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Reduced Order Observers Design for Systems with Delays in State Variables: Time and Frequency Domain cases

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***Abstract*— Both time and frequency domain new designs of reduced order observers for linear systems with constant time delay in state variable are addressed in this paper. The order of this observers permits to avoid redundancy between the measurements and the full order observer state. The time procedure design is based on Lyapunov Kravoskii stability theory where, after given the existence condition of such observers, the optimal gain implemented in the reduced order observer with internal delay design is obtained in terms of linear matrix inequalities (LMIs). The independent of internal delay and independent of delay cases are derived as particular cases. A design algorithm of reduced-order observer design is proposed. The frequency domain description is derived from the time domain one by applying the factorization approach. We propose some useful right coprime Matrix Fraction Descriptions (MFDs). A numerical example is given to illustrate the proposed approach.**

I INTRODUCTION

Time delay systems are commonly encountered in various engineering systems, such as chemical processes, long transmission lines in pneumatic and hydraulic systems [1], [2], [3]. The time delay usually results in unsatisfactory performance and is frequently a source of instability, so control of time-delay systems is practically important. This control is often realized with the assumption that the entire state vector is available through output measurement. Since this is not generally true in the practice, it is necessary to design observers which produces an estimate of this state vector.

Observer design theory for time-delay systems has been most widely considered in the last decade and several design methods have been proposed (reducing transformation technique, coordinate change approach, LMI method, ...) [4], [5], [6], [7], [8].... However, little research has been focused on design of reduced order observer for linear systems with delays in state variables.

In frequency domain, there is less literature in state estimation compared to that of time domain [9], [10], [12], [13], [14], [15], [16], [19], although it is the basis for most analysis performed on control systems . And it is well known that both linear optimal state feedback and optimal linear filtering problems can be formulated and solved in the frequency domain. A first solution for the filter transfer matrix was presented by MacFarlane [18], and it was demonstrated in [11] that optimal state feedback law and the optimal filter can be characterized by polynomial matrix that directly parameterizes the state feedback control and the observer in the frequency domain.

Motivated by these facts, a new time and frequency domain design procedure of the reduced order observers for time delay linear systems is presented. The time domain procedure is based on Lyapunov-Karavoskii stability theory, where we give the existence condition of such observers and the gain implemented in the design is obtained using (LMIs). The particular cases where the observer is independent of internal delay and of delay are also considered. The frequency domain procedure design is derived from time domain results, where we propose some suitable Matrix Fractions Descriptions (MFDs) and mainly, applying the factorization approach permits to give a polynomial description of the proposed observer.

The main reason of formulating the results of the time domain in the frequency one is the advantages that it presents for the observer-based control [17]. In fact, in this case, the compensator is driven by the input and the output of the system. So only the input-output behavior of the compensator (characterized by its transfer function) influences the properties of the closed-loop system. The additional degrees of freedom given by the frequency approach can then be used for robustness purpose for example [17]. Note that another interest of our approach, is the use of a reduced order observer (not a full order one), without estimating all the state of the system contrary to full order. The benefits of reduced order observer are for example reduction of the computation time of the observer and the capability to use it for large scale system by reducing the dimension of the observer.

II PROBLEM FORMULATION

Let's consider the following continuous-time linear time-delay system described by

$$\dot{x}(t) = A\,x(t) + A_d\,x(t-\tau) + B\,u(t) \quad (1a)$$
$$y(t) = C\,x(t) \quad (1b)$$
$$x(t) = \phi(t), t \in [-\tau, 0] \quad (1c)$$

where $x(t) \in \Re^n$ is the state vector, $y(t) \in \Re^p$ is the measured output vector, $u(t) \in \Re^r$ is the input vector. τ is the constant time-delay of the system. $\phi(t)$ is a continuous vector-valued initial function. Matrices A, A_d, B and C are known constant matrices of appropriate dimensions.

