

Simple Integral Sliding Fuzzy Type-2 Control For Non linear Systems

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Abstract—A simple sliding fuzzy type-2 controller scheme for nonlinear uncertain perturbed systems is proposed in the paper. To overcome the constraint on the knowledge of the system model, local models related to some operating points were used to synthesize a nominal fuzzy type-2 global model. The controller uses integral sliding mode concept and contains two parts. The first one leads to achieve finite time stabilization of the higher order input output dynamics without uncertainties. The second part has the object to reject bounded uncertainties throughout the entire response of the system. In order to eliminate the chattering effect, the *sign* function in this part control is substituted by a smooth variation fonction. Its structure conserves the characteristics of the original function. The sliding surface is computed using Slotine equation, this choice leads to reduce the relative degree of the system to one, which simplifies the design of the proposed method. The advantages of the controller are that its implementation is easy, the convergence time is chosen in advance and the robustness is ensured. A robot arm actuated by a DC motor is used as illustrative example. The obtained results show the good tracking performances and the applicability of the method.

I. INTRODUCTION

It has been shown that fuzzy systems can be considered as a universal approximators, and hence can approximate any continuous functions on a compact set to a given accuracy [2]. The main property that distinguishes fuzzy logic is its capacity of representing and modeling imprecision and uncertainty by giving each point in fuzzy set a value belonging to the interval $[0,1]$. However, the imprecision in such classical fuzzy system, which is called some times Type-1 fuzzy logic system (T1FLS) is not fully exploited and can deliver a non-satisfactory performance. Practically, three ways of uncertainty can be observed in the fuzzy logic system (FLS): (1) the used words in antecedents and consequents of rules can have different meaning to different people, (2) consequents obtained by polling a group of experts will often be different for the same rule and (3) both of training data and measurements used to activate the FLS are noisy [3],[9].

Recently, there has been a growing interest in using type-2 FLSs (T2 FLSs) in many applications as well as in control processes due to their ability to model uncertainties [5],[4],[1],[9]. With the development of T2 FLSs and their ability to handle uncertainty, utilizing type-2 FL Control Systems (T2 FLCs) has attracted a lot of interest in recent years. Although, to date, only IT2 FLSs have been applied for control applications, promising results have been reported,

Wu and Tan [22] designed an IT2 FLCs for a coupled-tank liquid-level system and showed that when the level of uncertainty increases, the IT2 FLCs outperforms its type-1 counterpart. Manceur et al. [9] designed a second order IT2 FC for an arm manipulator actuated by direct courant motor. In addition, Hagrais [5] applied IT2 FLC to mobile robot navigation in dynamic unstructured indoor and outdoor environments. All the IT2 FLCs implemented in [5] used much smaller rule bases than their type-1 counterparts, and it was concluded that IT2 FLCs provide a faster computation platform as well as enhanced performance results over their type-1 counterpart. In [1] many exemples and comparison results are done, case studies reveal also that an IT2 FLCs outperforms clearly its type-1 counterpart.

The sliding mode control (SMC) is a powerful method to control high-order nonlinear dynamic systems operating under uncertainty conditions [20]. The technique consists of two stages. First, a sliding surface, to which the controlled system trajectories must belong, is designed according to some performance criterion. Then, a discontinuous control is designed to force the system state to reach the sliding surface such that a sliding mode occurs on this manifold. When sliding mode is realized, the system exhibits robustness properties with respect to parameter perturbations and external matched disturbances [19]. In spite of claimed robustness properties, high frequency oscillations of the state trajectories around the sliding manifold known as chattering phenomenon [19] are the major obstacles for the implementation of SMC in a wide range of applications. A number of methods have been proposed to reliably prevent chattering: among them, the boundary layer solution [19], [15], tangent hyperbolic [16], observer-based solution [20], Adaptive fuzzy calculation method of control gains [9],[11] and higher order SMC [13].

