Nonlinear Predictive Control with Input Constraints of Robot Manipulator

K. Bdirina, R. Hadjer, M. Boucherit and D. Djoudi

Abstract— **Rigid link manipulators have attracted more and more attention from robot control theorists and robot users because of its various potential advantages. However, their nonlinear dynamics present a challenging control problem, since traditional linear control approaches do not easily apply. For a while, the difficulty was mitigated by the fact that manipulators were highly geared, thereby strongly reducing the interactive dynamic effects between links. At the same time more and more constraints, stemming for example from environmental and safety considerations, need to be satisfied. Often these demands can only be met when process constraints are explicitly considered in the controller. In this work, the one step ahead predictive control with input constraints has been applied to compute time optimal solutions for a two links manipulator operating in the horizontal plane subject to control angle positions in presence of input constraints (Torques amplitude). Tracking performances of the controller are investigated via some simulations, where the comparisons are done for the cases of unconstrained and constrained input.**

I. Introduction

The concept of predictive control is the creation of an anticipatory effect, this control structure, developed for linear systems, has experienced a real boom as advanced control technology since the 80s [1]. This growth is due to its robustness vis-à-vis the structured or unstructured uncertainties. In general, the dynamic model of physical processes is nonlinear and the establishment of predictive control laws for these processes requires minimizing the cost function online, which is an operation very complex [2]. To avoid this problem of online optimization, nonlinear predictive control several off-line have been proposed [3]–[4]–[5]. The prediction of tracking discard at one step is obtained using Taylor expansion of order ri of the output signal and reference, where ri is the relative degree of the i[th] system output, the solution of the minimization a quadratic criterion at one step establishes the control law.

In this paper a fixed one step ahead nonlinear predictive control with input constraints is applied to two links manipulator system. After presenting the principle of the one step ahead predictive control, a mathematical model is developed for our nonlinear system in the presence of input constraints to validate and test the proposed control, simulations were conducted through which the control performance is evaluated..

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II. Nonlinear Predictive Control

Consider the nonlinear system

$$\begin{cases} \dot{x} = f(x) + \sum_{i=1}^{m} g_i(x)u_i(t) \\ y(t) = h(x) \end{cases} \quad (1)$$

Where x(t) is the vector of state variables, u(t) is the control vector and y(t) is the output vector, the functions f, g, h are assumed to be real and have continuous partial derivatives. The classical goal in control is to impose the output of the controlled system to achieve a setpoint as quickly as possible [6]. In the predictive context, the predicted tracking error is minimized over a finite horizon. The model prediction of a nonlinear system is a continuous function that allows us to calculate the system output at future time (t + h), where h> 0 is the prediction horizon.

The predictive model output based on the Taylor series expansion is given by,

$$y(t+h) = y(t) + V_y(x,h) + \Lambda(h)W(x)u \quad (2)$$

Where

$$V_y(x,h) = (v_1(x,h) \quad v_2(x,h) \quad \dots. \quad v_m(x,h))^T;$$

With $v_i(x,h) = hL_f h_i(x) + \frac{h^2}{2!}L_f^2 h_i(x) + \cdots + \frac{h^{r_i}}{r_i!}L_f^{r_i} h_i(x)$

$$\Lambda(h) = diag\left(\frac{h^{r_1}}{r_1!}, \frac{h^{r_2}}{r_2!}, \dots, \frac{h^{r_m}}{r_m!}\right)$$

$$W(x) = (w_1 \quad w_2 \quad \cdots \quad w_m)^T;$$

With $w_i(x) = \left(L_{g1}L_f^{r_i-1}h_i(x) \quad \dots \quad L_{gm}L_f^{r_i-1}h_i(x)\right)$

A. Reference Trajectory

For the output y(t) of nonlinear system (1) can follow the reference trajectory $y_{ref}(t)$, it must be r differentiable, where r is the relative degree of the output y(t). This condition ensures the controllability of the output along the setpoint $y_{ref}(t)$ [7].

