^T S)^{-1} B^T(t) P(t)$
4. $\dot{P}(t) + P(t)(A_p(t) - B_p(t)\tilde{K}'_e(t)C_p) + (A_p(t) - B_p(t)\tilde{K}'_e(t)C_p)\ P(t)$ is negative definite.

Note that the matrix $\tilde{K}'_x(t)$, $\tilde{K}'_u(t)$, $\tilde{K}'_e(t)$ and *P(t)* as well as the matrices $S_{ij}(t)$ are not necessary for implementation of the control law.

The following theorem summarizes the stability of the approach DMRAC with augmented of the plant and the model.

Theorem 1 [7]

Let us consider the adaptive controller given by equation (6), With the adaptive rule defined by equation (7), if the following designed conditions are satisfies, then, the error output $y_p(t) - y_m(t)$ tends to zero asymptotically and all the states and the controls will be limited. [7]

1. $G_a(s) = G_p(s) + H(s)$ is ASPR, where $G_a(s)$ is the transfer function of the plant and $H(s)$ is the transfer function of the compensator.
2. A solution exist for the CGT problem (equation 4)
3. The compensator $H(s)$ is stable, it means $H(s)$ has all its roots in the left half-plane.

IV. The Incertitude Of The System With Time Delay

The Uncertainty on the parameters of the system is brought back to an additional dynamics in the form of a multiplicative or additive uncertainty. This dynamics is added to the nominal system presumably known or multiplied by this one [11].

The uncertainty of system modeling is generally divided into two categories, structured uncertainties and not structured uncertainties. For structured uncertainty, the model of uncertainty and the row of variation of its parameters are supposed to be known.

The structured uncertainty can be modeled in several ways of which additive uncertainty and multiplicative uncertainty.

Let us considered a non ASPR SISO plant of the form

$$G_p(s) = G(s)T(s) \qquad (12)$$

Where

$$G_p(s) = \frac{C_m s^m + C_{m-1}s^{m-1} + ... + C_0}{s^n + B_{n-1}s^{n-1} + ... + B_0} \qquad (13)$$

In which the coefficients B_{n-j} and C_{m-j} can take any values within the given bounds:

$$\underline{C}_{m-j} \le C_{i\,m-j} \le \overline{C}_{m-j} \qquad j = 0,1,\ldots,m$$

$$\underline{B}_{n-j} \le B_{n-j} \le \overline{B}_{n-j} \qquad j = 0,1,\ldots,n$$

And $T(s)$ is the time delay of the plant and is of the form

$$T(s) = e^{-T_d s} \tag{14}$$

Let $G_0(s)$ be the nominal plant obtained from $G(s)$ using the nominal values of the parameters. Then, defining

$$\Delta_a(s) = G(s) - G_0(s) \tag{15}$$

And $\Delta_m(s) = T(s) - 1$

(16)

The actual plant $G_p(s)$ can be rewritten as

$$G_p(s) = (G_0(s) + \Delta_a(s))(1 + \Delta_m(s)) \tag{18}$$

Defining

$$\Delta(s) = \Delta_m(s) + G^{-1}{}_0(s)\Delta_a(s)(1 + \Delta_m(s) \tag{19}$$

The actual plant $G_p(s)$ takes the following form

$$G_p(s) = G_0(s)(1 + \Delta(s)) \tag{20}$$

From (15) and (19), it is seen that the uncertainty $\Delta(s)$ is a function of plant parameters which vary in a range. Thus, in the design of a feedforward compensator, the worst case uncertainty should be taken into account. To this effect, the following optimization procedure is considered to determine the worst case uncertainty at each frequency.

Define a vector whose elements are plant parameters

$$V = [C_m \;\; C_{m-1} \ldots C_0 \;\;\; B_n \;\; B_{n-1} \ldots B_0]$$

Then

$$\max_V \left|\Delta_{\max}(j\omega)\right| \forall w$$

$$subject \;\; to : \begin{cases} \underline{C}_{m-j} \le C_{m-j} \le \overline{C}_{m-j} & j = 0,1,\ldots\ldots m \\ \underline{B}_{n-j} \le B_{n-j} \le \overline{B}_{n-j} & j = 0,1\ldots\ldots n \end{cases}$$

Where $\Delta(jw)$ is the perturbation given by (19). It is important to note that this optimization is performed for each frequency. Given the worst case uncertainty at each frequency, it is assumed that there exists a known rational function $W(s) \in RH_\infty$ such that

$$\left|W(j\omega)\right| \ge \left|\Delta_{\max}(j\omega)\right| \;\; \forall\omega \tag{21}$$

Now, the following assumptions are imposed on the plant

Assumption1

1. 1) Nominal plant $G_0(s)$ is known, minimum phase, and stable.