The aim of this paper is to present a sliding mode controller for uncertain SISO nonlinear systems. The main objective of this approach is to propose a controller for which the implementation is easy, the convergence time is finite and well-known in advance. The design uses the integral sliding mode concept [18]. Actually, the problem of the SMC based on this concept is formulated on higher order SM for SISO non linear systems [12]. That of SISO minimum-phase uncertain systems is formulated in input-output terms only through the differentiation of the sliding variable and it is equivalent to the finite time stabilization of higher order input-output linear dynamics with bounded uncertainties [8]. We propose to use the sliding surface given by Slotine [15], which allows us to reduce the relative degree of the system to one [13]. This choice reduces and simplifies the control

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method introduced by Laghrouche et al [7]. Indeed, in [7] the control strategy is acting on the high dynamic of the surface and its derivatives. The sliding manifold is in function of the system state variables, when we drift the surface brings also to drift this variables. This induce an inconvenience when we consider a real time application, because we need more informations which implies more sensors.

The proposed control will be composed of two parts. The first part is discontinuous, forces the establishment of a sliding mode on the integral sliding manifold and ensures the robustness of the closed loop. Since the *sign* function causes the chattering phenomenon, we propose to substitute it by a continuous function [6]. Its structure conserves the characteristics of the original function. The second one, which is obtained through optimal feedback control over finite time interval with fixed final states [14], is used to stabilize the sliding surface and its derivative. As in [7],[12], in this paper, this part is based an close-loop control and has two dynamics, at $t \in [0, t_f]$ will be associated a forcing term δ and after $t > t_f$ will be removed. Since the considered system is nonlinear and uncertain, we propose to use Takagi-Sugeno (T-S) fuzzy system to construct the nominal model. For this, we define some local models related to some operating points, which will be used as conclusion of this T-S system.

The paper is organized as follows. Section II gives an overview of the theory of the type-2 fuzzy systems. The problem formulation is presented in section III. The design procedure of the SMC based an integral sliding mode and its stability is discussed in section IV. Simulation example to demonstrate the performance of the proposed method is provided in section V. Section VI gives the conclusions of this paper.

II. TYPE-2 FUZZY LOGIC SYSTEM

Figure 1 depicts the structure of a T2FLS, it is quite similar to a T1FLS, the only difference being that the antecedent and/or consequent sets in a T2FLS are type-2, so that each rule output set is a type-2. There are five principal parts in a T2FLS: fuzzifier, rule base, inference engine, type-reducer and defuzzifier.

Fig. 1. The structure of T2FLS with its two out puts.

In an interval T2FLS with meet under minimum or product t-norm, the firing interval $W^i = [\underline{w}^i, \bar{w}^i]$ of a given i^{th} rule is an interval T1 set, it is determined by its left-most and right-most points \underline{w}^i and \bar{w}^i such that:

$$\begin{cases} \underline{w}^i = \left[\underline{\mu}_{\bar{F}_1}(\underline{x}_1) \times \dots \times \underline{\mu}_{\bar{F}_n}(\underline{x}_n) \right] \\ \bar{w}^i = \left[\bar{\mu}_{\bar{F}_1}(\underline{x}_1) \times \dots \times \bar{\mu}_{\bar{F}_n}(\underline{x}_n) \right] \end{cases} \quad (1)$$

The most commonly adopted IT2 FLSs utilize KarnikMendel (KM) algorithms for type reduction. KM algorithms compute the left and right end points needed to characterize IT2 FSs. In order to bypass the computational effort of KM algorithms, Wu and Mendel [23] developed uncertainty bounds

for IT2 FSs to approximate type reduction while achieving similar results. There exist many kinds of type reduction [21]. It is in extension of T1 defuzzification method and it is called type reduction because this operation takes us from the T2 output sets of the FLS to a T1 set. This set may then be defuzzified to obtain a single crisp number. In this paper, we will use the center of sets type reduction, which can be expressed in the case of interval T2 fuzzy sets as [21]:

$$\begin{aligned} Y_{tr} &= \int_{y_l \in [y_l^1, y_r^1]} \dots \int_{y_m \in [y_l^m, y_r^m]} \int_{w^1 \in [\underline{w}^1, \bar{w}^1]} \\ &\dots \int_{w^M \in [\underline{w}^M, \bar{w}^M]} 1 / \frac{\sum_i w^i y^i}{\sum_i w^i} \\ Y_{tr} &= [y_l, y_r] \end{aligned} \quad (2)$$

where Y_{tr} is the interval set determined by two end points y_l and y_r , $y^i \in Y^i = [y_l^i, y_r^i]$, Y^i the centroid of the T2 interval consequent set \bar{G}^i and $w^i \in W^i = [\underline{w}^i, \bar{w}^i]$ the firing interval. To defuzzify the type reduced set Y_{tr} , we just need to compute the average of y_l and y_r . So the defuzzified output of an interval singleton type-2 FLS is:

$$y = \frac{y_l + y_r}{2} \quad (3)$$

III. PROBLEM FORMULATION

Consider the n th-order uncertain nonlinear system of the form:

$$\begin{cases} \dot{x}^{(n)} = f(\underline{x}) + g(\underline{x}) \cdot u(t) + d \\ y = x \end{cases} \quad (4)$$

where $f(\underline{x})$ and $g(\underline{x})$ are two nonlinear continuous and uncertain bounded functions, $u \in \mathbb{R}$ and $y \in \mathbb{R}$ are the input and output of the system, respectively, d is an external unknown bounded disturbance. $\underline{x} = [x, \dot{x}, \dots, x^{n-1}]^T \in \mathbb{R}^n$ is the measurable state vector. We assume that the two functions $f(\underline{x})$ and $g(\underline{x})$ can be written respectively as the sum of a nominal function and an unknown bounded uncertainty:

$$\begin{cases} f(\underline{x}) = f_0(\underline{x}) + \Delta f(\underline{x}); |\Delta f(\underline{x})| < \Delta_f \\ g(\underline{x}) = g_0(\underline{x}) + \Delta g(\underline{x}); |\Delta g(\underline{x})| < \Delta_g \end{cases} \quad (5)$$

where Δ_f and Δ_g are two positive constants. Replacing (5) into (4), we obtain:

$$\begin{cases} \dot{x}^{(n)} = f_0(\underline{x}) + g_0(\underline{x}) \cdot u(t) + D \\ y = x \end{cases} \quad (6)$$

where

$$D = \Delta f(\underline{x}) + \Delta g(\underline{x})u + d \quad (7)$$

The nominal model of the system can be obtained by identification or by approximation using a fuzzy system. We consider the second case, because it allows us to exploit linguistic information from the expert human. The approximation can be made either using a T-S classical fuzzy system or an adaptive one. This later can certainly get good results but requires a long computing time where we consider more than two inputs. Moreover, the linearisation techniques permit to transform the dynamics of the system into a local models around certain operating points. Thus, to calculate the nominal model, Takagi and Sugeno proposed in [17]

a T1 fuzzy dynamic model formed by rules. However, as known in literature, the knowledge used to construct such rules are uncertain. This uncertainty leads to obtain rules whose premises or consequences uncertain, which can create uncertainties in the membership functions. As a T2FS can take in account this ncertainties, in many works the T-S fuzzy modeling was extended by introducing the concept of T2F logic [9],[11],[12],[10]. The nominal IT2F model related to the i^{th} rule will be given as follows :

IF x is \tilde{H}_1^i and \dot{x} is \tilde{H}_2^i and..... and $x^{(n-1)}$ is \tilde{H}_n^i THEN

$$x^{(n)} = A_i(n, 1 : n)\underline{x}_i + B_i(n)u \quad (8)$$

Where $\tilde{H}_j^i, (j = 1, 2, \dots, n)$ is the j^{th} interval T2 fuzzy set of the i^{th} rule, $A_i(n, 1 : n) = A_i(n, j)_{1 \leq j \leq n}$. For a given pair (\underline{x}, u) , the fuzzy model resulting of the system appears as a weighted average of the local models. If we use the product as an interference engine, the method of center set for the reduction type and center of gravity for defuzzification, the output fuzzy system will be giving by:

$$x^{(n)} = \frac{\sum_{i=1}^r w^i [A_i(n, 1 : n)\underline{x}]}{\sum_{i=1}^r w^i} + \frac{\sum_{i=1}^r w^i [B_i(n)u]}{\sum_{i=1}^r w^i} \quad (9)$$

