Solving Reactive Power Dispatch by Multiobjective Particle Swarm Optimization using Matrix Computation

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Abstract— **This paper proposes a new approach of optimization in order to resolve multi-objective reactive power dispatch problem. This approach is called matrix multiobjective particle swarm which incorporates vector function as objective function and uses matrix computation to develop the Pareto front unlike the existing multi-objective algorithms which use an external archive. We apply this approach to resolve multi-objective reactive power dispatch problem of IEEE 10-bus system. Numerical tests are encouraging.**

I. Introduction

Energy production and distribution companies have to ensure energy supply at various levels of consumption with a good quality and, at the same time, in minimum cost. The objective of the optimal multi-objective reactive power dispatch (OMORPD) is to minimize three objective functions: compensation devices cost, real power loss and deviation voltage profile while satisfying operational constraints.

Particle swarm optimization (PSO), originated by Kennedy and Eberhart in 1995, was inspired by the choreography of a bird flock. Then, PSO becomes well-known as an efficient optimization algorithm used, in first time, for solving single objective problems and improved for solving multi-criteria problems. In fact, there are many modifications of PSO and a lot of work for adapting PSO to problems with multiple objectives [1, 2]. For example, the study presented in [3] proposes multi-objective PSO with time variant inertia and acceleration coefficients where inertia weight and PSO algorithm parameters expressions depend to iteration number. Other study developed in [4] proposes multi-objective PSO with dynamic population size. Others studies presented in [5, 6, 7] which use hybridation technique.

Unlike all these studies which use inertia weight to develop Pareto front and an external archive to save non-dominated solution, we will modify this algorithm for causing it to use matrix computation, then this algorithm incorporates function vector as objective function and uses Pareto dominance for selecting best solutions and updating Pareto set.

The proposed method has already been implemented to calculate the OMORPD of IEEE 10-bus. The results and the CPU times of algorithm executing will be presented in the paper.

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II. Problem Formulation

A. Objective functions

Compensation devices cost: The compensation cost is formed by the fixed installation cost and the purchase cost [8, 9].

$$F_1 = \sum_{j=1}^{N_C} C_{fi} + C_{ci}\left|Q_{gi}\right|, \qquad (1)$$

where C_{fi} is the fixed installation cost of the reactive power sources at the ith bus ($). C_{ci} is the cost by MVAR of the compensation devices at the ith bus ($/MVAR). Q_{ci} is the reactive compensation at the ith bus (MVAR). N_c is the number of possible bus for the installation of the compensation devices.

Voltage deviation: The voltage deviation at the ith bus is represented by the following function [15]:

$$F_2 = \sum_{i=1}^{N_c}\left(V_i - V_i^{ref}\right)^2 , \qquad (2)$$

where V_i^{ref} is the reference voltage. In normalized values, $V_i^{ref} = 1$.

Real power loss: This objective is to minimize the real power loss in transmission lines [10, 9]. It can be expressed as:

$$F_3 = \sum_{i=1}^{N} V_i \sum_{j=1}^{N} V_j Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij}) , \qquad (3)$$

where V_i and δ_i are the size and the argument of the voltage at ith bus. Y_{ij} and θ_{ij} are the size and the argument the nodal admittance matrix.

B. Problem constraints

Equality constraints: These constraints are represented by load flow equations. They are expressed as follows:

$$P_{gi} - P_{di} - V_i \sum_{j=1}^{N} V_j Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij}) = 0 \qquad (4)$$

$$Q_{gi} - Q_{di} - V_i \sum_{j=1}^{N} V_j Y_{ij} \sin(\delta_i - \delta_j - \theta_{ij}) = 0 \quad (5)$$

for i= 1,…,N, P_{gi} and Q_{gi} are the generator real and reactive power respectively, P_{di} and Q_{di} are the load real and reactive power respectively.