Therefore we can apply the Taylor expansion of order r to the reference signal:

$$y_{ref}(t+h) = y_{ref}(t) + d(t,h) \qquad (3)$$

Where

$$d(t,h) = (d_1(t,h) \quad d_2(t,h) \quad \dots \quad d_m(t,h))^T$$

With $d_i(t,h) = h\dot{y}_{refi} + \frac{h^2}{2!}\ddot{y}_{refi} + \dots + \frac{h^{r_i}}{r_i} y_{refi}^{(r_i)}$

In case this is not checked, a trajectory model of exponential type is used to generate the reference trajectory $y_{ref}(t)$ from the setpoint $y_d(t)$ [5]. The reference trajectory $y_{ref}(t)$ is in this case the solution of differential equation:

$$y_{ref}(t) + \gamma_1 \frac{dy_{ref}}{dt} + \gamma_2 \frac{dy_{ref}}{dt} \dots + \gamma_r \frac{dy_{ref}}{dt} = y_d \qquad (4)$$

B. One Step Ahead Predictive Control

The objective of one step ahead predictive control is to find a control law u(t) which coincides the output y (t) with the reference trajectory y_{ref} (t) at time (t + h) [3]–[8]. So the criterion is to minimize the following functional:

$$J_1(y, y_{ref}, R, Q, u) = \frac{1}{2}\left\|y(t+h) - y_{ref}(t+T)\right\|_Q^2 + \frac{1}{2}\|u(t)\|_R^2 \qquad (5)$$

By replacing equations (3) and (2) in (5) the cost function is then written:

$$J_1(y, y_{ref}, R, Q, u) = \frac{1}{2}\left\|(y(t) + V_y(x,h) + \Lambda(h)W(x)u) - (y_{ref}(t) + d(t,h))\right\|_Q^2 + \frac{1}{2}\|u(t)\|_R^2 \qquad (6)$$

Where Q ϵ $\mathbf{R}^{m\times m}$ is a definite positive matrix and R ϵ $\mathbf{R}^{m\times m}$ is a positive semi definite matrix. The optimal solution is then obtained by minimizing the criterion (6) for the nonlinear system (1) compared to control vector u (t)

$$u(t) = -[(\Lambda W)^T Q \Lambda W + R]^{-1} (\Lambda W)^T Q \left[e(t) + V_y(x,h) - d(t,x)\right] \qquad (7)$$

e (t) is the tracking error

$e(t+h) = y(t+h) - y_{ref}(t+h)$

C. One Step Ahead Predictive Control with Input Constraints

The cost function given by (6), can be written in quadratic form expressed as

$$J = \frac{1}{2}u^T E u + u^T F \qquad (8)$$

Where

$$E = (\Lambda W)^T Q \Lambda W + R$$

$$F = (\Lambda W)^T Q \left[e(t) + V_y(x,h) - d(t,x)\right]$$

E, *F*: are compatible matrices and vectors in the quadratic programming problem. Without loss of generality, *E* is symmetric and positive definite.

As the optimal solutions will be obtained using quadratic programming, the constraints need to be decomposed into two parts to reflect the lower limit and the upper limit with opposite sign. In case of input constraints which are the most commonly encountered among all constraint types, we demand that

$$U_{min} \leq u(k) \leq U_{max}$$

Or in a matrix form

$$\begin{bmatrix} -I \\ I \end{bmatrix} u \leq \begin{bmatrix} -U_{min} \\ U_{max} \end{bmatrix}$$

With

I: is (m×m) identity matrix

m: Number of inputs

So the objective of one step ahead predictive control, which allow us to find the control law in a way that coincides the output with the desired trajectory and that satisfied the input constraints, can be expressed by the following functional:

$$J = \frac{1}{2}u^T E x + u^T F \qquad (9)$$

$$Dx \leq \gamma$$

Where

$$D = \begin{bmatrix} -I \\ I \end{bmatrix}; \quad \gamma = \begin{bmatrix} -U_{min} \\ U_{max} \end{bmatrix}$$

Once the problem of control is translated in a quadratic function (9), a simple quadratic programming algorithm can be used to find the control law; in this work Lagrange method is chosen.