If we denote by $f_0(\underline{x}) = \left[\frac{\sum_{i=1}^r w^i A_i(n, 1 : n)\underline{x}}{\sum_{i=1}^r w^i} \right]$ and

$g_0(\underline{x}) = \left[\frac{\sum_{i=1}^r w^i B_i(n)}{\sum_{i=1}^r w^i} \right]$, the fuzzy nominal model will be giving by:

$$x^{(n)} = f_0(\underline{x}) + g_0(\underline{x})u \quad (10)$$

where the variable w^i is the firing interval, which is determined by its most left and right points as given by (1). The reduction type algorithm given by karnik and Mendel is detailed in [21].

The aim of our study is to determine a control law based on first order and integral SMC, so that the system output y follows a reference trajectory y_r and the tracking error $e = y - y_r$ converges to zero in the presence of uncertainties and disturbances. For this and in order to get more precision, there is an additional constraint to satisfy, which is to fulfill the sliding variable $\sigma(\underline{x}, t) = 0$ in finite time and to keep it exactly by discontinuous feedback control. The first order SMC approach allows the finite time stabilization to zero of the sliding variable $\sigma(\underline{x}, t) = 0$ and its first time derivative is achieved by defining a suitable discontinuous control function.

IV. CONTROL DESIGN

In [12],[7] the control problem is formulated in higher order, the advantage of this method will be kept and formulated for the case of the systems with relative degree one ($r = 1$).

The derivative of the sliding surface s is in function of the systems variables and the control variable, it is given by :

$$\dot{\sigma} = \phi(\cdot) + \gamma(\cdot)u \quad (11)$$

The first order SMC with respect to the sliding variable s is equivalent to the finite time stabilization of :

$$\begin{cases} z = \sigma \\ \dot{z} = \phi(\cdot) + \gamma(\cdot)u \end{cases} \quad (12)$$

The control law is composed of two parts. The first one is named ideal control. It is continuous and stabilizes in finite time (12) at the origin when there are no uncertainties. In fact, this control part is used in order to generate trajectories that the system is forced to track. The second part, named integral sliding mode control, provides the complete compensation of uncertainties for $t \geq 0$ and ensures that control objectives are reached.

A. Continuous Control Part Design

Consider system (12), according to some assumptions it can be trivially rewritten as [7],[12]:

$$\begin{cases} z = \sigma \\ \dot{z} = \underbrace{\phi(\cdot) + [\gamma(\cdot) - 1]u}_{\psi} + u \end{cases} \quad (13)$$

which can be written under the form :

$$\begin{cases} z = \sigma \\ \dot{z} = \psi(\cdot) + u \end{cases} \quad (14)$$

Let define the global control by :

$$u = u_0 + u_{dis} \quad (15)$$

with u_0 being the ideal control, and u_{dis} being the integral sliding mode control. Consider now the particular case $\beta(\cdot) = 0$. Then, as no control part u_{dis} is necessary in order to compensate the uncertainties, the control law u reads as $u = u_0$. One gets

$$\dot{z} = az + bu_0 \quad (16)$$

In [7] and [12] the problem is formulated in high order SMC ($r > 1$), in these case a and b have the form :

$$a = \begin{bmatrix} 0 & 1 & \dots & 0 & \dots \\ \vdots & \ddots & \ddots & \ddots & \\ 0 & \ddots & \ddots & \dots & 1 \\ 0 & \ddots & \ddots & \ddots & 0 \end{bmatrix}_{r \times r}, \quad b = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}_{r \times 1}$$