Inequality constraints: Voltage and the reactive production at the load buses are restricted by upper and lower limits:

$$V_i^{\min} \le V_i \le V_i^{\max}. \quad (6)$$

$$Q_{gi}^{\min} \le Q_{gi} \le Q_{gi}^{\max}. \quad (7)$$

The reactive power at i^{th} bus is limited.

III. MULTIOBJECTIVE OPTIMIZATION

We are interested in solving problems of the type:

Minimize: $F(x) = \left[f_1(x), f_2(x), ..., f_{Nobj}(x) \right]$ (8)

Subject to: $g_j(x) \le 0; j = 1, ..., M$

Where $x = \left[x_1, x_2, ..., x_n \right]$ is an n dimensional vector, each $x_i = \left(i = 1, ..., n, n \ge 1 \right)$ can be real-valued, integer-valued or boolean-valued. $f_i(x) = \left(i = 1, ..., Nobj, Nobj \ge 1 \right)$, $g_j(x), \left(j=1,...,M, M \ge 0 \right)$ are linear or nonlinear arbitrary functions.

The family of optimal solutions of this MO problem is composed of all those potential solutions such that the components of the corresponding objective vectors cannot be simultaneously improved. This is known as the concept of Pareto optimality. In a minimization problem, Pareto dominance and Pareto optimality are defined as follows [11]:

Definition 1 (Pareto dominance): A given vector $x = \left[x_1, x_2, ..., x_n \right]$ is said to dominate $y = \left[y_1, y_2, ..., y_n \right]$ if and only if $\forall i \in \{1, 2, ..., n\}, x_i \le x_1^{'}$ and $\exists i \in \{1, 2, ..., n\}, x_i < y_i$.

Definition 2 (Pareto optimality): For a general MO problem, a given solution $x^* \in F$ (where F is the feasible solution space) is Pareto optimal if and only if there is no $x \in F$ that dominate x^*.

IV. MATRIX MULTI-OBJECTIVE PARTICULE SWARM OPTIMIZATION

In this paper, we develop a novel approach in order to better explore the search space then improve final result and speed up the convergence of the algorithm.

Unlike the existing studies presented in [3, 5, 6, 7] which apply the PSO algorithm for solving multiobjective problems, the AMOPSO approach doesn't use an external archive to save the non-dominated solutions but it updates all the best solutions at each iteration. AMOPSO approach is based on the improvement of the population of the best positions, denoted Xmobest, which converges toward the Pareto optimal set. The population in developing Xpart benefit from the experiences of the entire developed population.

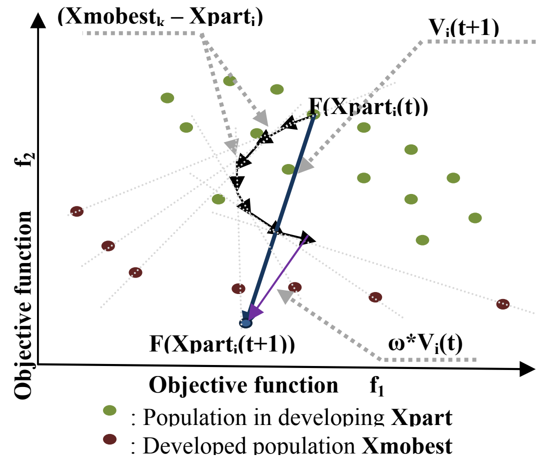

Figure 1. Two objective space of the case of two-objective problem

We use the matrix representation of these two populations Xmobest and Xpart.

