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Output Regulation of PV Power System via a TS Fuzzy Model-based Approach

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Abstract— **This paper presents the fuzzy control of switching Photovoltaic (PV) DC-DC power converters subject to parameter uncertainties and the state variables unavailable for measurements based on the Takagi- Sugeno (TS) fuzzy modeling approach. First, we derive the TS model for the switching converter. Second, a nonlinear robust fuzzy controller is designed accordingly. The TS fuzzy model is adopted for fuzzy modelling of the nonlinear system and establishing fuzzy state observers. The concept of General Design Approach (GDA) is employed to design fuzzy control and fuzzy observers from the TS fuzzy models. The proposed algorithm regulate the produced voltage and able to maintain the system stable during the parameter uncertainties. The design procedures are applied to a dynamics model of typical PV power control system using a DC-DC buck converter to illustrate the effectiveness of the proposed control techniques.**

I. INTRODUCTION

Recent years, the differential geometric control methods have led to a new class of control technique for nonlinear systems which are affine in the control inputs [1]-[3]. However, the effects of system stability are the mismatch between mathematical models and real systems, especially when the controlled system is uncertain nonlinear. Therefore, to overcome this problem, it is important to design a robust fuzzy controller for the efficient control system design and performance improvement. The parameter uncertainties may come with the change of environment (temperature change, pressure change, load change and etc.), which make the values of the parameters deviate from the nominal values and may worsen system performance or even cause instability of the system. May be considered the mentioned parameter uncertainties are measurable or unmeasurable depending on the sensors implementation. For example, the change of the temperature can be measured by thermal sensors. Some other parameter changes such as the resistance change of a power converter, ageing of components and measurement error are difficult to measure and are treated as unmeasurable parameter uncertainties.

Fuzzy controllers, which derive the signal of the control based on expert knowledge in terms of linguistic rules, can be applied to tackle this class of systems. Moreover, if sensors cannot be used due to some physical constraints or cost considerations, the parameter uncertainties will become unmeasurable.

Feedback linearization and Nonlinear Internal Model Control (NIMC) based on Takagi-Sugeno (TS) models are investigated in [4]. References [5]-[7], proposed a robust fuzzy feedback linearization control to stabilize uncertain systems and derived sufficient conditions to guarantee the robust stability of the closed-loop system with bounded parametric uncertainties. In [8], the tracking control problem has been considered for a class of nonlinear systems with the unknown system and gain functions, and the TS type fuzzy logic systems have been used to approximate unknown system function and a direct robust adaptive fuzzy tracking control (DRAFC) algorithm and direct adaptive fuzzy controller is also developed in [9]. Various stability conditions have been obtained through the employment of Lyapunov stability theory [10]-[12] and other methods [13]-[15].

It is well-known that switching DC-DC power converters are highly nonlinear system with uncertain parameters owing to the uncertain input voltage of Photovoltaic (PV) and output load during the operation. Consequently, the control (regulation) of DC-DC switching power converters can be a difficult task. In addition, the observer design is a very important problem in control systems. Since in many practical nonlinear control systems, state variables are often unavailable. As application examples of the fuzzy controllers, we consider DC to DC converters to illustrate the feasibility of the control design and demonstrate its high performance. Along with rapidly grown electronic technology [16]-[18], DC-DC switching converters are widely used in DC power supplies and DC motor drive applications. The output voltage of the DC-DC converters must be regulated to a desired level, in presence of output load and input voltage fluctuations.

In this paper, we consider the basic pulse-width-modulation (PWM) buck converter [16]-[18]. The main contribution of this paper is some sufficient conditions in the LMIs format and a systematic design procedure for the controller design for a general nonlinear system with parametric uncertainties and state variables unavailable for measurements for continuous-time TS fuzzy systems. Specifically, this paper proposes solutions to the robust stabilization problem for a class of nonlinear systems with norm-bounded parametric uncertainties. Based on the exact fuzzy modeling technique [19], the TS fuzzy systems are classified into three families based on the input matrices and an unique controller synthesis procedure is developed for each family.

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To illustrate the merits of the proposed controllers, they are applied to PV Power Control System Using a DC-DC Buck Converter subject to uncertain parameters and, state variables unavailable for measurements.

In Appendix we include the derivation of the proposed algorithms. Some sufficient conditions for stabilization of the continuous-time TS fuzzy models are derived, subject to system parametric uncertainties. The stability conditions for nominal nonlinear systems given in [20] are extended here to general nonlinear systems with parametric uncertainties and state variables unavailable for measurements, which are likewise formulated in the LMIs format.