In this work, our study is addressed only for the case of the systems with relative degree is one ($r = 1$). In this case : $a = [0]$ and $b = [1]$, (16) becomes :

$$\dot{z} = u_0 \quad (17)$$

The control objective is to drive the state of (17) to $z = 0$ at the fixed final time $t = t_f < +\infty$, with $z(0)$ being a bounded initial state value. This objective can be done through the use of optimal control laws. The control law u_0 ensures the minimization of the following criterion :

$$J = \frac{1}{2} \int_0^{t_f} zqz + u_0^2 dt \quad (18)$$

with $t_f < +\infty$, $|z(0)| < +\infty$ and q a positive constant definite under the fixed final state constraint $z(t_f) = 0$. A control law u_0 minimizing the criteria (18) and driving (17) to $z(t) = 0$ at $t = t_f$ for an initial bounded condition $z(0)$ is given by (with $0 \leq t \leq t_f < +\infty$)

$$u_0 = -b^T M z(t) + b^T \delta(t) \quad (19)$$

with $\delta(t)$ and M are defined by

$$\begin{cases} \dot{\delta} = -(a^T - M b b^T) \delta \\ 0 = M a + a^T M - M b b^T M + q \end{cases} \quad (20)$$

Initial condition $\delta(0)$ of $\delta(t)$ is selected in order to satisfy the terminal condition $z(t_f) = 0$. The control law u_0 is defined in order to drive in finite time the system (17) to $z(t) = 0$ at $t = t_f$, then, the state variables of (17) converge exactly to the origin at $t = t_f$. The control law u_0 will maintain the required equilibrium state $z = 0$ even after the terminal time, i.e. $t > t_f$, provided that the forcing term $\delta(t)$ is removed at the terminal time, i.e. $u_0 = -b^T M z$ for $t > t_f$ [14]. Since $a = 0$ and $b = 1$, in order to reach in a finite time t_f the origin $z = 0$, and to maintain the system (17) at this point for $t > t_f$, the solution that we propose is :

$$u_0 = \begin{cases} -\sqrt{q}z(t) + \delta(t) & \text{for } 0 \leq t \leq t_f \\ -\sqrt{q}z(t) & \text{for } t > t_f \end{cases} \quad (21)$$

B. Integral Sliding Manifold

The basic idea consists in determining a sliding manifold such that the state trajectories start on this manifold at the initial time $t = 0$, which induces a sliding mode without reaching phase [19]. One has (for $1 \leq i \leq r - 1$)

$$\begin{cases} z = \sigma \\ \dot{z} = \psi(\cdot) + u_0 + u_{dis} \end{cases} \quad (22)$$

u_{dis} is a discontinuous function designed in order to exactly reject the perturbation $\psi(\cdot)$ throughout the entire response of the system. In order to reach this objective, the integral sliding mode control is used [18],[20]. Let $\sigma \in \mathbb{R}$ the sliding manifold defined as :

$$\sigma = \sigma_0 + \zeta \quad (23)$$

where σ_0 may be designed as the linear combination of the state variables of (22). The term $\zeta \in \mathbb{R}$ induces the integral term and will be determined below. In order to determine the motion equations on the sliding manifold, the equivalent control method is used [18]. The time derivative of σ reads as :

$$\dot{\sigma} = \frac{\partial \sigma_0}{\partial z}(u_0) + \frac{\partial \sigma_0}{\partial z}(u_{dis} + \psi(\cdot)) + \dot{\zeta} \quad (24)$$

A sufficient condition ensuring $\dot{\sigma} = 0$ for $t \geq 0$ is that the equivalent control u_{eq} of u_{dis} reads as :

$$u_{eq} = -\psi(\cdot) \quad (25)$$

and $\dot{\zeta} = -\frac{\partial \sigma_0}{\partial z}(u_0)$, $\zeta(0) = -\sigma_0(z(0))$, where $\zeta(0)$ is determined based on the requirement $\sigma(0) = 0$.