Let N be the population size. Then:

$$Xmobest = \left(Xmobest_1, Xmobest_2, ..., Xmobest_N \right) \quad (9)$$

$$Xpart = \left(Xpart_1, Xpart_2, ..., Xpart_N \right) \quad (10)$$

Also, the calculation of the evaluation of these two populations will use a matrix based. We will apply the vector function F = $[f_1, f_2, …, f_{Nobj}]^T$ on the matrix of population. Then:

$$F(Xpart) = \left(F(Xpart_1), F(Xpart_2), ..., F(Xpart_N) \right) \quad (11)$$

$$F(Xpart_i) = \left(f_1(Xpart_i), f_2(Xpart_i), ..., f_{Nobj}(Xpart_i) \right)^T \quad (12)$$

In our approach, the population in developing doesn't follow a single best position but it moves toward each non dominated position. Fig. 1 shows the two objective space of the case of two-objective problem.

Then, the expression of the population velocity, at iteration t, is as follows:

$$S(t) = \omega \cdot S(t-1) + \sum_{k=1}^{N} \rho_k \left(A_N \left(Xmobest_k(t-1) \right) - Xpart(t-1) \right) \quad (13)$$

$$Xpart(t) = Xpart(t-1) + S(t) \quad (14)$$

where $A_N(X) = \underbrace{(X, ..., X)}_{Nelement}$. ω and ρ_k are the acceleration coefficients.

AMOPSO algorithm does not use an external archive for saving best solutions but improves best positions set (developed population Xmobest) at each iteration. Then each

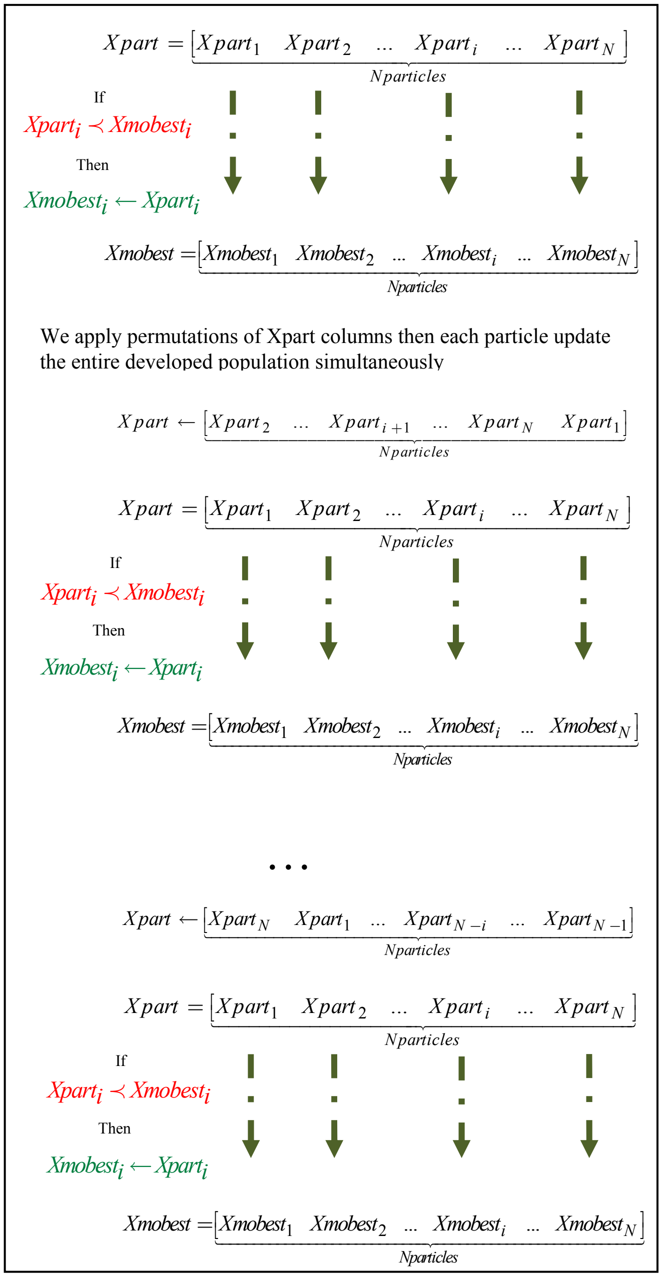

Figure 2. Steps of developed population updating at iteration t

particle in motion improves the position of the entire developed population if it has a better position. Fig. 2 shows the steps of developed population updating at iteration t.
The study of the convergence conditions of our approach and initialization technique are developed in [12].