The rest of this paper is structured as follows: Section II provides the TS fuzzy model and fuzzy observer. The modified algorithm of fuzzy controller is given in section III. Section IV shows the stability and robustness conditions for the proposed algorithm and the calculation of state feedback gains of the proposed fuzzy controller and the gains of the observer are presented. Section V shows the TS fuzzy plant model of the PWM buck converter which is equivalent to its state space averaged model. Section VI shows the simulation results of the PWM buck converter. A conclusion will be drawn in Section VII.

II. TS FUZZY MODEL AND FUZZY OBSERVER

A. TS Fuzzy Model

The continuous fuzzy dynamic model given in [19], is described by fuzzy IF-THEN rules, which represent local linear input-output relations of uncertain nonlinear systems. The i^{th} rule of this fuzzy model is given by.

Plant Rule i: IF $q_1(x(t))$ is N^i_1 AND … AND $q_\psi(x(t))$ is N^i_ψ

Then $\dot{x}(t)=(A_i+\Delta A_i)\,x(t)+(B_i+\Delta B_i)u(t)$

$$y(t)=C_i x(t) \qquad 1,2,\ldots,p \tag{1}$$

where N^i_Ω is a fuzzy set $\Omega=1,2,\ldots,\psi$, $i=1,2,\ldots,p$, $x(t)\in\kappa^{nx1}$ is the state vector, $u(t)\in\kappa^{mx1}$ is the input vector, $\Delta A_i\in\kappa^{nxn}$ and $\Delta B_i\in\kappa^{nxm}$ are the uncertainties of the constant system matrices $A_i\in\kappa^{nxn}$ and $B_i\in\kappa^{nxm}$ respectively, $C_i\in\kappa^{gxn}$ is the output matrix, p is the number of IF-THEN rules, $q_1(t),\ldots,q_\psi(t)$ are the premise variables. The plant dynamics is then described by:

$$\dot{x}(t)=\sum_{i=1}^{p} h_i(q(t))[(A_i+\Delta A_i)\,x(t)+(B_i+\Delta B_i)u(t)]$$

$$y(t)=\sum_{i=1}^{p} h_i(\,q(t))C_i x(t) \tag{2}$$

Where $w_i(q(t))=\prod_{\Omega=1}^{\psi} N^i_\Omega(q(t))$, $h_i(\mathrm{q(t)})=\dfrac{w_i(q(t))}{\sum_{i=1}^{p} w_i(q(t))}$

, $h_i(q(t))>0,\ \sum_{i=1}^{p}\mathrm{h_i}\ (\mathrm{q(t)})=1$

we classify the TS fuzzy systems into three families based on how diverse their input matrices.

First, we consider the family of the TS fuzzy systems with the common input matrix property:

$$B_1=\ldots=B_p=B \tag{3}$$

The plant dynamics is then described by,

$$\dot{x}(t)=\sum_{i=1}^{p} h_i(q(t))[(A_i+\Delta A_i)\,x(t)+(B+\Delta B)u(t)\,] \tag{4}$$

Second, we consider the family of the TS fuzzy systems with the input matrices are not all the same

$$B_1\neq\ldots\neq B_p\neq B \tag{5}$$

The overall plant dynamics is the same (2).

Third, we consider the family of the TS fuzzy systems with the input matrices on one-dimensional cone:

$$B_1/\alpha_1=\ B_2/\alpha_2=\ldots=\ B_p/\alpha_p=B \tag{6}$$

where $\alpha_1,\ldots,\alpha_p>0$ and $B\in\kappa^{\mathrm{nxm}}$, thus from (2) the plant dynamics is given by,

$$\dot{x}(t)=\sum_{i=1}^{p} h_i(q(t))(A_i+\Delta A_i\,)\,x(t)+\left\{\sum_{i=1}^{p}\alpha_i h_i(q(t))\right\}(B+\Delta B)u(t) \tag{7}$$

B. TS Fuzzy Observer

In order to estimate the system states, a fuzzy observer which shares the same antecedents as the fuzzy plant model is used as proposed in [21]. Its i-th rule is given by:

Rule i: IF IF $q_1(x(t))$ is N^i_1 AND … AND $q_\psi(x(t))$ is N^i_ψ

Then $\dot{\hat{x}}(t)=A_i\hat{x}(t)+B_i u(t)+K_i(y(t)\text{-}\hat{y}(t))$

$$\hat{y}(t)=C_i\hat{x}(t) \qquad 1,2,\ldots,p \tag{8}$$

Where $\hat{\mathrm{x}}(\mathrm{t})$ is the estimated state vector and $K_i\in\kappa^{nxg}$ is the fuzzy observer gain in each rule to be designed. The inferred observer states are governed by:

$$\dot{\hat{x}}(t)=\sum_{i=1}^{p} h_i(q(t))[A_i\hat{x}(t)+B_i u(t)+K_i C(x(t)\text{-}\hat{x}(t))]$$

$$\hat{y}(t)=\sum_{i=1}^{p} h_i(\,q(t))C_i\hat{x}(t) \tag{9}$$

For the first family, The inferred observer states are governed by:

$$\dot{\hat{x}}(t)=\sum_{i=1}^{p} h_i(q(t))[A_i\hat{x}(t)+Bu(t)+K_i C(x(t)\text{-}\hat{x}(t))] \tag{10}$$

For the second family, The overall observer states are the same (9).

For the last family, The inferred observer states are governed by:

$$\dot{\hat{x}}(t) = \sum_{i=1}^{p} h_i(q(t))A_i\,\hat{x}(t) + \left\{\sum_{i=1}^{p} \alpha_i h_i(q(t))\right\}Bu(t) + \sum_{i=1}^{p} h_i(q(t))K_i C(x(t)-\hat{x}(t)) \quad (11)$$

III. THE PROPOSED MODIFIED ALGORITHM

In this section, a unique controller synthesis procedure is developed for each of the TS family. It is more reliable and has larger freedom on finding the fuzzy controller to deal with a wide range of uncertainties as follow,

A. *Fuzzy Controller*

A fuzzy controller with *c* fuzzy rules is to be designed for the plant. The j^{th} rule of the fuzzy controller is given by [19],[20]:

Controller Rule *j:* IF $g_1(x(t))$ is M^j_1 AND … AND $g_\eta(x(t))$ is M^j_η

$$\text{Then } \; u(t) = -G_j x(t) + r(t) \quad (12)$$

where $M^j_{ʃ}$ is a fuzzy term of rule *j* corresponding to the function $g_{ʃ}\ (x(t))$, $ʃ = 1,2,\ldots,\ \eta$, $j=1,2,\ldots,c$, η is a positive integer; $G_j \in \kappa^{mxn}$ is the feedback gain of rule *j*, $r(t) \in \kappa^{mx1}$ is the reference input vector (set-point).

The inferred output of the fuzzy controller

$$u(t) = \sum_{j=1}^{c} \mu_j(g(t))[-G_j x(t) + r(t)] \quad (13)$$

From (9) the fuzzy controller in (13) becomes,

For the first family, the inferred output of the fuzzy controller is given by,

$$u(t) = \sum_{j=1}^{c} \mu_j(g(t))[-G_j \hat{x}(t) + r(t)] \quad (14)$$

$$D_j(g(t)) = \prod_{i=1}^{\eta} M_i^{\,j}(g(t)), \;\; \mu_j(x(t)) = \frac{D_j(g(t))}{\sum_{j=1}^{c} D_j(g(t))}$$

$$,\mu_j(g(t)) > 0, \sum_{j=1}^{c} \mu_j(g(t)) \;\; = 1$$

For the second family, the overall output of the fuzzy controller is given by,

$$\dot{u}(t) = \; Z_k \; u(t) + \sum_{j=1}^{c} \mu_j(g(t))[-G_j \hat{x}(t) + r(t)] \quad (15)$$

where Z_k is constant

For the last family, the fuzzy controller is given by,

$$u(t) = \frac{\sum_{j=1}^{c} \mu_j(g(t))[-G_j \hat{x}(t) + r(t)]}{\sum_{j=1}^{c} \alpha_j \mu_j(g(t))} \quad (16)$$

B. *General Design Approach (GDA)*

It is used when the membership functions are known and the rule antecedents of the TS fuzzy plant model and the fuzzy controller are different [20]. In order to carry out the analysis, the closed-loop fuzzy system should be obtained first.

$$\text{Let} \qquad e(t) = x(t) - \hat{x}(t) \quad (17)$$

From systems (4), (10), (14) and (17), $\mu_j(g(t))$ is written as μ_j, and $h_i(q(t))$ as h_i.