D. Stability

Let h be a positive real number and Q a positive definite matrix. Predictive optimal control described in (7) linearizes the dynamics of the tracking error and allows a continuation of the asymptotic trajectory reference if and only if $r \leq 4$ for i = 1, 2, ... m. [7] –[9]..

III. One Step Ahead Predictive Control of Rigid Manipulator with Input Constraints

The robot system is described by nonlinear model [9], with q being the joint angles, τ being the joint inputs.

Rigid robot manipulator can be generally expressed as

$$\ddot{q}(t) = -M(q)^{-1}\big(C(q,\dot{q})\dot{q} + G(q)\big) + M(q)^{-1}\tau(t) \quad (10)$$

Optimal control is applied to the robot manipulator to solve the problem of tracking a reference trajectory for the joint positions, the reference trajectory is a specified by continuous function $q_{ref}(t)$ for tϵ[t0, tf]. The problem consists to find a control law of Torque τ (t) such that the tracking error is minimized at the instant (t+h), where h is an increment of time, in a way that the predicted output q (t + h) coincides with the set-point q_{ref} (t + h). Thus, the tracking error at time (t + h) will be defined by:

$$e(t+h) = q(t+h) - q_{ref}(t+h) \quad (11)$$

The easiest way to predict the influence of the torque τ (t) on the output q(t + h) is to use the Taylor expansion [10] –[11] to second order (r = [2.2]), we get:

$$q(t+h) = q(t) + h\dot{q}(t) + \frac{h^2}{2}\ddot{q}(t) \quad (12)$$

From equation (10), we deduce the model predictive output at time (t + h):

$$q(t+h) = q(t) + V(q,\dot{q},h) + \frac{h^2}{2}W(q)\tau(t) \quad (13)$$

Where

$$V(q,h\dot{q},h) = h\dot{q}(t) - \frac{h^2}{2}M^{-1}(q)(C(q,\dot{q})\dot{q} + G(q))$$

$$W = M(q)^{-1}$$

Similarly, we apply the Taylor expansion of order 2 to the reference signal q_{ref} (t + h), we have:

$$q_{ref}(t+h) = q_{ref}(t) + h\dot{q}_{ref}(t) + \frac{h^2}{2}\ddot{q}_{ref}(t)$$

$$= q_{ref}(t) + d(t,h) \quad (14)$$

The dynamic tracking at time (t + h) depending on the input torque can be then written:

$$e(t+h) = e(t) + V(q,\dot{q},h) - d(t,h) + \frac{h^2}{2}W(q)\tau(t) \quad (15)$$

The aim of control in this case is to find the torque τ (t) which forwards to the output *q (t)* the reference trajectory $q_{ref}(t)$ at time (t + h) while seeking the least possible actuators. This then leads to an optimization criterion of the form:

$$J_l(e,\tau,h) = \tfrac{1}{2}e(t+h)^T Q e(t+h) + \tfrac{1}{2}\,\tau(t)^T R \tau(t) \quad (16)$$

From the first derivative of the cost function *J,* we find the optimal solution for the control signal as:

$$\tau(t) = -\left[\frac{h^4}{2}W(q)^T Q W(q) + R\right]^{-1}\left(\frac{h^2}{2}W(q)\right)^T Q\big(e(t) + V(q,\dot{q},h) - d(t,h)\big) \quad (17)$$

In case of input constraints the matrix E and F of the quadratic cost function (9) are

$$E = \frac{h^4}{2}W(q)^T Q W(q) + R$$

And

$$F = \left(\frac{h^2}{2}W(q)\right)^T Q\big(e(t) + V(q,\dot{q},h) - d(t,h)\big)$$

Consider the manipulator similar to the fig. 1; the manipulator consists of two rigid bodies of masses m_1 and m_2, lengths $l_1$and l_2. The angles of the joints are respectively *q1* and *q2* and a load mass m_L of diameter l_L.