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However, Since the p outputs of system (1) can be measured directly ((1b)), it suffices to build a reduced order observer of order $(n-p)$ for the remaining $(n-p)$ linear combinations $z(t)$ of the system states. Let,

$$z(t) = Tx(t) \tag{2}$$

where $z(t) \in \Re^{n-p}$.

In the sequel of the paper we make the following assumption.

Assumption 1:

1) $rank \begin{bmatrix} T \\ C \end{bmatrix} = n$

2) $\begin{bmatrix} T \\ C \end{bmatrix}$ is non singular.

Following assumption 1, let

$$\begin{bmatrix} T \\ C \end{bmatrix}^{-1} = \begin{bmatrix} H_1 & E_1 \end{bmatrix} \tag{3}$$

which gives the following relations:

$$\begin{bmatrix} T \\ C \end{bmatrix} \begin{bmatrix} H_1 & E_1 \end{bmatrix} = \begin{bmatrix} I_{n-p} & 0 \\ 0 & I_p \end{bmatrix} \tag{4}$$

So, the full state estimation $\hat{x}(t)$ of $x(t)$, can be obtained by the relation

$$\begin{aligned} \hat{x}(t) &= \begin{bmatrix} T \\ C \end{bmatrix}^{-1} \begin{bmatrix} \hat{z}(t) \\ y(t) \end{bmatrix} &(5)\\ &= H_1\hat{z}(t) + E_1 y(t) &(6) \end{aligned}$$

where $\hat{z}(t)$ is the output of the proposed reduced order observer for system (1), as given in the following section.

III TIME DOMAIN DESIGN OF THE REDUCED ORDER OBSERVER

We aim to design an observer of order $(n-p)$ for system (1), of the form:

$$\begin{aligned} \dot{\epsilon}(t) &= N\epsilon(t) + N_d\epsilon(t-\tau) + Hu(t) + D_1 y(t) \\ &+ \ D_d y(t-\tau) &(7a)\\ \hat{z}(t) &= \epsilon(t) + Ey(t) &(7b) \end{aligned}$$

where $\hat{z}(t) \in \Re^{n-p}$ is the estimate of $z(t)$ (see (2)).

The problem of the reduced order observer design to be solved in the paper can be stated as follows.

Problem : Given the system (1) and the reduced order observer (7), design the observer matrices N, N_d, H, D_1, D_d and E such that $\hat{z}$ asymptotically converges to z.

III-A REDUCED ORDER OBSERVER CONDITIONS OF CONSTANT TIME-DELAY LINEAR SYSTEMS

Define $e(t)$ as the time estimation error, using (2) and (7b), it is given by

$$\begin{aligned} e(t) &= z(t) - \hat{z}(t) &(8a)\\ &= (T - EC)x(t) - \epsilon(t) &(8b)\\ &= \psi x(t) - \epsilon(t) &(8c) \end{aligned}$$

with

$$\psi = T - EC \tag{9}$$

then, we have the following theorem.

Theorem 1: The function observer (7) is a reduced order observer for linear system (1), if the following equations are satisfied:

$i)$ $\dot{e}(t) = Ne(t) + N_d e(t-\tau)$ is asymptotically stable.

$ii)$ $\psi A - N\psi - D_1 C = 0$

$iii)$ $\psi A_d - N_d\psi - D_d C = 0$

$iv)$ $H = \psi B$

Proof 1: The proof is omitted due to the lack of place.

III-B REDUCED ORDER OBSERVER DESIGN OF CONSTANT TIME-DELAY LINEAR SYSTEMS

Using the definition of ψ, conditions $ii) - iii)$ of theorem 1 can be written as:

$$NT + K_1 C + ECA = TA \tag{10}$$

$$N_d T + K_2 C + ECA_d = TA_d \tag{11}$$

where

$$K_1 = D_1 - NE \tag{12}$$

and

$$K_2 = D_d - N_d E \tag{13}$$

Notice that once matrix E is determined, matrix H is immediately deduced from condition $iv)$ of theorem 1.