when the input matrices are all the same, we have the fuzzy control system

$$\dot{x}(t) = \sum_{i=1}^{p}\sum_{j=1}^{c} h_i \mu_j((A_i + \Delta B) - (B + \Delta B)G_j)x(t)$$

$$+ \sum_{i=1}^{p}\sum_{j=1}^{c} h_i \mu_j[(B + \Delta B)\mathrm{G}_j e(t) + (B + \Delta B)r(t)]$$

$$\dot{e}(t) = \sum_{i=1}^{p}\sum_{j=1}^{c} h_i \mu_j(A_i - K_i C_i + \Delta BG_j)e(t)$$

$$+ \sum_{i=1}^{p}\sum_{j=1}^{c} h_i \mu_j[(\Delta A_i - \Delta BG_j)x(t) + \Delta Br(t)] \quad (18)$$

Therefore, the augmented system can be expressed as the following form:

$$\dot{X}(t) = \sum_{i=1}^{p}\sum_{i=1}^{c} h_i \mu_j[(\mathrm{E}_{\mathrm{ij}} + \Delta E_{\mathrm{ij}})\mathrm{X(t)} + (S + \Delta S)r(t)] \quad (19)$$

where

$$X(t) = \begin{bmatrix} x(t) \\ e(t) \end{bmatrix}, \mathrm{E}_{\mathrm{ij}} = \begin{bmatrix} H_{ij} & BG_j \\ 0 & (A_i - K_i C_i) \end{bmatrix},$$

$$\Delta E_{\mathrm{ij}} = \begin{bmatrix} \Delta H_{ij} & \Delta BG_j \\ \Delta H_{ij} & \Delta BG_j \end{bmatrix}, S = \begin{bmatrix} B \\ 0 \end{bmatrix}, \Delta S = \begin{bmatrix} \Delta B \\ \Delta B \end{bmatrix}$$

when the input matrices are different**,** using (2), (9), (15) and (17), the fuzzy control system is given by,

$$\dot{X}(t) = \sum_{i=1}^{p}\sum_{j=1}^{c} h_i \mu_j[(E_{ij} + \Delta E_i)X(t) + Qr(t)\,] \quad (20)$$

where

$$E_{ij}=\begin{bmatrix} A_i & B_i & 0 \\ -G_j & Z_k & G_j \\ 0 & 0 & A_i-K_iC_i \end{bmatrix}, \mathit{\Delta} E_i=\begin{bmatrix} \mathit{\Delta} A_i & \Delta B_i & 0 \\ 0 & 0 & 0 \\ \mathit{\Delta} A_i & \Delta B_i & 0 \end{bmatrix},$$

$$Q=\begin{bmatrix} 0 \\ 1 \end{bmatrix}, \; X(t)=\begin{bmatrix} x(t) \\ u(t) \\ e(t) \end{bmatrix}$$

when the input matrices are on one dimensional cone, using (7), (11), (16) and (17), the fuzzy control system is given by (19)

$$\dot{X}(t)=\sum_{i=1}^{p}\sum_{j=1}^{c} h_i\mu_j[(E_{ij}+\mathit{\Delta} E_{ij})x(t)+(S+\mathit{\Delta} S)r(t)\ ] \tag{21}$$

where E_{ij}, $\mathit{\Delta} E_{ij}$, S and $\mathit{\Delta} S$ are given as in (19).

IV. STABILITY AND ROBUSTNESS CONDITIONS FOR THE PROPOSED ALGORITHM

A. Derivation of the Stability and Robustness Conditions

In this section, we develop stability conditions to the robust analysis of closed-loop nonlinear systems, where the analysis conditions are cast in terms of LMIs. The analysis procedures for uncertain fuzzy control systems under GDA are given in appendix. The stability and robustness analysis under GDA is summarized in the following lemma and theorem.

Lemma: the fuzzy control system as given by (19) with parameter uncertainty is stable if

$$\Gamma[\mathrm{TH_{ij}T^{-1}}]\leq-\left\|T\Delta \mathrm{H_{ij}}T^{-1}\right\|\mathrm{max}-\delta \tag{22}$$

Also from (20), the system is stable if

$$\Gamma[\mathrm{TH_{ij}T^{-1}}]\leq-\left\|T\Delta \mathrm{H_{i}}T^{-1}\right\|\mathrm{max}-\sigma \tag{23}$$

Also from (21), the system is stable if

$$\Gamma[\mathrm{TH_{ii}T^{-1}}]\leq-\left\|T\Delta \mathrm{H_{ii}}T^{-1}\right\|\mathrm{max}-\varphi \tag{24}$$

The analysis given in the appendix indicates that $\left\|x(t)\right\|$ will go to its steady state faster if we use larger values of φ,δ,σ. Thus the system performance with a larger φ,δ,σ is better than with smaller φ,δ,σ but from lemma, the robust area is smaller for a larger φ,δ,σ than a smaller φ,δ,σ.