2. 2) Actual plant $G_p(s)$ is stable.

3. 3) $\min \rho_p \ge \rho_m$, where ρ_p and ρ_m are the relative degrees of the actual plant and the nominal plant, respectively.

4. 4) $\Delta(jw)$ Satisfies (21).

Now consider the following augmented plant with the parallel feedforward compensator

$$G_a(s) = G_p(s) + H(s).$$

The following theorem gives the design conditions for a parallel feedforward compensator $H(s)$ so that the ASPR conditions of the augmented plant are satisfied in the presence of plant perturbations and time delay. [5]

Theorem 2 [5]

If $H(s)$ is designed according to the following conditions, then the augmented plant $G_a(s) = G_p(s) + H(s)$ with plant perturbations will be ASPR.

1. $H(s)$ is stable with relative degree one.

2. The augmented nominal plant $G_0(s) + H(s)$ is ASPR.

3. $\tilde{\Delta}(s) \in RH_\infty$ et $\left\|\tilde{\Delta}(s)\right\|_\infty \prec 1$

Where $\tilde{\Delta}(s) = \dfrac{G_0(s)W(s)}{G_0(s) + H(s)}$ is the uncertainty of the augmented plant

With regard to the design conditions for a feedforward compensator, the following design procedure is proposed

Design procedure 1

1. The order of a feedforward compensator can be determined from the fact that the sufficient order of a feedforward compensator is equal to the order of a plant.

2. compensator parameters are determined from the following optimization procedure:

$$\underset{X}{\min imize}\left\|\tilde{\Delta}(jw)\right\|_{\infty}$$

$$subject\ to:\ \ \mathrm{Re}\,al[roots(z(s))] < 0$$

Where $z(s)$ the zero polynomial of the augmented nominal plant and X is a vector whose elements are compensator parameters. All the conditions of theorem 1 will be satisfied using the design procedure given above.

V. Simulation Results

Let the transfer function of the real system given by:

$$G_p(s) = \frac{K_{22}e^{-T_{22}s}}{\tau_{22}s+1}$$

Let us note that for simulation we considered the following Cases:

TABLE I

The Cases Considered For Simulation.

Case	K_{22}	τ_{22}	T_{22}
1	-1	40	50
2	-50	40	50
3	-15	40	50

The transfer function of the reference model is given by

$$G_m(s) = \frac{1}{1s+1}$$

The Simulations with the three cases, show that a compromise is obtained for the compensator

$$H(s) = \frac{120}{s+2}$$

The modeling error or the additive uncertainty is given by:

$$\Delta_a(s) = -\frac{K_{22}+15}{40s+1}\,.$$

The multiplicative uncertainty is given by:

$$\Delta_m(s) = e^{-T_{22}s} - 1$$

Then the combined uncertainty, using equation (19), is

$$\Delta(s) = e^{-T_{22}s}\left(\frac{K_{22}+30}{15} - 1\right)$$

In this simulation, the control objective is to force the system output to track a desired model reference.

The augmented system with the additive uncertainty $G_a(s)$ is of relative degree one and it is minimal phase for the three cases of simulation, it is then ASPR (Almost Strictly Positive Real) and thus the error convergence is guaranteed. The application of this adaptive controller to the real system leads to an error asymptotically stable. The input for the reference model is a square signals of amplitude -10 and of period 10 seconds.

The Fig.1, Fig.2 and Fig.3 represent the outputs of the system and the reference model, the perfect tracking is visible in permanent mode, the representation of the control is given in Fig. 4, Fig.5 and Fig.6; we see that they are bounded.