First, notice that equations (10)-(11) can be rewritten as follows:

$$[N \;\; K_1] \begin{bmatrix} T \\ C \end{bmatrix} = (TA - ECA) \tag{14}$$

and,

$$[N_d \;\; K_2] \begin{bmatrix} T \\ C \end{bmatrix} = (TA_d - ECA_d) \tag{15}$$

then we can express matrices N, K_1, N_d and K_2 in terms of matrix E. In fact, using equation (3) we get:

$$[N \;\; K_1] = [TAH_1 - ECAH_1 \;\; TAE_1 - ECAE_1] \tag{16}$$

and,

$$[N_d \;\; K_2] = [TA_dH_1 - ECA_dH_1 \;\; TA_dE_1 - ECA_dE_1] \tag{17}$$

It is then obvious that

$$N = TAH_1 - ECAH_1 \tag{18}$$

$$K_1 = TAE_1 - ECAE_1 \tag{19}$$

$$N_d = TA_dH_1 - ECA_dH_1 \tag{20}$$

$$K_2 = TA_dE_1 - ECA_dE_1 \tag{21}$$

The second step of the proposed resolution method is to transform equation (20) to the following matrix form:

$$[N_d \quad E]\begin{bmatrix} I_{n-p} \\ CA_dH_1 \end{bmatrix} = TA_dH_1 \quad (22)$$

Our purpose is now to resolve equation (22). Let us set,

$$[N_d \quad E] = X \quad (23)$$

$$\begin{bmatrix} I_{n-p} \\ CA_dH_1 \end{bmatrix} = \Sigma \quad (24)$$

$$TA_dH_1 = \Theta \quad (25)$$

therefore (22) becomes

$$X\Sigma = \Theta \quad (26)$$

This equation has a solution X if and only if

$$rank\begin{pmatrix} \Sigma \\ \Theta \end{pmatrix} = rank\ \Sigma \quad (27)$$

and a general solution of (26), if it exists, is given by

$$X = \Theta\Sigma^+ - Z(I - \Sigma\Sigma^+) \quad (28)$$

where Σ^+ is a generalized inverse of matrix Σ given by (24) and Z is an arbitrary matrix of appropriate dimensions, that will be determined in the sequel using LMI approach.

Once matrix X is determined, it is easy to give the expressions of matrices N_d and E.

In fact,

$$N_d = X\begin{pmatrix} I_{n-p} \\ 0 \end{pmatrix} = A_{11} - ZA_{22} \quad (29)$$

where

$$A_{11} = \Theta\Sigma^+\begin{pmatrix} I_{n-p} \\ 0 \end{pmatrix} \quad (30)$$

$$A_{22} = (I - \Sigma\Sigma^+)\begin{pmatrix} I_{n-p} \\ 0 \end{pmatrix} \quad (31)$$

$$E = X\begin{pmatrix} 0 \\ I_p \end{pmatrix} = B_{11} - ZB_{22} \quad (32)$$

where

$$B_{11} = \Theta\Sigma^+\begin{pmatrix} 0 \\ I_p \end{pmatrix} \quad (33)$$

$$B_{22} = (I - \Sigma\Sigma^+)\begin{pmatrix} 0 \\ I_p \end{pmatrix} \quad (34)$$

Matrices N_d and E are consequently known. Matrix N is immediately deduced from (18). In fact,

$$N = C_{11} - ZC_{22} \quad (35)$$

where

$$C_{11} = TAH_1 - \Theta\Sigma^+\begin{pmatrix} 0 \\ I_p \end{pmatrix}CAH_1 \quad (36)$$

$$C_{22} = -(I - \Sigma\Sigma^+)\begin{pmatrix} 0 \\ I_p \end{pmatrix}CAH_1 \quad (37)$$

So, once matrices N, N_d and E are determined, values of matrix D_1 and D_d are deduced from (19) and (21) by taking into account (12) and (13).

$$D_1 = TAE_1 - ECAE_1 + NE \quad (38)$$

and

$$D_d = TA_dE_1 - ECA_dE_1 + N_dE \quad (39)$$

Hence all observer matrices are determined if and only if the matrix Z is known.