Theorem. The TS fuzzy system (19) is asymptotically stabilizable if there exist symmetric and positive definite matrix *P*, some matrix G_j, (*j=1,2,…,c*), such that the following LMIs are satisfied

$$Q_{11}A_i^T+A_iQ_{11}-(B_iM_j)^T-(B_iM_j)<0 \tag{25}$$

$$A_i^TP_2+P_2A_i-(Y_iC_i)^T-(YC_i)<0 \tag{26}$$

B. Calculation of State Feedback Gains and Observer Gains

In this sub-section, the calculation of K_i of the observer gain and G_j of the fuzzy controller that satisfies the stability and robustness conditions is formulated as an LMIs problem. Let $T^T=T$, $P=T^T$ [20]. The equilibrium of a fuzzy control system (19) is asymptotically stable at large using the control law (14) if there exists a common positive definite matrix *P* such that

$$PE_{ij}+E_{ij}{}^TP<0 \qquad \forall i,j \tag{27}$$

Equation (27) for a common $P=P^T$ forms a set of Bilinear Matrix Inequalities (BMIs). The BMIs in (27) should be transformed into pure LMIs as follows: For the convenience of design, Assuming $P=diag(P_1,P_2)$. By multiplying (27) from left and right by $Q_{11}=P_1^{-1}$ and applying the change of variables $M_j=G_jQ_{11}$ and $Y_i=P_2K_i$, the LMIs (25) and (26) is obtained.

V. PV PWM BUCK CONVERTER MODEL AND TS FUZZY MODEL

To show the effectiveness of the proposed controller design techniques, PV power control system with parametric uncertainties are simulated. We will design fuzzy controllers for the model which depend on the input matrix.

A. PV Power Control System Using a DC-DC Buck Switching Converters Modelling

In this example, a PV power control system using a DC-DC buck switching converters will be used as a plant [18], [22]-[24] as shown in Fig. 1. The switching DC-DC PWM buck converter is a highly nonlinear system with uncertain parameters owing to the uncertain input voltage and output load during the operation. Consequently, the control (regulation) of the switching DC-DC PWM buck converter can be a difficult task, especially when the range of operating condition is large. The switching DC-DC PWM buck converter is a step-down power converter. Referring to Fig. 1, the input voltage v_{ph} is regulated at a certain output voltage. Regulation of the output voltage is achieved by controlling the duty ratio of the PWM signal, practically set to range (0.1 to 0.9), of the MOSFET. The system dynamics is governed by the following nonlinear state-space averaged model [18], [22]-[24].

$$\begin{aligned} \dot{x}(t)&=Ax(t)+B(x)u(t)+B_r \\ y(t)&=Cx(t) \end{aligned} \tag{28}$$

where

$$x(t)=\begin{bmatrix} i_L \\ v_C \end{bmatrix}=\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix},$$

$$A=\begin{bmatrix} -\frac{1}{L}[R_L+\frac{RR_{Co}}{R+R_{Co}}] & -\frac{R}{L(R+R_{Co})} \\ \frac{R}{C_o(R+R_{Co})} & -\frac{1}{C_o(R+R_{Co})} \end{bmatrix},$$

$$B(x)=\begin{bmatrix}\frac{1}{L}(-V_D-v_{pv}+R_M i_L)\\ 0\end{bmatrix},\ B_r=\begin{bmatrix}-\frac{V_D}{L}\\ 0\end{bmatrix},$$

$$C=\begin{bmatrix}\frac{RR_{Co}}{(R+R_{Co})} & \frac{R}{(R+R_{Co})}\end{bmatrix}$$

where, v_o=v_C and i_L are the capacitance voltage C_o and the inductance current L, respectively, u is the duty ratio of the Pulse-Width-Modulated (PWM) signal to control the switching MOSFET, R_M, R_{Co} and R_L are the internal resistances on the power MOSFET, the capacitance C_o and the inductance L, respectively, V_D is the forward voltage of the power diode, R is the load resistance, v_{pv}=V_{in} is the input of the converter and the output of the PV. Notice that system (1) is nonlinear since the input vector is state-dependent.

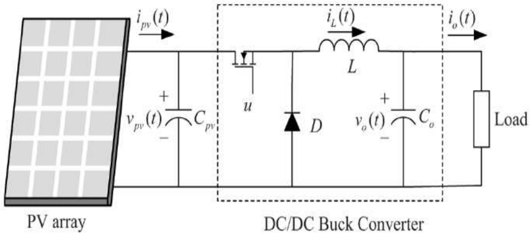

Fig. 1. PV power control system using a DC-DC buck converter.