A. *Flow chart of AMOPSO for ORPD*

Fig. 3 shows the flow chart of Matrix Multi-objective Particle Swarm Optimization applying to reactive power dispatch.

V. SIMULATION AND RESULTS

The used network is an IEEE test network, including 10 buses, 5 thermal generators and 13 lines (see Fig. 4) [9].

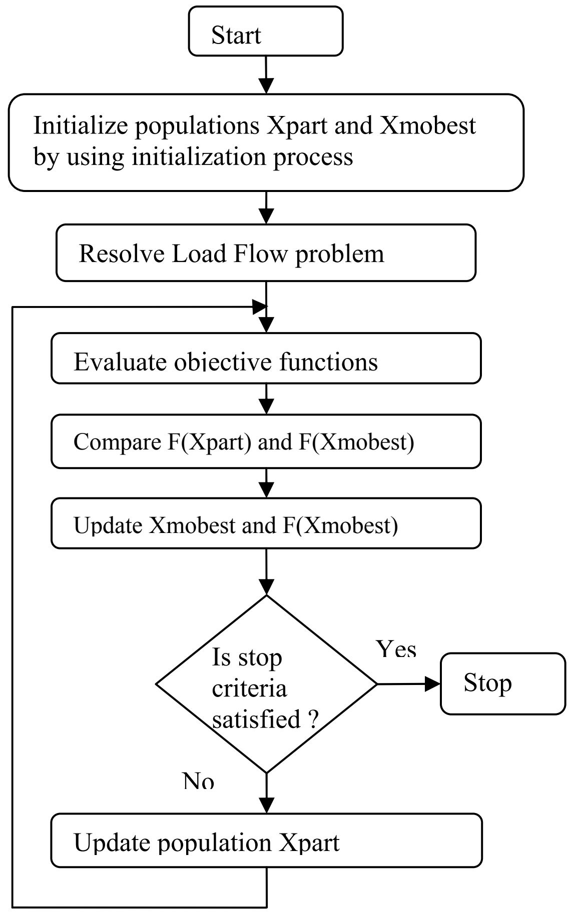

Figure 3. Flow chart of AMOPSO for ORPD

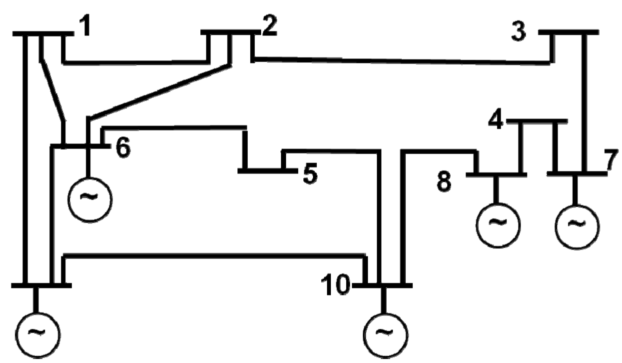

Figure 4. Single line diagram of IEEE 10-bus test system

The data of the lines and buses and the coefficients of the compensation devices cost function and the voltage values wanted of the load buses are given also in article [9].

We will apply classical PSO algorithm and our matrix PSO approach to resolve ORPD for one objective then we will compare results and CPU times.

For bi-objective and three objective ORPD problems, we will only apply matrix MOPSO.

A. *Mono-objective optimization*

Compensation devices cost:

Table 1 gives minimal compensation devices cost with corresponding voltage deviation and real power losses.