In this present work, since $r = 1$, it allows to simplify the implementation of the method :

$$\sigma_0 = z \longrightarrow \dot{\sigma} = u + \psi(\cdot) + \dot{\zeta} \quad (26)$$

In this case, condition (25) holds if

$$\dot{\zeta} = -u_0, \zeta(0) = -z(0) \quad (27)$$

C. Design of the Discontinuous Control u_{dis}

The control law u_{dis} is designed in order to ensure that the sliding motion on $\sigma = 0$ is guaranteed for $t \geq 0$ in spite of uncertainties and disturbances.

$$u_{dis} = -\beta \text{sign}(\sigma) \quad (28)$$

where the gain β is a positive constant tuned such that η -attractiveness condition is satisfied :

$$\sigma \dot{\sigma} \leq -\eta |\sigma|, \eta > 0 \quad (29)$$

In order to ensure a tracking trajectories and to reduce the relative degree of the system to one ($r = 1$), the surface given by Slotine [15] is used to compute σ_0 :

$$\begin{aligned} \sigma_0 &= \left(\frac{\partial}{\partial t} + \lambda \right)^{(n-1)} e \\ &= \sum_{k=0}^{n-1} \frac{(n-1)!}{k!(n-k-1)!} \left(\frac{\partial}{\partial t} \right)^{(n-k-1)} \lambda^k e \end{aligned} \quad (30)$$

$\lambda > 0$, the derivative of σ_0 can be expressed as:

$$\dot{\sigma}_0 = \underbrace{\delta_{\sigma_0} - y_r^{(n)} + f_0(\underline{x}) + g_0(\underline{x})u + D}_{\nabla} \quad (31)$$

which ∇ has the same form as given by equation (11), with $r = 1$, $\psi(\cdot) = \delta_{\sigma_0} - y_r^{(n)} + f_0(\underline{x})$, $\gamma(\cdot) = g_0(\underline{x})$ and $\delta_{\sigma_0} = \sum_{k=1}^n \frac{(n-1)!}{k!(n-k-1)!} \left(\frac{\partial}{\partial t} \right)^{(n-k-1)} \lambda^k \dot{e}$.

From (23),(24),(25),(28) and (31) the derivative of the sliding surface can be reads as :

$$\dot{\sigma} = -\beta \text{sign}(\sigma) + D \quad (32)$$

By substituting (32) into (29), we get :

$$\begin{aligned} \sigma(-\beta \text{sign}(\sigma) + D) &\leq -\eta |\sigma| \\ -\beta + |D| &\leq -\eta \\ \implies \beta &\geq \eta + |D| \end{aligned} \quad (33)$$

if we take in account the expression of D (equation (7)), (33) becomes :

$$\beta \geq \eta + |\Delta f(\underline{x}) + \Delta g(\underline{x})u + d| \quad (34)$$

In order to ensure the stability and asymptotic convergence in finite time of the control law and the stability of the closed loop, β is chosen so that :

$$\beta \geq \eta + |\Delta_f + \Delta_g u_M + d_M|, \quad M = \max \quad (35)$$

The presence of a sign function in equation (28) causes much chattering. In order to avoid the chattering effect, the discontinuous function is replaced by a continuous smooth variation function [6]:

$$\text{sign}(\sigma_0) \xrightarrow{\text{substitution}} \text{conti}(\sigma_0) = \frac{\sigma_0}{|\sigma_0| + \tau} \quad (36)$$

$conti(\sigma_0) = sign(\sigma_0)$ when σ_0 is equal to zero and when σ_0 is large $conti(\sigma_0)$ goes to zero, meaning that the effects of the discontinuous function are reduced. Where :

$$\tau = \sigma_0(0) + \int |\sigma_0| \quad (37)$$

$\sigma_0(0)$ is the initial condition.

V. ILLUSTRATIVE EXAMPLE

In this section, the proposed control is used to control an arm robot with single joint, of mass m and languor l . This arm is actuated by a DC motor as shown in Fig 2.