B. TS Fuzzy Model of PV Power Control System Using a DC-DC Buck Converters

According to the modeling approach, the dynamical equation (28) can be exactly represented by TS fuzzy model with the following rules with the i_L as the premise variable, where $i_L(t)\in\{i_{L\min},i_{L\max}\}\in\{8,12\}$.

The membership functions are defined as follows :

$$F_1(i_L(t))=\frac{i_{L\max}-i_L(t)}{i_{L\max}-i_{L\min}},\ F_2(i_L(t))=1-F_1(i_L(t))) \tag{29}$$

The degree of membership function for $q_1(t)$ is depicted in Fig.2. Consequently, the TS-fuzzy plant model having 2 rules and can be written as follow (i=1,2),

Rule i: IF $i_L(t)$is F_i

Then $\dot{x}(t)=(A_i+\Delta A_i)\,x(t)+(B_i+\Delta B_i)\,u(t)+B_r$ (30)

The system dynamics is described by

$$\dot{x}(t)=\sum_{i=1}^{2}\mu_i[(A_i+\Delta A_i)\,x(t)+(B_i+\Delta B_i)u(t)+B_r] \tag{31}$$

where $x(t)\in\kappa^{2x1}$and $u(t)\in\kappa^{1x1}$ are the state vectors and the control input, respectively.

where

$$A_1=A_2=\begin{bmatrix}-\frac{1}{L}[R_L+\frac{RR_{Co}}{R+R_{Co}}] & -\frac{R}{L(R+R_{Co})}\\ \frac{R}{C_o(R+R_{Co})} & -\frac{1}{C_o(R+R_{Co})}\end{bmatrix},$$

$$B_1=\begin{bmatrix}\frac{1}{L}(-V_D-v_{pv}+R_M i_{L\min})\\ 0\end{bmatrix},$$

$$B_2=\begin{bmatrix}\frac{1}{L}(-V_D-v_{pv}+R_M i_{L\max})\\ 0\end{bmatrix}$$

VI. Simulations and Results

In this section, considers the problem of regulating the output voltage of a PV power control system using a DC-DC buck switching converters via fuzzy control. The response of $v_C(t)$ of the system, under V_{in}=30V, $V_{in(ref)}$=12V , and R is changing from 6Ω to16Ω and then back to 6 Ω, is shown in Fig. 2. The control signal of duty ratio is shown in Fig. 3. The overshoot due to the change of load is less than 200 mV.

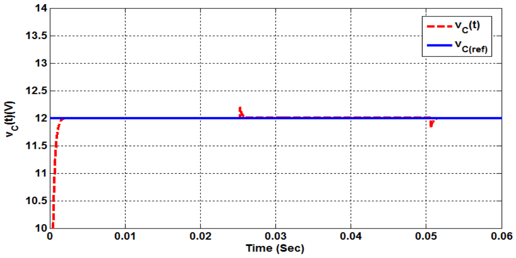

Fig. 2. Response of v_C (t) of the switching DC-DC PWM buck when R = 6 Ω →16 Ω →6Ω and V_{in}=30V

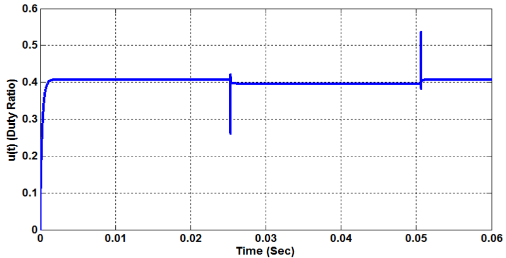

Fig. 3. Control signal response of $u(t)$ of the fuzzy controller applying to the switching DC-DC PWM buck converter when R = 6 Ω →16 Ω →6Ω and V_{in}=30V

The response of $v_C(t)$ of the system when R = 6Ω and, V_{in} is changing from 30V to 19V and then back to 30V, is shown in Fig. 4. The overshoot due to the change of line voltage is still less than 200 mV.

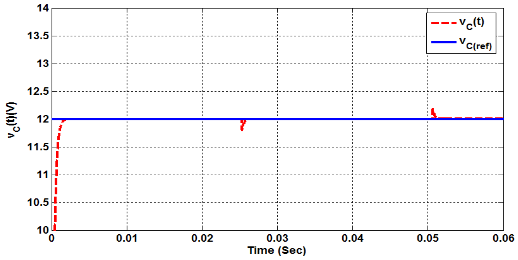

Fig. 4. Output voltage response of fuzzy controlled buck converter subject to an input voltage changing from 30V→19V→30V and R=6Ω.