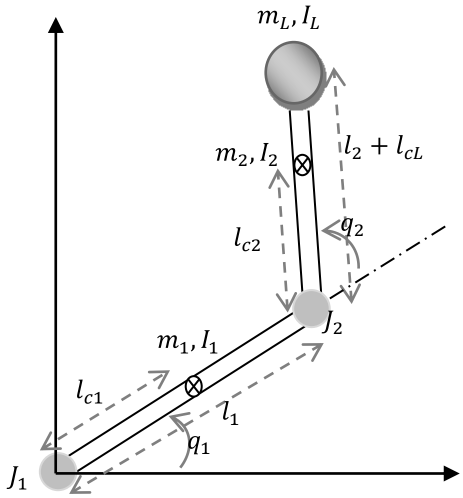


Figure 1. Schematic of Two axis robot

The dynamic model is described by equation (10) with the following components [9] – [10]:

$$A_{11} = m_2 l_1^2 + m_l l_1^2 + I_1 + I_2 + I_l + J_1 + 2(m_2 l_1 (l_{c2} + l_2) + m_l l_1 (l_2 + l_{cl})) \cos q_2$$
$$A_{12} = A_{21} = I_2 + I_l + (m_2 l_1 (l_{c2} + l_2) + m_l l_1 (l_2 + l_{cl})) \cos q_2$$
$$A_{22} = I_2 + I_l + J_2$$
$$c_{11} = -\dot{q}_2 (m_2 l_1 (l_{c2} + l_2) + m_L l_1 (l_2 + l_{cL})) \sin q_2;$$
$$c_{12} = -(\dot{q}_1 + \dot{q}_2)(m_2 l_1 (l_{c2} + l_2) + m_L l_1 (l_2 + l_{cL})) \sin q_2;$$
$$c_{21} = \dot{q}_1 (m_2 l_1 (l_{c2} + l_2) + m_L l_1 (l_2 + l_{cL})) \sin q_2;$$
$$c_{22} = 0.$$
$$g_1 = (m_1(l_{cl} + l_1) + m_2 l_1 + m_L l_1) g \cos q_1 + (m_2(l_{c2} + l_2) + m_L(l_2 + l_{cl})) g \cos(q_1 + q_2)$$
$$g_2 = (m_2(l_{c2} + l_2) + m_L(l_2 + l_{cl})) g \cos(q_1 + q_2)$$

J1 and J2 represent the inertia of the motors. The numerical values of the parameters of the represented manipulator are given in the table below

Link 1	$l_1 =$ 0.45 m	$m_1 =$ 100 Kg	$l_{c1} =$ 0.15 m	$I_1 =$ 6.25 Kg m^2	$J_1 =$ 4.77 Kg m^2
Link 2	$l_2 =$ 0.20 m	$m_2 =$ 25 Kg	$l_{c2} =$ 0.15 m	$I_2 =$ 0.61 Kg m^2	$J_2 =$ 3.58 Kg m^2
		$m_L =$ 40 Kg	$l_{CL} =$ 0.15 m	$I_L =$ 7.68 Kg m^2	

Table 1. Physical parameters of two links manipulator

Where

I_L is the inertia of the terminal link (where payload inertia is included)

l_{cl} is the center of gravity of the load.

The reference models chosen in continuous time are:

$$q_{ref} = \begin{vmatrix} q_{ref1} \\ q_{ref2} \end{vmatrix}$$

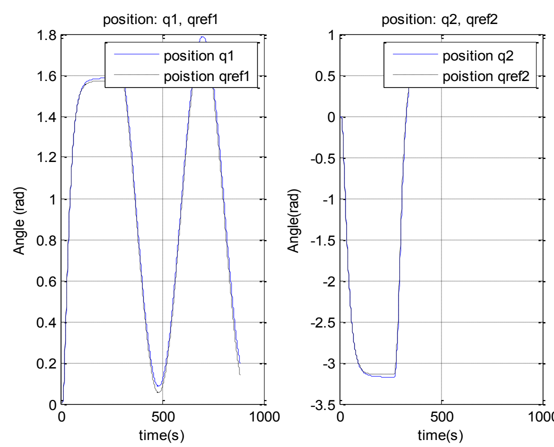
Where:

$$q_{refi}(s) = \frac{\omega^2}{s^2 + 2\xi\omega s + \omega^2} r_i(s) \ldots\ldots i = 1,2 \qquad (18)$$