Fig. 2. Arm robot actuated by a DC motor

α means the angle rotation of the arm relative to the vertical. The dynamic of the robot is given by the 3rd order equation:

$$\alpha^{(3)} = f_0(\alpha, \dot{\alpha}) + g_0(\alpha, \dot{\alpha})u + D \quad (38)$$

$$\begin{cases} f_0(\alpha, \dot{\alpha}) = -\frac{R}{L}\ddot{\alpha} - \left(\frac{K_b N^2 K_t}{ml^2 L} + \frac{g}{l} \cos \alpha\right) \dot{\alpha} - \frac{Rg}{lL} \sin \alpha \\ g_0(\alpha, \dot{\alpha}) = \frac{K_t N}{ml^2 L} \end{cases}$$

where $\alpha^{(3)}$, $\ddot{\alpha}$ and $\dot{\alpha}$ are the time derivatives of the angle α . g , L , R , N , K_b and K_t are respectively the gravity and the motor parameters whose signification is given in Table (1). $D = d + \Delta f + \Delta g$ means the sum of unknown external disturbances and model uncertainties.

TABLE I
MODEL PARAMETERS

| Parameter | Notation | Value |
|-----------------------|----------|--------------|
| Arm mass | m | 2 Kg |
| Gravity | g | 9.81 m/s |
| Arm languor | l | 0.5 m |
| Resistance | R | 1.5 Ω |
| Inductance | L | 0.05 H |
| Constant EFM | K_b | 0.2 |
| Constant torque motor | K_t | 0.3 |
| Reduction ratio | N | 60 |

As in [12],[11] the nominal fuzzy model is obtained using 2 rules describing the system dynamics around the operating points 0 and $\pm \frac{\pi}{2}$.

Rule1: IF $x_1(t)$ is close to 0

THEN

$$\begin{cases} \dot{x}(t) = A_1(3, 1 : 3)x(t) + B_1(3)u(t) \\ y(t) = Cx(t) \end{cases} \quad (39)$$

Rule2: IF $x_1(t)$ is close to $\pm \pi/2$

THEN

$$\begin{cases} \dot{x}(t) = A_2(3, 1 : 3)x(t) + B_2(3)u(t) \\ y(t) = Cx(t) \end{cases} \quad (40)$$

where:

$$A_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\frac{Rg}{lL} & -\frac{g}{l} - \frac{K_b N^2 K_t}{ml^2 L} & \frac{R}{L} \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 0 \\ \frac{K_t N}{ml^2 L} \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\frac{Rg}{lL} & -\frac{K_b N^2 K_t}{ml^2 L} & \frac{R}{L} \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 0 \\ \frac{K_t N}{ml^2 L} \end{bmatrix}$$

$$C = [1 \ 1 \ 0]^T \text{ and } x(t) = [\alpha \ \dot{\alpha} \ \ddot{\alpha}]^T.$$

The control objective is to control the state $x_1(t)$ of the system to track the reference trajectory $y_r(t) = \sin(2t)$. Suppose that structural uncertainty, which represents the variation in the mass in this example, is in the form $dm = 0.02m(\sin(2t) + \sin(3t))$ while the external perturbation is assumed to be $d = \sin(3t)$.

According to the sliding surface given by Slotine [15] and since the system is of order ($n=3$), the sliding surface is given by $\sigma = \ddot{e} + 2\lambda\dot{e} + \lambda^2 e$. e is the tracking error position ($e = x_1 - y_r$), \dot{e} is the tracking error velocity ($\dot{e} = x_2 - \dot{y}_r$). To construct the nominal model as given by (10), we define three IFT2 sets depending on the variable $x_1(t)$. The membership functions involved in building the model are given by the following equations:

$$\begin{aligned} \mu_{\tilde{H}_1^i} &= 1 / (1 + \exp(1.8(x_1(t) + (0.61 + c_1)) / (\frac{\pi}{16}))) \\ \mu_{\tilde{H}_2^i} &= \exp\left(-\left(x_1(t) / \left(\frac{\pi}{4.5 + c_2}\right)\right)^2\right) \\ \mu_{\tilde{H}_3^i} &= 1 / (1 + \exp(-1.8(x_1(t) + (0.61 + c_1)) / (\frac{\pi}{16}))) \end{aligned}$$

where c_1 and c_2 designate an area of uncertainty, in the case of our application $c_1 \in [0, 0.2]$ and $c_2 \in [0, 1]$. Similarly the antecedent sets of the two adaptive systems are obtained by adding an area of uncertainty. Fig. 3 shows the shape fuzzy sets of the nominal model. The time

