Higher Order Fuzzy Sliding Mode Control

A. Msaddek, A. Gaaloul, and F. M'sahli,

***Abstract*— Higher order sliding mode controller (HOSMC) is a robust control scheme used to overcome the chattering phenomenon which appears, generally, with standard sliding mode controller (SMC). Such controller is presented, usually, under tow parts: A continuous part ensuring the reaching phase and a discontinuous part which allows establishing the sliding mode. This paper presents a comparative study between three well known techniques of HOSMC and presents a higher order fuzzy sliding mode control. The proposed controller keeps the continuous part and replaces the gain of the discontinuous part by a fuzzy controller. Numerical simulations are developed to show the effectiveness of the proposed approach.**

***Index Terms*— Nonlinear systems, Sliding mode control, Higher-order, Robustness, Fuzzy logic.**

I. Introduction

The sliding mode control (SMC) has amply demonstrated its effectiveness through theoretical and practical studies. Its main areas of application include robotics and electrical machinery [4], [6], [15]. Such control technique is well known with its robustness against external matched disturbances, parametric variations, modeling uncertainties and nonlinearities (hysteresis, friction, etc.) that often characterize dynamical systems. Indeed, SMC is able to overcome these barriers in regulation and tracking. The robustness property is achieved by using a high frequency switching to steer the states of a system into the sliding surface [16].

The high-frequency switching leads, generally, to the appearing of an undesirable chattering of the control input. As a result, a large energy is lost in electric actuators and a rapid wear of mechanical actuators [4]. To overcome this problem, several solutions have been proposed in the literature. Among them, the "$sign$ "function is replaced by the saturation function or the sigmoid function in order to obtain a smoother control signal [4]. Besides, fuzzy logic may be used together with SMC [14].

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The most interesting way to get rid of the chattering phenomenon consists of enforcing a higher-order sliding mode (HOSM). The main objective of SMC of order ρ (ρ -SMC) is to obtain a finite time convergence onto the manifold $S^{\rho} = \left\{ s = \dot{s} = \ldots = s^{(\rho-1)} = 0 \right\}$, s is the sliding variable. So, the control acts on s and its higher derivatives to force the sliding variable and its ρ -1 first time derivatives to zero in finite time [8].

In [8], [9], [10] and [11], authors proposed HOSMC that does not depend on the dynamics of the system and which guarantees the robustness of the closed loop system. Laghrouche et al. [7] developed a controller based on the minimization of a quadratic criterion using the concept of sliding mode control with integral action. This allows stabilizing in finite time a system of high order on the sliding surface. Besides, it permits to choose in advance the convergence time to the sliding surface. Although these algorithms are general, a priori accurate knowledge of the initial conditions of the system limit seriously the applicability of these approaches.

Defoort et al. [5] proposed a finite time HOSMC for a class of multivariable nonlinear systems. In such work, authors remove the drawbacks devoted in [7] and presented a simple method for adjusting the synthesis parameters of the control law to achieve the desired performances. Recently, a second order sliding mode controller based on a nonlinear sliding surface is proposed to control uncertain linear systems with matched uncertainty [13]. The stability of the nonlinear sliding manifold is guaranteed and the chattering of the control input is reduced. The most advantage of this control strategy consists to obtain smooth states trajectories of the system around the sliding surface. This allows solving partially the problem of chattering phenomenon. However, this solution can't remove totally the discontinuous oscillations of the control signal.

In this paper, we compare three most important works dealing with higher order sliding mode controller proposed by Levant [10], Laghrouche et al. [7] and Defoort et al. [5]. Such algorithms are applied to solve a tracking problem of the trajectory of a car. Furthermore, we evaluate the robustness of such controllers against parameters uncertainties and external disturbances. Simulations results developed in this work show the effectiveness of the integral HOSMC proposed in [5]. To improve this latter strategy of control, we proposed a higher order fuzzy sliding mode control (HOFSMC). Such controller is represented by two parts: The first is done by the continuous part of the controller proposed by Defoort et al [5] used to stabilize the system to the sliding surface. The second discontinuous part of our controller is designed to reject the effect of the matched disturbances. The gain of the controller is variable and controlled by the fuzzy logic. This allows overcoming the chattering phenomenon. The proposed approach is applied to a model of a car in order to ensure a robust tracking of a prescribed reference trajectory. Simulation results show the effectiveness of the proposed controller.

This paper is organized as follows. The next section is devoted to the presentation of the higher order sliding mode controllers. In section 3, we described a higher order fuzzy sliding mode controller where the main contribution is introduced. In section 4, we presented the simulations results. Conclusions are reported in the last section of the paper

II. HIGHER ORDER SLIDING MODE CONTROLLERS

Consider a nonlinear dynamical system described by

$$\begin{aligned} \dot{x} &= f(x,t) + g(x,t)u \\ s &= s(x,t) \end{aligned} \qquad (1)$$

where $x = [x_1, \ldots, x_n]^T \in X$ is the state variable of the system with X an open set of $\Re^n$ and $u \in U$ the control input is a feature possibly discontinuous and bounded, depending on time and the system state, with U is an open set $\Re$, $f(x,t)$ and $g(x,t)$ are sufficiently differentiable vector fields.