Table 1. Minimal compensation devices cost

Minimal compensation devices cost ($) 8857.9	Reactive power at load buses (pu)				
	Q_{g1}	Q_{g2}	Q_{g3}	Q_{g4}	Q_{g5}
Corresponding voltage deviation (pu) 0.0944	0	0	0	0	0
Corresponding real power loss (pu) 2.0696					

Voltage deviation:

Table 2 gives minimal voltage deviation with corresponding compensation devices cost and real power losses.

Table 2. Minimal voltage deviation

Minimal voltage deviation (pu) 0.0032	Reactive power at load buses (pu)				
	Q_{g1}	Q_{g2}	Q_{g3}	Q_{g4}	Q_{g5}
Corresponding compensation devices cost ($) 12235	0.0378	0.3599	0.1653	0.0406	0.0318
Corresponding real power losses (pu) 2.0407					

Real power loss:

Table 3 gives minimal real power losses with corresponding compensation devices cost and real power losses.

Table 3. Minimal real power loss

Minimal real power loss (pu) 1.8394	Reactive power at load buses (pu)				
	Q_{g1}	Q_{g2}	Q_{g3}	Q_{g4}	Q_{g5}
Corresponding compensation devices cost ($) 34846	0.9930	0.9911	0.9663	0.9834	0.9559
Voltage deviation (pu) 1.1262					

We note that if we try to optimize one objective function, we obtain its minimum value but the values of the two other objective functions are deteriorating

B. *Bi-objective optimization*

Figs. 5, 6 and 7 show Pareto front for bi-objective functions of compensation devices cost/real power loss, compensation devices cost/ voltage deviation and voltage deviation/real power loss using matrix MOPSO.