Fig. 3. Interval type-2 antecedent membership functions of $x_1(t)$ defined for the fuzzy nominal model.

derivative of s reads as $\dot{\sigma} = \phi(\cdot) + \gamma(\cdot)u$ with

$\phi(\cdot) = -\dot{y}_r^{(3)} + f_0(x) + 2\lambda\ddot{e} + \lambda^2 e$ and $\gamma(\cdot) = g_0(x)$. Let $z = \sigma$, since $r = 1 \implies a = [0]$ and $b = [1]$. Then as shown in subsection IV.A, the SMC is equivalent to the finite time stabilization of :

$$\dot{z} = az + bu + [\psi(\cdot) + (\gamma(\cdot) - 1)u] \quad (41)$$

Let $t_f = 0.2s$. The state is initialized at $x(0) = [1 \ 0 \ 0]^T$, the control gain and the slope of sliding surface are chosen to be respectively $\beta = 250$ and $\lambda = 20$, which implies that $z(0) = -320$. As mentioned in [14], the initial condition of δ are computed in order that the sliding variable s equal 0 at exactly $t = t_f = 0.2s$.

q is stated as $q = 10$, then one gets $M = \sqrt{q} = 3.1623$. The second step consists in determining $\delta(0)$ such that $z(t_f) = 0$, knowing that only initial bounded condition $z(0)$ is known. From first line of (21), one gets :

$$\delta(t) = e^{A_m t} \delta(0) \quad (42)$$

with $A_m = -[A^T - MBB^T] = \sqrt{q}$. The initiale condition of z

is calculated as follows [7],[12]:

$$z(0) = \left[-\int_0^{t_f} e^{\sqrt{q}t} e^{\sqrt{q}t} dt \right] \delta(0) \quad (43)$$

$H = \left[\int_0^{t_f} e^{\sqrt{q}t} e^{\sqrt{q}t} dt \right]$ is the partial reachability gramian. Then, the initial condition $\delta(0)$ ensuring that $z(t_f) = 0$ can be derived from $\delta(0) = -H^{-1}z(0)$. We get $\delta(0) = 795.9240$.

Fig. 4. (a) s versus time(s), (b) \dot{s} versus time(s)

Fig. 4 (a,b) and Fig. 5(a) display the convergence of s , \dot{s} and the quadratic error e to zero in exactly $0.2s$. Fig. 5(b)

Fig. 5. (a) Trajectory of the quadratic tracking error position; (b) Control input u (v)

displays the control signal which is smooth and free from chattering and abrupt variations. Fig. 6 shows a good tracking Fig. 6. (a) The state $x_1(t)$ (solid line) and its reference y_r (dashed line);(b) The state $x_2(t)$ (solid line) and its reference \dot{y}_r (dashed line)

of position and velocity trajectories. The state $(x_1(t); x_2(t))$ converge quickly to their references $(y_r; \dot{y}_r)$.

VI. CONCLUSION

In this paper, we have presented a type-2 fuzzy sliding mode controller for a perturbed uncertain nonlinear systems. The control is generated from a nominal fuzzy type-2 model, who exploits the linear local models of the system. These local models are obtained by linearisation around some operating points. The controller design uses the integral sliding mode concept. It is able to steer to zero in finite time the sliding surface and its derivatives, the convergence time can be fixed in advance. In order to eliminate the chattering effect, the *sign* function in this part control is substituted by a smooth variation fonction. Its structure conserves the characteristics of the original function. The sliding surface is computed using Slotine equation, this choice leads to reduce the relative degree of the system to one, which simplifies the design of the proposed method. A simulation example has been presented to illustrate the effectiveness and the robustness of the proposed approach. Currently we are working on the generalization of this approach to multi-input multi-output systems with real-time implementation.

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