So, under theorem 1, condition (27) and by using (29) and (35) the dynamics of the observer error reads

$$\begin{aligned} \dot{e}(t) &= Ne(t) + N_de(t-d) \quad (40) \\ &= (C_{11} - ZC_{22})e(t) + (A_{11} - ZA_{22})e(t-d) \end{aligned}$$

Based on Theorem 1 and Lyapunov Karavoskii stability theory, we can give the independent of delay conditions for the stability of the reduced order observer observation error, to get the gain matrix Z which parameterizes the observer matrices.

Theorem 2: The function observer in the form of (7) is a reduced order observer one for system (1) if there exist matrices $P = P^T > 0$, $Q = Q^T > 0$, and Y satisfying the following Linear Matrix Inequalities (LMI):

$$\begin{bmatrix} L_{11} & L_{12} \\ L_{12}^T & -Q \end{bmatrix} < 0 \quad (41)$$

with, $L_{11} = PC_{11} + (PC_{11})^T + Q - YC_{22} - (YC_{22})^T$
$L_{12} = PA_{11} - YA_{22}$

Then the gain Z is given by $Z = P^{-1}Y$.

Proof 2: The proof is omitted due to the lack of place.

Based on the above analysis, an algorithm for reduced order observer design can be given as follows.

Algorithm for Reduced Order Observer design for Constant Time-Delay linear Systems

1) Check that conditions 1) and 2) of assumption 1 are satisfied for a matrix T. Then, compute the values of H_1 and E_1 from (3).
2) Compute Σ and Θ from (24) and (25).
3) Deduce the values of matrices A_{11}, A_{22}, C_{11} and C_{22} from (30), (31), (36) and (37).
4) Verify that rank condition (27) is satisfied. Then, resolution of the proposed LMI (41), gives the gain matrix Z.
5) Compute N_d and N from (29) and (35).
6) Compute E, D_1 and D_d respectively from (32), (38) and (39).
7) After computing ψ from (9), the value of matrix H implemented in the design of reduced order observer can be deduced from iv) of Theorem 1.

In order to obtain observers that are easier to implement, we consider the following particular cases.

III-C PARTICULAR CASE 1: REDUCED ORDER OBSERVER DESIGN : INDEPENDENT OF INTERNAL DELAY OBSERVERS

In this section we present the application of the above results to the particular case where the observer (7) is independent of internal delay ($N_d = 0$).

Therefore, equations (18)-(21) become,

$$N = TAH_1 - ECAH_1 \tag{42}$$

$$K_1 = TAE_1 - ECAE_1 \tag{43}$$

$$0 = TA_dH_1 - ECA_dH_1 \tag{44}$$

$$D_d = TA_dE_1 - ECA_dE_1 \tag{45}$$

Let us define the following matrices:

$$E = X_1 \tag{46}$$

$$CA_dH_1 = \Sigma_1 \tag{47}$$

$$TA_dH_1 = \Theta_1 \tag{48}$$

After verifying that,

$$rank \begin{pmatrix} \Sigma_1 \\ \Theta_1 \end{pmatrix} = rank\ \Sigma_1 \tag{49}$$

the observer matrices are obtained, as follows:

$$E = \Theta_1\Sigma_1^+ - Z_1(I - \Sigma_1\Sigma_1^+) \tag{50}$$

So,

$$N = TAH_1 - \Theta_1\Sigma_1^+CAH_1 + Z_1(I - \Sigma_1\Sigma_1^+)CAH_1 \tag{51}$$

and

$$D_1 = TAE_1 - ECAE_1 + NE \tag{52}$$

with matrix D_d is given by (45).