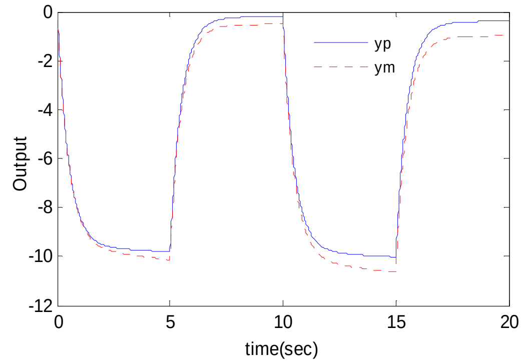

Fig. 1. The output of the plant and reference model .-Case (1)-

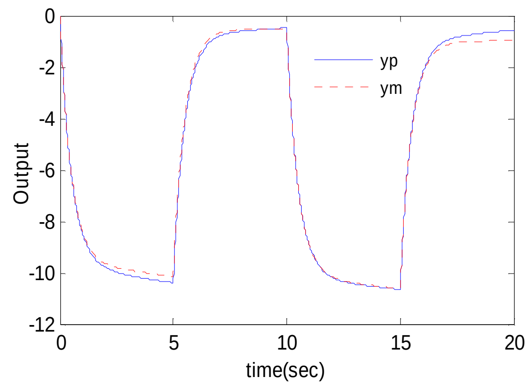

Fig. 2. The output of the plant and reference model .-Case (2)-

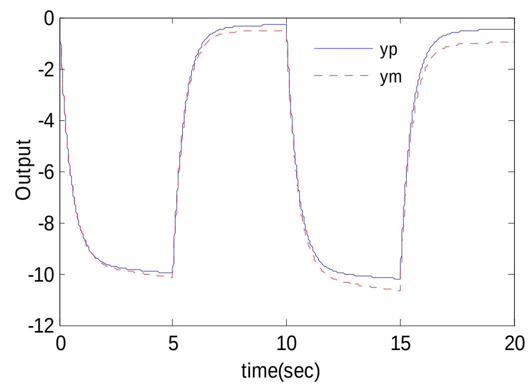

Fig. 3. The output of the plant and reference model .-Case (3)-

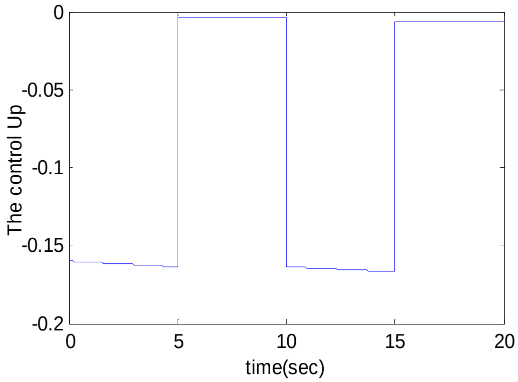

Fig. 4. The Control Up.

-Case (1)-

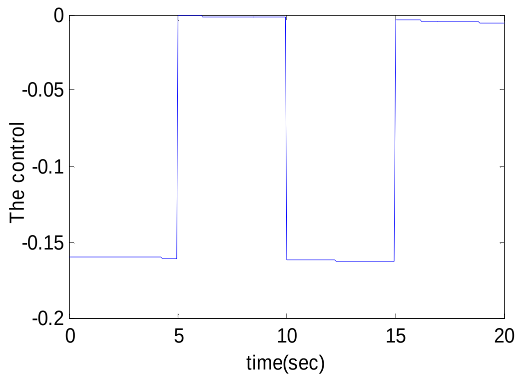

Fig. 5. The control Up.

-Case (2)-

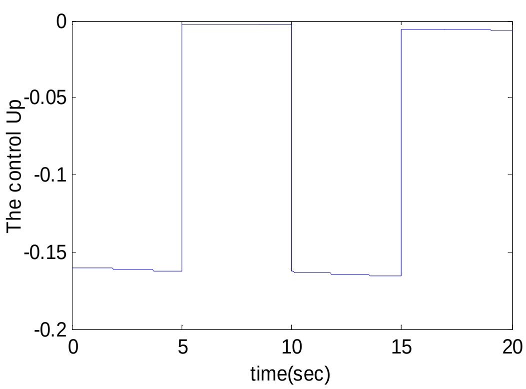

Fig. 6. The control Up.

-Case (3)-

VI. Conclusion

In this paper, we have presented an extension of direct model reference adaptive control (DMRAC) to the Non ASPR systems with uncertainties in both plant parameters and time delay, direct model reference adaptive control is considered when the plant-model matching conditions are violated due to large changes in the plant or incorrect knowledge of the plant's mathematical structure. Because of the mismatch, the plant can no longer track the original reference model, but may be able to track a modified reference model that still provides satisfactory performance.

A systematic design method based on an optimization procedure for robust stability analysis for a feedforward compensator was developed. This easily implementable design procedure enables the augmented plant to satisfy the ASPR conditions in the presence of variations in plant parameters and time delay elements. Hence, the applicability of direct adaptive controllers has now been extended to systems with time delays. Simulation results demonstrate the viability of the DMRAC algorithm designed using this new method. Further works must be carried for non linear systems.

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