Assumption 1. System (1) admits a $\rho \in N$ constant and known relative degree with respect to the sliding variable $s(x,t)$.

A. Quasi-Continuous Sliding Mode Conroller

To eliminate the effect of chattering appeared, generally, with the use of standard sliding mode control, Levant [11], proposed a robust homogenous higher order sliding mode controller. This control technique allows to stabilize the system after a finite transient time on the sliding surface defined by $S^\rho = \{s = \dot{s} = \ldots = s^{(\rho-1)} = 0\}$. Such quasi-continuous higher order sliding mode controller is described as follows for $\rho \leq 4$

$$u = -\alpha\, sign(s) \qquad \text{for } \rho = 1 \quad (2)$$

$$u = -\alpha \left(\dot{s} + |s|^{1/2} sign(s)\right) / \left(|\dot{s}| + |s|^{1/2}\right) \quad \text{for } \rho = 2 \quad (3)$$

$$u = -\alpha \frac{\left[\ddot{s} + 2\left(|\dot{s}| + |s|^{2/3}\right)^{-1/2}\left(\dot{s} + |s|^{2/3} sign(s)\right)\right]}{\left[|\ddot{s}| + 2\left(|\dot{s}| + |s|^{2/3}\right)^{1/2}\right]} \qquad \text{for } \rho = 3 \quad (4)$$

$$\varphi_{3,4} = \dddot{s} + 3\left[\ddot{s} + \left(|\dot{s}| + 0.5|s|^{3/4}\right)^{-1/3}\left(\dot{s} + 0.5|s|^{3/4} sign(s)\right)\right] \times \left[|\ddot{s}| + \left(|\dot{s}| + 0.5|s|^{3/4}\right)^{2/3}\right]^{-1/2}$$

$$N_{3,4} = |\dddot{s}| + 3\left[|\ddot{s}| + \left(|\dot{s}| + 0.5|s|^{3/4}\right)^{2/3}\right]^{1/2}$$

$$u = -\alpha\, \varphi_{3,4} / N_{3,4} \qquad \text{for } \rho = 4 \quad (5)$$

where $\alpha > 0$

B. Intergral Sliding Mode Control

Laghrouche et al. [7] proposed a sliding mode control with integral action. This control strategy allows choosing in advance the convergence time to the sliding surface.

The system (1) can be written as follows:

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = z_3 \\ \vdots \\ \dot{z}_{\rho-1} = z_\rho \\ \dot{z}_\rho = \phi(x,t) + \varphi(x,t)u \end{cases} \qquad (6)$$

with

$$\begin{cases} \phi = \bar{\phi} + \delta_\phi \\ \varphi = \bar{\varphi} + \delta_\varphi \end{cases}$$

where $z = \begin{bmatrix} z_1 & z_2 & \cdots & z_\rho \end{bmatrix}^T = \begin{bmatrix} s & \dot{s} & \cdots & s^{(\rho-1)} \end{bmatrix}^T$, $\bar{\phi}$ and $\bar{\varphi}$ are nominal known parts, δ_φ and δ_ϕ are unknown parts, including disturbances and uncertainties.

Assumption 2. The nominal part $\bar{\varphi}$ is assumed invertible.

The control is given by [7]

$$u = \bar{\varphi}^{-1}\left((w_0 + w_1) - \bar{\phi}\right) \qquad (7)$$

with

$$w_0 = \begin{cases} -B^T M z(t) + B^T \delta(t) & for \;\; 0 \leq t \leq t_F \\ -B^T M z(t) & for \;\; t > t_F \end{cases}$$

and

$$w_1 = -\alpha\, sign(\sigma) \qquad (8)$$

where $\delta(t)$ and M are the solution of the following equations

$$\begin{aligned} \dot{\delta} &= -\left(A^T - MBB^T\right)\delta \\ 0 &= MA + A^T M - MBB^T M + Q \end{aligned} \qquad (9)$$

where the matrix Q is symmetric positive definite and the matrices A and B are given, respectively, by

$$A = \begin{bmatrix} 0 & 1 & \cdots & 0 & \cdots \\ \vdots & \ddots & \ddots & \ddots & \ddots \\ 0 & \ddots & \ddots & \cdots & 1 \\ 0 & \ddots & \ddots & \ddots & 0 \end{bmatrix}_{\rho\times\rho}, \quad B = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}_{\rho\times 1}$$

The sliding variable is given by

$$\sigma = z_\rho + \xi$$

with

$$\dot{\xi} = -w_0, \quad \xi(0) = -z_\rho(0)$$

The gain of the discontinuous part satisfies the following property

$$\alpha > \frac{C_0 + (K_M + 1)w_{0M} + \eta}{K_m} \tag{10}$$

with $|\phi| \le C_0$, $K_m \le \varphi \le K_M$, $\eta > 0$ and $|w_0| < w_{0M}$ where K_m and K_M are two positive constants.