The matrix gain Z_1, which is equivalent to matrix Z of equation (28) in the general case, can be obtained from any pole placement method, such that the reduced order observer is asymptotically stable.

III-D Particular case 2: REDUCED ORDER OBSERVER DESIGN : INDEPENDENT OF DELAY OBSERVERS

In this section we present the application of the above results to the particular case where the reduced order observer (7) is independent of delay ($N_d = 0$, $D_d = 0$). Therefore, in this case one has following equations,

$$N = TAH_1 - ECAH_1 \tag{53}$$

$$K_1 = TAE_1 - ECAE_1 \tag{54}$$

$$0 = TA_dH_1 - ECA_dH_1 \tag{55}$$

$$0 = TA_dE_1 - ECA_dE_1 \tag{56}$$

the desired matrices can be obtained, similar to previous case, as follows:

$$E = \Theta_1\Sigma_1^+ - Z_1(I - \Sigma_1\Sigma_1^+) \tag{57}$$

So,

$$N = TAH_1 - \Theta_1\Sigma_1^+CAH_1 + Z_1(I - \Sigma_1\Sigma_1^+)CAH_1 \tag{58}$$

and

$$D_1 = TAE_1 - ECAE_1 + NE \tag{59}$$

IV FREQUENCY DOMAIN DESIGN

In this section, we propose an easier technique of designing a reduced order observer for systems with delays in state variables, described in the frequency domain by applying the factorization approach [22] (see also [13]) and [23].

IV-A REVIEW OF FACTORIZATION APPROACH: CASE WITHOUT DELAY

A transfer matrix $H(s)$ is said to be a $RH\infty$ - matrix if $H(s)$ is a real rational matrix which is stable and proper. Furthermore, a state-space realization of

$$H(s) = C(sI - A)^{-1}B + D \tag{60}$$

is denoted by

$$\begin{bmatrix} A & B & C & D \end{bmatrix} \tag{61}$$

A double coprime factorization or Matrices Fractions Descriptions (MFDs) of $H(s)$ reads

$$H(s) = N(s)M^{-1}(s) = \hat{M}^{-1}(s)\hat{N}(s) \tag{62}$$

where $N(s), M(s)$ are right coprime and $\hat{M}(s)$, $\hat{N}(s)$ are left coprime.

Following [22], matrices implemented in factorizations (62) can be computed as follows:

$$M(s) = \begin{bmatrix} A_{R_1} & B & R_1 & I \end{bmatrix} \tag{63}$$

$$N(s) = \begin{bmatrix} A_{R_1} & B & C + DR_1 & D \end{bmatrix} \tag{64}$$

$$\hat{M}(s) = \begin{bmatrix} A_R & -R & C & I \end{bmatrix} \tag{65}$$

$$\hat{N}(s) = \begin{bmatrix} A_R & B - RD & C & D \end{bmatrix} \tag{66}$$

where R_1 and R are chosen such that $A_{R_1} = A + BR_1$ and $A_R = A - RC$ are stable.

IV-B REVIEW OF FACTORIZATION APPROACH: CASE WITH DELAY

Consider a transfer function matrix $G(s) = G_0(s)e^{-\tau s}$, where $G_0 \in IR(s)$ is a strictly proper rational transfer matrix, with the state-space realization $G(s) = C(sI - A)^{-1}Be^{-\tau s}$, and τ is a constant time delay. The double coprime factorization of $G(s)$ can be written as ([23]), see also ([20]- [21]):

$$G(s) = G_1(s)G_2^{-1}(s) = \hat{G}_1^{-1}(s)\hat{G}_2(s) \tag{67}$$

The four matrices above can be calculated by the standard algorithms in the state-space construction and are given below

$$\begin{aligned} G_2(s) &= K(sI - A - BK)^{-1}B + I && (68)\\ G_1(s) &= C(sI - A - BK)^{-1}Be^{-\tau s} && (69)\\ \hat{G}_1(s) &= C(sI - A - L_1C)^{-1}L_1 + I && (70)\\ \hat{G}_2(s) &= C(sI - A - L_1C)^{-1}Be^{-\tau s} && (71) \end{aligned}$$

where K and L_1 are chosen such that $det(sI - A - BK)$ and $det(sI - A - L_1C)$ are stable.