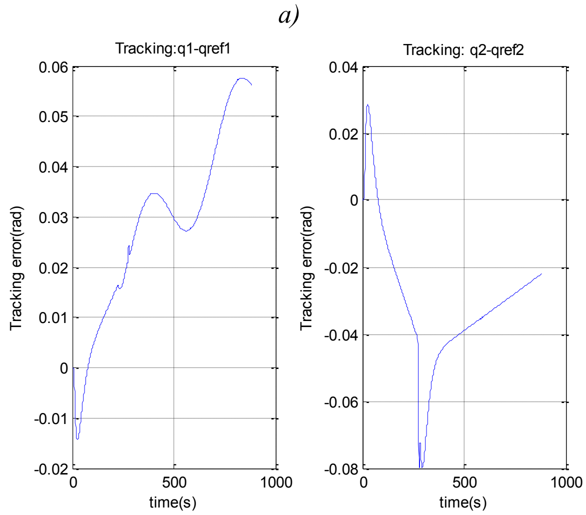
The nonlinear predictive controller is used to track these desired trajectories

$$r_1(t) = \begin{cases} \frac{\pi}{2} & si\ 0 \le t \le 2.5\ s \\ 0.9(1 - \cos(1.26t)) & si\ t > 2.5\ s \end{cases}$$

$$r_2(t) = \begin{cases} -\pi & si\ \ 0 \le t \le 3\ s \\ \frac{\pi}{4} & si\ \ t > 3\ s \end{cases}$$



a)



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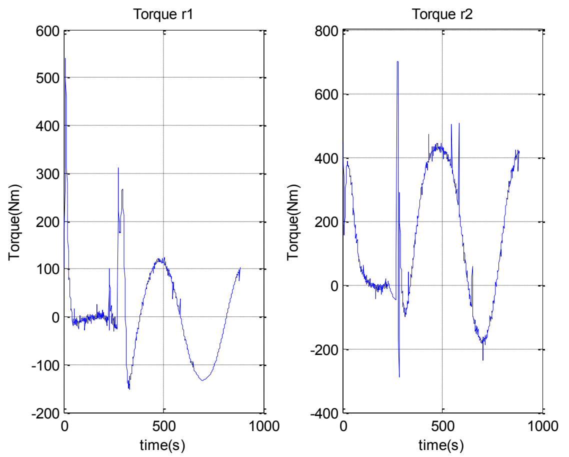
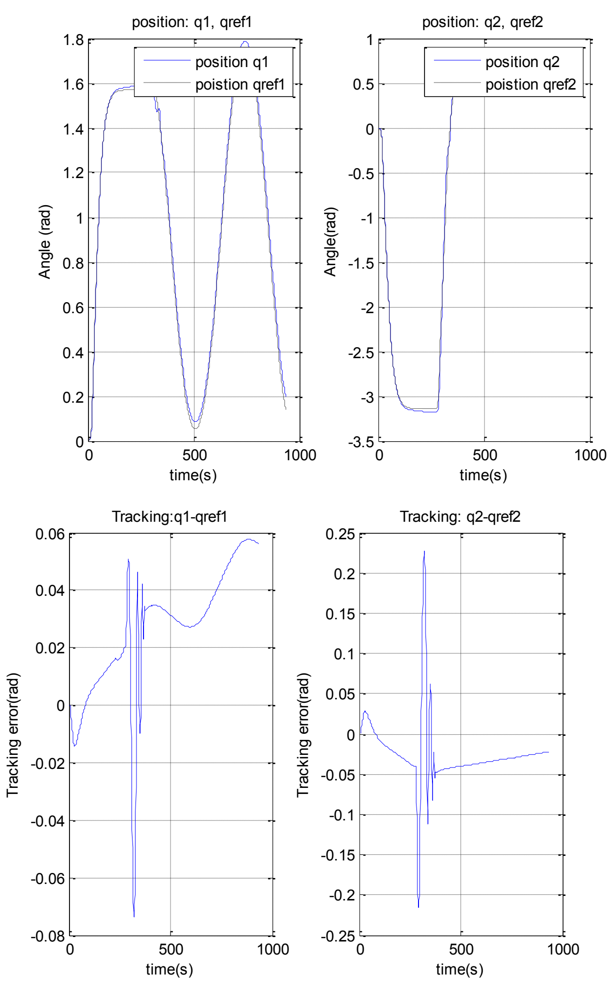