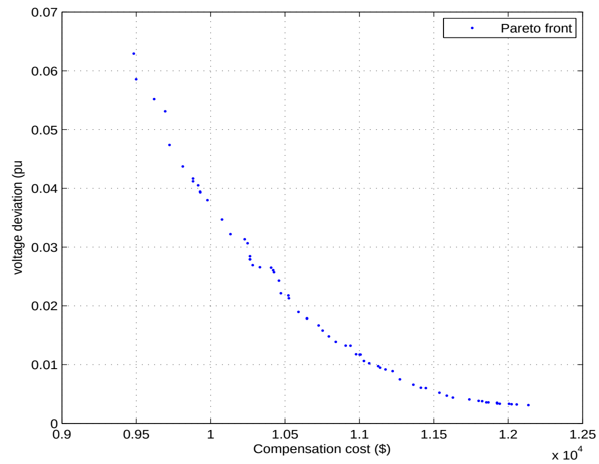

Figure 5. Pareto front of compensation device cost/ voltage deviation

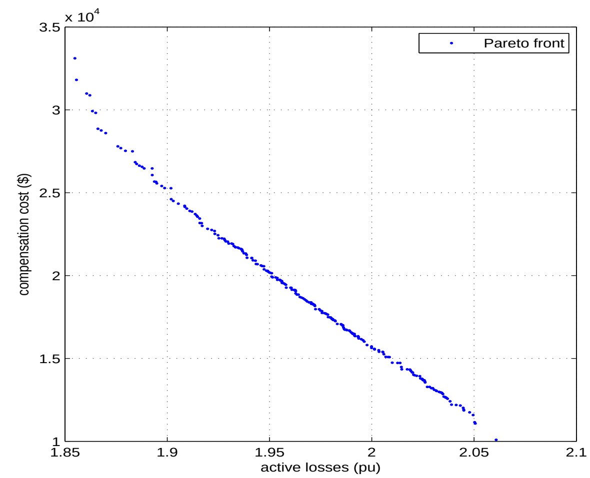

Figure 6. Pareto front of compensation device cost/ Real power loss

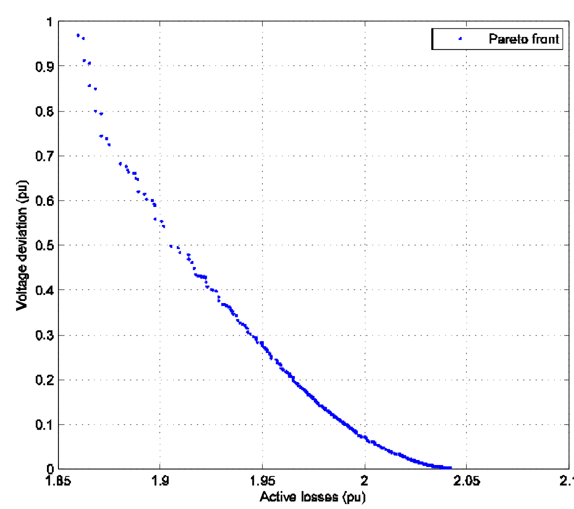

Figure 7. Pareto front of voltage deviation/ real power loss

Tables 4, 5 and 6 give principal points from these Pareto front.

Table 4. Compensation devices cost/ Voltage deviation

Compensation devices cost($)	Voltage deviation (pu)	Reactive power at load buses (pu)				
		Q_{g1}	Q_{g2}	Q_{g3}	Q_{g4}	Q_{g5}
9628	0.0637	0.0045	0.0265	0.0169	0.0025	0.0945
11333	0.0073	0.0588	0.1854	0.1927	0.0003	0.0285
12185	0.0031	0.1387	0.2482	0.1507	0.0592	0.0292

Table 5. Compensation devices cost/ Real power loss

Compensation devices cost($)	Real power loss (pu)	Reactive power at load buses (pu)				
		Q_{g1}	Q_{g2}	Q_{g3}	Q_{g4}	Q_{g5}
10089	2.0	0.1412	0.0265	0.0169	0.0025	0.0945
17736	1.976	0.0023	0.1854	0.1927	0.0003	0.0285
22220	1.927	0.9178	0.2482	0.1507	0.0592	0.0292

Table 6. Voltage deviation/ Real power loss

Voltage deviation (pu)	Real power loss (pu)	Reactive power at load buses (pu)				
		Q_{g1}	Q_{g2}	Q_{g3}	Q_{g4}	Q_{g5}
0.0032	2.0418	0.1242	0.2022	0.1862	0.0862	0.0494
10.8381	4.0291	0.3442	0.9051	0.7279	0.1944	0.0150
0.8565	1.8657	0.9106	0.8907	0.9853	0.8269	0.4470

We note that the points obtained are not comparable and that the dispatcher who decides which is the best solution for a specified electrical network.

C. Three-objective optimization

Fig. 8 shows Pareto surface for three-objective functions of compensation devices cost, real power loss and voltage deviation using matrix MOPSO.

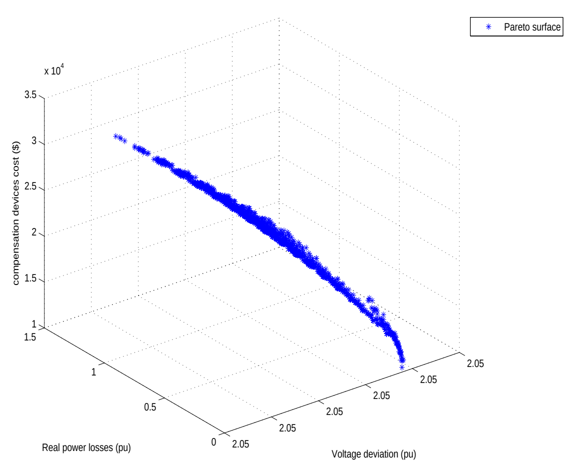

Figure 8. Pareto surface of compensation cost/ voltage deviation/ real power loss

Table. 7 gives principal points from this pareto surface.