IV-C FREQUENCY DOMAIN SYNTHESIS

The next theorem presents a polynomial description of the proposed reduced order observer for constant time delay linear systems , which is based mainly on the factorization approach.

Theorem 3: Consider the following right coprime factorization based on Matrix Fraction Descriptions (MFDs)

$i)$

$$(sI - N_1(s))^{-1}D_de^{-\tau s} = N_2(s)M_2^{-1}(s) \quad (72)$$

where

a) $N_1(s) = N + N_de^{-\tau s}$

b) The two polynomial matrix $N_2(s)$ and $M_2(s)$ have the specification to be right coprime. These transfer functions, as mentioned above, can be calculated from the factorization approach, as in ([20]- [21]).

$ii)$

$$(sI - N_1(s))^{-1}H = N_3(s)M_3^{-1}(s) \quad (73)$$

$iii)$

$$(sI - N_1(s))^{-1}D_1 + E = N_4(s)M_4^{-1}(s) \quad (74)$$

Note that, polynomial matrices $N_3(s)$, $M_3(s)$, $N_4(s)$ and $M_4(s)$ can be computed from [22].

Then, a frequency domain representation of the reduced order observer (7) of order $(n - p)$, related to system (1) is given by,

$$\begin{aligned} \hat{z}(s) &= N_3(s)M_3^{-1}(s)\ u(s)\\ &+ [N_2(s)M_2^{-1}(s) + N_4(s)M_4^{-1}(s)]\ y(s) \quad (75) \end{aligned}$$

where, polynomial matrices implemented in the proposed MFDs, are given as follows:

$$\begin{aligned} N_2(s) &= (sI - N_1(s) - D_dK)^{-1}D_de^{-\tau s} && (76)\\ M_2(s) &= K(sI - N_1(s) - D_dK)^{-1}D_d + I && (77)\\ N_3(s) &= (sI - N_{1R_1}(s))^{-1}H && (78)\\ M_3(s) &= R_1(sI - N_{1R_1}(s))^{-1}H + I && (79)\\ N_4(s) &= (I + ER_2)(sI - N_{1R_2}(s))^{-1}D_1 + E && (80)\\ M_4(s) &= R_2(sI - N_{1R_2}(s))^{-1}D_1 + I && (81) \end{aligned}$$

R_1, R_2 and K are such that $N_{1R_1}(s) = N_1(s) + HR_1$, $N_{1R_2}(s) = N_1(s) + D_1R_2$ and $det(sI - N_1(s) - D_dK)$ are stable .

Proof 3: The proof is omitted due to the lack of place.

V NUMERICAL EXAMPLE

Consider the system (1) with the following numerical values:

$A = \begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix}$, $A_d = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$.

1 - Time domain reduced order observer design

By applying the proposed algorithm, we obtain the following results:

1)

$$T = \begin{bmatrix} 1 & 2 \end{bmatrix}, H_1 = \begin{bmatrix} 0 \\ -0.5 \end{bmatrix}, E_1 = \begin{bmatrix} 1 \\ -0.5 \end{bmatrix}$$

2)

$$\Sigma = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \Theta = 2$$

3) $A_{11} = 1$, $A_{22} = \begin{bmatrix} 0.5 \\ -0.5 \end{bmatrix}$, $C_{11} = -2$, $C_{22} = \begin{bmatrix} 0.25 \\ -0.25 \end{bmatrix}$