Fig. 5 shows the simulated $v_C(t)$ response of the fuzzy controlled buck converter subject to significant changes of load resistance and input voltage, respectively. Fig. 7 shows the responses of the fuzzy controller. The results simulated by MATLAB show the short settling time, small overshoot, and nearly zero steady state error due to the variation of line voltage as shown in Fig. 6. The overshoot is less than 100 mV, i.e., 0.83% to the output voltage.

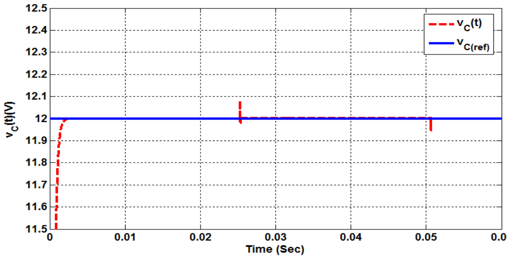

Fig. 5. Output voltage response of the basic converter subject to an input voltage changing from 30V→19V→30V V and a load changing from 6 Ω →16 Ω →6Ω.

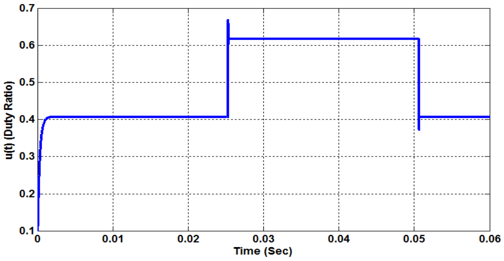

Fig. 6. Control signal response of $u(t)$ of the fuzzy controller applying to the switching DC-DC PWM buck converter when input voltage changing from 30V→19V→30V V and a load changing from 6 Ω →16 Ω →6Ω.

It can been seen that the system responses provided by the proposed fuzzy controller are better than those by the PI controller. The stability and robustness of an uncertain multivariable fuzzy control system that is designed based on a small parameter uncertainty approach have been investigated. The analyses are simple and systematic. Stability and robustness conditions under three design approaches of the fuzzy controller have been derived. The resulting fuzzy controller is capable of tackling multivariable nonlinear systems subject to large parameter uncertainties. Application examples on stabilizing or regulating uncertain nonlinear a switching DC-DC-PWM buck converter, has been given to illustrate the stabilizability and robustness property of the proposed fuzzy controller.

VII. CONCLUSION

In this paper, the TS fuzzy modeling for nonlinear system is firstly applied to PV power control system using a DC-DC buck switching converters. Based on TS fuzzy plant model, a nonlinear fuzzy controller has been designed with guaranteed stability and good dynamic behavior for a PWM buck converter. When the regulated converter is subject to significant changes of input voltage from PV and the load resistance, the fuzzy controller can offer much better transient responses than a conventional PI controller. The designed controller can indeed tolerate admissible (norm bounded and structured) parametric uncertainties, namely, it can globally asymptotically stabilize the closed-loop TS fuzzy system subject to all admissible parametric uncertainties. Some convenient sufficient conditions for robust stabilization of the TS fuzzy model, containing parametric uncertainties and state variables unavailable for measurements, were derived for continuous-time case. The conditions are formulated in the LMIs format. Simulation results have verified and confirmed the effectiveness of the new approach in controlling nonlinear systems with parametric uncertainties. The approach presented in this paper has virtually generalized and extended some existing fuzzy control methods based on the TS model and the LMIs criterion. It is believed that this approach is useful for the control of ill-modelled, uncertain, nonlinear, and complex system.

APPENDIX

when the input matrices are not all the same and on a one-dimensional cone

Consider the Taylor series [20].

$$X(t+\Delta t)=X(t)+\dot{X}(t)\Delta t+o(\Delta t) \qquad \text{(A.1)}$$

Where $o(\Delta t)=X(t+\Delta t)-X(t)-\dot{X}(t)\Delta t$ is the residual and $\Delta t>0$

$$\lim_{\Delta t\to 0^+} (o(\Delta t)/\Delta t)=0 \qquad \text{(A.2)}$$

From (21) and (A.1), and multiplying a transformation matrix $T\square R^{nxn}$ of rank n to both sides, and taking the norm on both sides, we have

$$\lim_{\Delta t\to 0^+}(\|T(X(t+\Delta t))\|-\|TX(t)\|)/\Delta t \le$$

$$\lim_{\Delta t\to 0^+}\{\sum_{i=1}^{p}\sum_{j=1}^{c}h_i\mu_j\,(\|I+TH_{ij}T^{-1}\Delta t\|-1)\|TX(t)\|\}/\Delta t$$

$$+im_{\Delta t\to 0^+}\{\left\|\sum_{i=1}^{p}\sum_{j=1}^{c}h_i\mu_j\,[T\Delta H_iX(t)+TSr(t)]\Delta t\right\|+\|To(\Delta t)\|\}/\Delta t \qquad \text{(A.3)}$$