C. *Higher Order Integral Sliding Mode Control*

Consider system (6) which can be rewritten as follows:

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = z_3 \\ \quad\vdots \\ \dot{z}_{\rho-1} = z_\rho \\ \dot{z}_\rho = v(x,t) + (1 + \varsigma(x,t))w \end{cases} \tag{11}$$

with

$$u = \bar{\varphi}^{-1}(w - \bar{\phi}) \quad \text{and} \quad \begin{cases} v = \delta_\phi - \delta_\varphi \bar{\varphi}^{-1}\bar{\phi} \\ \varsigma = \delta_\varphi \bar{\varphi}^{-1} \end{cases}$$

The functions $v(x,t)$ and $\varsigma(x,t)$ may include the uncertainties of the system.

Assumption 3. The functions $v(x,t)$ and $\varsigma(x,t)$ are bounded. In addition, there is a positive function $a(x)$ and b a positive constant $0 < b \le 1$, such that:

$$\begin{cases} |v(x,t)| \le a(x) \\ |\varsigma(x,t)| \le 1 - b \end{cases} \tag{12}$$

To stabilize the uncertain system (11) in finite time, the control law is given by [1]

$$\begin{cases} w(z) = w_{nom}(z) + w_{disc}(z, z_{aux}) \\ \dot{z}_{aux} = -w_{nom}(z) \end{cases} \tag{13}$$

with

$$w_{nom}(z) = -k_1 sign(z_1)|z_1|^{v_1} - \cdots - k_\rho sign(z_\rho)|z_\rho|^{v_\rho} \tag{14}$$

where $k_1 \ \cdots \ k_\rho$ are positive constants chosen such that the polynomial $p^\rho + k_\rho p^{\rho-1} + \cdots + k_2 p + k_1$ is Hurwitz, and $v_1 \ \cdots \ v_\rho$ satisfy

$$v_{i-1} = \frac{v_i v_{i+1}}{2v_{i+1} - v_i}, \quad i = 2 \ \cdots \ \rho \tag{15}$$

with $v_\rho = v$ and $v_{\rho+1} = 1$, $v \in (1-\varepsilon, 1)$, $\varepsilon \in (0, 1)$.

The discontinuous part is given by

$$w_{disc}(z, z_{aux}) = -G(z) sign(\sigma) \tag{16}$$

with

$$\sigma(z,t) = z_p(t) + z_{aux}(t) \tag{17}$$

The gain $G(z)$ is a positive function which satisfies the fallowing property

$$G(z) \ge \frac{(1-b)|w_{nom}(z)| + a(x) + \eta}{b}, \quad \eta > 0 \tag{18}$$

Theorem I [2], [3]. Let the uncertain nonlinear system (1) of relative degree ρ with respect to the sliding variable $s(x,t)$. The control law

$$u = \bar{\varphi}^{-1}\left(w_{nom}(z) + w_{disc}(z, z_{aux}) - \bar{\phi}\right) \tag{19}$$

allows to stabilize the system (1) in finite time at the sliding surface $S^\rho = \{s = \dot{s} = \cdots = s^{(\rho-1)} = 0\}$. Therefore, a sliding mode of order ρ is established with respect to the sliding variable $s(x,t)$, provided that Assumption (2) and (3) are verified.

Remark : To implement the controller (4), (5), (7) and (19), a finite time differentiator [12] is used to estimate the successive derivatives $(\dot{s}, \ddot{s}, \cdots, s^{(\rho-1)})$ of the sliding variable (s).

III. HIGH ORDER FUZZY SLIDING MODE CONTROL

In this section, we refer to the fuzzy logic to improve the sliding mode control developed by Defoort et al [4]. The basic idea is to cancel the discontinuous control if the system reaches the sliding surface. This allows eliminating the phenomenon of chattering in the absence of disturbances. In presence of the latter, the new control law must verify the condition of attractiveness. So that, the system can returns back to the sliding surface.