4) Resolving LMI (41) gives

$$Z = \begin{bmatrix} -0.7428 & -2.4928 \end{bmatrix}$$

5) Then using equations (29) and (35)

$$N = -2.4375, \quad N_d = 0.1250,$$

6) Equations (32), (38) and (39) give

$$E = 1.8750, \quad D_1 = -1.2578, D_d = -0.7656,$$

7) Finally H is computed using theorem 1 and is given by,

$$H = -0.8750$$

2 - Frequency domain reduced order observer design

For that, let $\tau = 0.2$. By using the factorization approach as mentioned in section (IV) (see ((75)-(74))) with $K = 1$, $R_1 = 1$ and $R_2 = -1$, matrix $N_1(s) = -2.4375 + 0.1250e^{-0.2s}$ and the polynomial matrices implemented in the reduced order observer design are easily computed.

So, the frequency domain description of the observer is given by

$$\begin{aligned}\hat{z}(z) &= \frac{-0.8750}{s-1250e^{-0.2s}+2.4375}\,u(s)\\ &+ \frac{-0.7656e^{-0.2s}}{s-0.1250e^{-0.2s}+2.4375}\,y(s)\\ &+ \frac{1.8750s-0.2344e^{-0.2s}+3.3125}{s-0.1250e^{-0.2s}+2.4384}\,y(s)\end{aligned}$$

Finally, figures 1 and 2 show the time and frequency domain behavior of the proposed reduced order observer for linear system with delays in state variables (1) and so, the effectiveness of our approach.

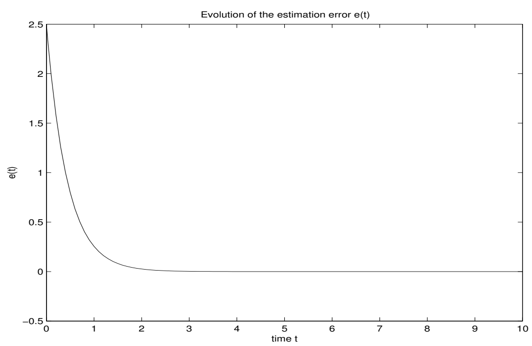

Fig. 1. Evolution of the time domain estimation error e(t)

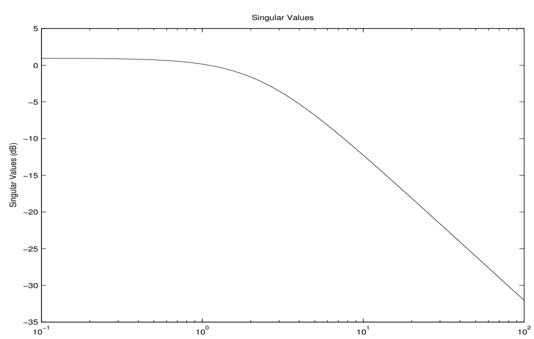

Fig. 2. A singular value plot of the frequency estimation error e(s)

VI CONCLUSIONS AND FUTURE WORKS

In this paper new reduced order observers for linear systems with constant delay in state variable has been designed in time and frequency domains. The time domain method is based on Lyapunov Karavoskii stability theory where we give the existence condition of such observers and an algorithm that summarizes the main steps of the observer design. Then, based on time domain results and by applying the factorization approach, a frequency domain description of the reduced order observer is derived. Further works will concern the design of reduced order filters for systems where measurements are affected by disturbances and therefore, and the observer-based compensator design as it is the main motivation of the frequency domain approach.

REFERENCES

[1] : *Niculescu.I.S*, "Delay effects on stability, A robust control approach", Springer Verlag, 2001.

[2] : *Malek-Zavarei.M and Jamshidi.M*, " Time delay systems: analysis, optimization and applications",North-Holland, 1987.

[3] : *Hale.K.J and Lunel.V.M.S*, "Introduction to functional differential equations", New York: Springer-Verlag, 1993.