Where $\|\cdot\|$ denotes the L_2 norm for vectors and L_2 induced, from (A.2) and (A.3), we obtain

$$d\|TX(t)\|/dt \le \sum_{i=1}^{p}\sum_{j=1}^{c}h_i\mu_j\,\Gamma[TH_{ij}T^{-1}]\|TX(t)\|$$

$$+\left\|\sum_{i=1}^{p}\sum_{j=1}^{c}h_i\mu_j\,[T\Delta H_iX(t)+TSr(t)]\right\| \qquad \text{(A.4)}$$

Where $\Gamma[TH_{ij}T^{-1}]=\lim_{\Delta t\to 0^+}\{(\|I+TH_{ij}T^{-1}\Delta t\|-1)/\Delta t$

$$=\lambda_{\max}(\{TH_{ij}T^{-1}+(TH_{ij}T^{-1})^*\}/2) \qquad \text{(A.5)}$$

Where $\lambda_{max}(.)$ is the largest eigenvalue, and * is the conjugate transpose. From (A.4)

$$d\|TX(t)\|/dt \le \sum_{i=1}^{p}\sum_{j=1}^{c} h_i \mu_j \,(\Gamma[TH_{ij}T^{-1}] + \|T\Delta H_i T^{-1}\|)\|TX(t)\| + \|[TSr(t)]\| \qquad (A.6)$$

if $\Gamma[TH_{ij}T^{-1}]$ satisfies the following inequality:

$$\Gamma[TH_{ij}T^{-1}] \le -\|T\Delta H_i T^{-1}\|_{\max} - \delta \qquad \forall i, j \qquad (A.7)$$

Where $\|T\Delta H_i T^{-1}\|_{\max}$ is the maximum value of $\|T\Delta H_i T^{-1}\|$, δ is a nonzero positive constant. From (A.6) and (A.7), and multiply the both sides by $\exp(\delta(t-t_o))$, we obtain

$$\left(\frac{d\|TX(t)\|}{dt} + \delta\|TX(t)\|\right)\exp(\delta(t-t_o)) \le \|[TSr(t)]\|\exp(\delta(t-t_o))$$

$$\frac{d}{dt}(\|TX(t)\|\exp(\delta(t-t_o)) \le \|[TSr(t)]\|\exp(\delta(t-t_0)) \qquad (A.8)$$

Where $t_0<t$ is an arbitrary initial time. Based on (A.8) there are two cases to investigate the system behavior $r=0$ and $r \ne 0$. If the condition (A.7) is satisfied the closed loop system (21) is stable, and $\|X(t)\| \to 0$ as $t \to \infty$.

1. $r(t)=0$

$$\|TX(t)\| \le \|TX(t_o)\|\exp(-\delta(t-t_o)) \qquad (A.9)$$

Since δ is positive value, $\|X(t)\| \to 0$ as $t \to \infty$

2. $r \ne 0$, from (A.8)

$$\|TX(t)\| \le \|TX(t_o)\|\exp(-\delta(t-t_o)) + \{\|[T\widehat{S}r(t)]\|/\delta\}(1-\exp(-\delta(t-t_o))) \qquad (A.10)$$

Where $\|[T\widehat{S}r(t)]\| \le \max_i \|[TSr(t)]\|_{\max} \le \|[TSr(t)]\|$

From (A.10) is bounded if r is bounded, then the system (21) is also bounded. Then the system is stable.

The analysis procedures for uncertain fuzzy control systems for second family and third family are similar to those under first family

when the input matrices are not all the same, σ is governed by let

$$\Gamma[TH_{ij}T^{-1}] \le -\|T\Delta H_i T^{-1}\|_{\max} - \sigma \qquad \forall i \qquad (A.14)$$

when the input matrices are all the same, φ is governed by let

$$\Gamma[TH_{ii}T^{-1}] \le -\|T\Delta H_i T^{-1}\|_{\max} - \varphi \qquad \forall i \qquad (A.15)$$

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