Table 7. Compensation devices cost/ Voltage deviation/ Real power loss

Compensation devices cost($)	Real power loss (pu)	Voltage deviation (pu)	Reactive power at load buses (pu)				
			Q_{g1}	Q_{g2}	Q_{g3}	Q_{g4}	Q_{g5}
10508	2.056	0.0230	0.054	0.189	0.028	0.034	0.004
18923	1.967	0.2053	0.222	0.689	0.935	0.020	0.025
17796	1.976	0.1537	0.192	0.521	0.936	0.026	0.005

The values in the table 7 illustrate some points of the Pareto surface but the dispatcher can choose other compromise depending on the structure of a specified electrical network.

VI. CONCLUSIONS AND FUTUR WORK

In this paper, we have presented a novel multi-objective approach using PSO algorithm and based on matrix computation, called AMOPSO.

Then, this approach has been applied to resolve multi-objective reactive power dispatch problem. The results obtained by simulation show that the proposed approach is efficient for solving this problem, and generally, for solving multi-objective problems.

In future, we will develop this approach for causing it to resolve stochastic multi-objective problems and we will apply it to resolve multi-objective reactive power dispatch and multi-objective load flow problems of electric network integrating renewable energies.

REFERENCES

[1] Dian Palupi Rini, Siti Mariyam Shamsuddin and Siti Sophiyati Yuhaniz, “Particle Swarm Optimization: Technique, System and Challenges”, International Journal of Computer Applications (0975 – 8887), pp. 19-27, 2011.

[2] Margarita Reyes-Sierra and Carlos A. Coello Coello, “Multi-Objective Particle Swarm Optimizers: A Survey of the State-of-the-Art”, International Journal of Computational Intelligence Research. ISSN 0973-1873 Vol.2, No.3 pp. 287–308 , 2006.

[3] Praveen Kumar Tripathi, Sanghamitra Bandyopadhyay and Sankar Kumar Pal, “Multi-Objective Particle Swarm Optimization with time variant inertia and acceleration coefficients”, Information Sciences 177, pp.5033–5049, 2007.

[4] Wen-Fung Leong and Gary G. Yen, “Dynamic Population Size in PSO-based Multiobjective Optimization” Proceedings of IEEE Congress on Evolutionary, Computation Sheraton Vancouver Wall Centre Hotel, Vancouver, BC, Canada, pp. 6182-6189, 2006.

[5] Leticia Cagnina, Susana Esquivel and Carlos A. Coello Coello, “A Particle Swarm Optimizer for Multi-Objective Optimization”, JCS&T Vol. 5 No. 4, pp.204-210, 2005.

[6] Carlos A. Coello Coello, “Handling Multiple Objectives With Particle Swarm Optimization”, IEEE TRANSACTIONS ON EVOLUTIONARY COMPUTATION, VOL. 8, NO. 3, pp.256-279, 2004.

[7] Xiaodong Li, “Better Spread and Convergence: Particle Swarm Multiobjective Optimization Using the Maximin Fitness Function “,K. Deb et al. (Eds.): GECCO 2004, LNCS 3102, pp. 117–128, 2004.

[8] Lotfi Krichen, houssem Ben Aribia, Hsan HADJ ABDALLAH and Abderrazak Ouali,"ANN for multi-objective optimal reactive compensation of a power system with wind generators", Electic Power System Research 78, 2008.

[9] Houssem BEN ARIBIA and Hsan HADJ ABDALLAH,"Multi Objectives Reactive Dispatch Optimization of an Electrical Network", Leonardo Journal of Sciences 2007.

[10] M. A. Abido, "Multiobjective Optimal VAR Dispatch Using Strength Pareto Evolutionary Algrithm", IEEE Congress on Evolutionary Computation, 2006.

[11] Eckart Zitzler, "Evolutionary Algorithms for Multiobjective Optimization: Methods and Applications", Ph.D. Thesis Swiss Federal Institute of Technology Zurich, 1999.

[12] H. Bilil, R. Ellaia and M. Maaroufi "On convergence of Matrix Multi-objective Particle Swarm", working paper, Engineering Mohammadia School, 2011.