[4] : *Pearson. E. A and Fiagbedzi. A. Y*, "An observer for time lag systems", IEEE Trans. on Automatic. Control, vol. 34, no. 4, pp. 775-777, July 1989.

[5] : *Hou.M, Zitek.P, and Patton.J.R*, "An observer design for linear time-delay systems", IEEE Trans. on Automatic Control, vol. 47, no. 1, pp. 121-125, January 2002.

[6] : *Darouch.M*, "Linear functional observers for systems with delays in state variables", IEEE Trans. on Automatic Control, vol. 46, no. 3, pp. 491-496, March 2001.

[7] : *Darouch.M*, "Unknown inputs observers design for delay systems ", Asian Journal of Control, Vol. 9, No. 4, pp. 426-434, December 2007.

[8] : *Fu.Yan-Ming, Duan. Guang-Ren, and Song. Shen-Min*, "Design of Unknown Input Observer for Linear Time-delay Systems ", International Journal of Control, Automation, and Systems, vol. 2, no. 4, pp. 530-535, December 2004.

[9] : *Yao.X.Y and Zhang.M.Y*, "Parameterization of observers for time delay systems and its application in observer design", IEE Proc. Control Theory Appl., vol. 143, no. 3, pp. 225- 232, May 1996.

[10] : *Yao.X.Y, Zhang.M.Y and Kovacvic. R*, "Functional observer and state feedback for input time-delay systems", Int. J. Control, Vol. 66, No. 4, 603-617, 1997.

[11] : *Anderson B D O and $Ku\check{c}eraV$*, "Matrix fraction construction of linear compensators", IEEE Transaction on Automatic Control.Processing, 30, 1112-1114 1985.

[12] : *Ding X, Guo L and Frank M P* , "Parameterization of Linear Observers and Its Application to Observer Design", IEEE Transaction on Automatic Control.Processing, 39, 1648-1652 1994.

[13] : *Ding X, Frank M P and Guo L*, "Robust Observer Design Via Factorization Approach", Proceedings of the 29th Conference on Decision and Control Honolulu. Hawaï December 1990.

[14] : *Hippe P, Wurmthaler CH*, "Optimal reduced-order estimators in the frequency domain: the discrete-time case", International Journal of Control 1990; Vol 52: pp 1051-1064.

[15] : *Hippe P*, "Design of reduced-order optimal estimators directly in the frequency domain", International Journal of Control 189, Vol 50: pp 2599-2614, 1989.

[16] : *Hippe P*, "Design of observer based compensators: the polynomial approach", Kybernetika,125-150, 1991.

[17] : *Hippe P and Deutscher J*, "Design of observer-based compensators : From the time to the frequency domain", Springer-Verlag, London, 2009.

[18] : *MacFarlane A G J*, "Return-difference matrix properties for optimal stationary Kalman-Bucy filter", Proceedings of the Institution of Electrical Engineers, 118, 373-376 1971.

[19] : *Markez H J*, "A frequency domain approach to state estimation", Journal of the Franklin Institute 2003; 340: 147-157 .

[20] : *Nobuyama. E, and Kitamori. T*, "Spectrum assignment and parametrization of all stabilizing compensators for time-delay systems", Proceedings of the 29th Conference on Decision and Control, Honolulu, Hawaii, U.S.A., pp. 3629- 3634.

[21] : *Nobuyama. E*, "Robust stabilization of time-delay systems via reduction to delay-free model matching problems", Proceedings of the 31st Conference on Decision and Control, Tucson, Arizona, U.S.A.,pp. 357-358.

[22] : *Vidyasagar.M*, "Control Systems Synthesis: A Factorization Approach", M.I.T,Press, Cambridge, MA,1985.

[23] : *Yao.X.Y, Zhang.M. Y and Kovacevic. R*, "Functional observer and state feedback for input time-delay systems", International Journal of control 1997, vol. 66, NO. 4, 603-617.