(c)

Figure 2. One step ahead predictive control of two links robot: (a) Angle positions and references, (b) tracking errors (c) applied torques Constraints: τ_{max}=700Nm, τ_{min}=-700Nm



(a)

(b)

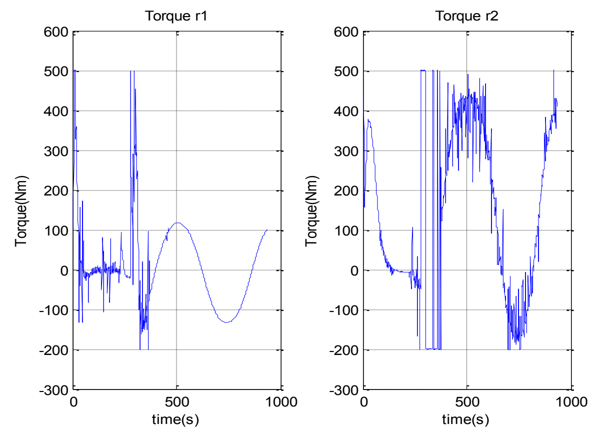


Figure 3. One step ahead predictive control of two links robot: (a) Angle positions and references, (b) tracking errors (c) applied torques Constraints : τ_{max}=500Nm, τ_{min}= -200Nm

The initial displacements and velocities are chosen as: q1(0)=q2(0)=0°, $\dot{q}_1(0) = \dot{q}_2(0) = 0$, the choice of the damping coefficient and natural angular frequency of the filter is as follows:

$$\xi=1,\ w_1=w_2=5\text{rad/s}$$

The nonlinear controller has been tested by simulation where we have represented two cases, in the first one torque constraints are supposed to be τ_{max}=700Nm, τ_{min}=-700Nm and τ_{max}=500Nm, τ_{min}=-200Nm in the second one .

The control parameters $\mathbf{Q}=10^5\ \mathbf{I}_n$, $\mathbf{R}=10^{-12}\ \mathbf{I}_n$ and h is set to 0.005. Simulation results are show in Figures 2 and 3. These Figures give the angular positions ($q1(t),q2(t)$) , angle references ($q_{ref}1(t),q_{ref}2(t)$) and the position tracking errors also the applied torques.

The simulation results clearly show the effectiveness of the one step ahead controller with input constraints in terms of references tracking (angle positions) and constraints respecting (Torques amplitude) where it's very clear from torques figures that the value of these one, in the two cases, still in the interval limited by the minimal and the maximal values. Note here the presence of undulation in the torque signal; this problem can be solved by imposing constraints of incremental variation of input signal in control objective. Finally the major drawback of one step ahead predictive control is the need to perform several simulations to obtain numerical values of optimal parameters settings.

VII. Conclusion

This work has focused on the contribution to the development of original control structure: the one step ahead predictive control with input constraints: a strategy based on nonlinear predictive model, where optimization mechanism of the quadratic criterion is used to extract the control law. The principle of the one step ahead predictive control, is based on the Taylor series expansion of the predicted output and the reference where the control law is obtained by minimizing the quadratic error between them. This approach has been applied to compute time optimal solutions for a nonlinear system that of two links manipulator operating in the horizontal plane subject to control angle positions in presence of input constraints (Torques amplitude). Concerning the one step ahead control with input constraints, the basic idea is to translate the constraints problems to linear inequalities then parameterize them using the same parameter vector u as the ones used in the design of predictive control, in order to relate to the original model predictive control problem, where the optimal solution is found by solving a quadratic optimization problem.

The simulation results clearly show the effectiveness of this approach in terms of references tracking (angle positions) and respecting input constraints. The control objective is achieved with good accuracy and finally the major drawback of one step ahead predictive control is the need to perform several simulations to obtain numerical values of optimal parameters settings.

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