The control part $w(z)$ proposed by Defoort et al. [4] is done by

$$w(z) = w_{nom}(z) + w_{disc}(z, z_{aux})$$

with

$$w_{disc}(z, z_{aux}) = -k\left(\frac{2}{\pi}\arctan\left(\frac{z_\rho + z_{aux}}{0.001}\right)\right)$$

Our approach consists of substituting the gain k by a fuzzy gain k_{flou} as follows

$$w_{disc}(z, z_{aux}) = -k_{flou}(z) w_{dnor}(z, z_{aux})$$

with

$$w_{dnor}(z, z_{aux}) = \left(\frac{2}{\pi}\arctan\left(\frac{z_\rho + z_{aux}}{0.001}\right)\right)$$

The principle of synthesis of fuzzy controller of gain k_{flou} is described by the following general rules

- If the sliding variable (s) and its first derivatives converge to the sliding surface then the gain value k_{flou} is decreasing.
- If the sliding variable (s) and its first derivatives diverge from the sliding surface, the gain value k_{flou} increases.

To get an idea of the divergence or convergence to the sliding surface we used the absolute values of the sliding variable and its first derivatives $\left(|s|, |\dot{s}|, \dots, |s^{(\rho-1)}|\right)$.

The explanatory structure of the proposed higher order fuzzy sliding mode control is presented by the following diagram

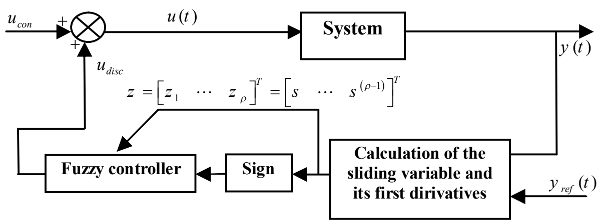

Fig 1. Block diagram of fuzzy sliding mode control

IV. SIMULATION EXAMPLE

To make the comparison between the different control techniques presented previously we use the model of a car to develop numerical simulations.

A. Mathematical Model of the Car

Consider a simple kinematic model of a car given by

$$\begin{cases} \dot{x} = v\cos\theta \\ \dot{y} = v\sin\theta \\ \dot{\theta} = v\dfrac{\tan\psi}{L} \\ \dot{\psi} = u \end{cases} \tag{20}$$

where x and y are Cartesian coordinates of the rearaxle middle point, θ is the orientation angle, v is the longitudinal velocity, L is the length between the two axles and ψ is the steering angle (figure 2).

The task is to steer the car to the trajectory $y = y_{ref}$, y is assumed to be measured in real time. Note that the actual control here is ψ and $\dot{\psi} = u$ is used as a new control in order to avoid discontinuities of ψ.

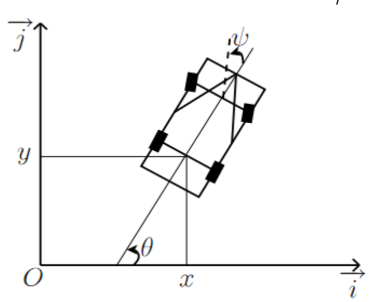

Fig. 2. Kinematic car model [7].

$L = 5m$, $x = y = \psi = \theta = 0$, at $t = 0$. Define the sliding variable $s = y - y_{ref}$. The relative degree of the system is 3. Indeed, one has

$$s^{(3)} = \left(\frac{v^2}{L}\left(1+\tan^2(\psi)\right)\cos(\theta)\right)u - \left(\frac{v^3}{L^2}\tan^2(\psi)\sin(\theta) + y_{ref}^{(3)}\right) \tag{21}$$

Consequently, a 3rd order SMC is designed.

First, we assume that the system is not noisy. So, we consider that $v = \text{const} = 10\ m/s$ and y is measured by a sensor perfectly non-noisy.

In the second part, we suppose that the system is uncertain. Indeed, we suppose that the longitudinal velocity is variable, i.e. $v(t) = 10 + 0.5\sin(t)\ m/s$ and the output y is measured by a sensor affected by disturbances of the form $0.5\sin(2t)$, t is the time variable.

B. Controllers Synthesis

Applied to the model of the car, the quasi-continuous sliding mode control [10] is given by

$$u = -\frac{\left[z_2 + 2\left(|z_1| + |z_0|^{2/3}\right)^{-1/2}\left(z_1 + |z_0|^{2/3} sign z_0\right)\right]}{\left[|z_2| + 2\left(|z_1| + |z_0|^{2/3}\right)^{1/2}\right]}$$

with

$$\dot{z}_0 = v_0 \quad v_0 = -14.7361|z_0 - s|^{2/3} sign(z_0 - s) + z_1$$
$$\dot{z}_1 = v_1 \quad v_1 = -30|z_1 - v_0|^{1/2} sign(z_1 - v_0) + z_2$$
$$\dot{z}_2 = -440 sign(z_2 - v_1)$$

where z_0, z_1 and z_2 are the estimation of s, $\dot{s}$ and $\ddot{s}$, respectively, obtained using the robust finite time differentiator [12].

Now, the algorithm proposed by Laghrouche et al [7] is given by (5). The components of the controller are

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad M = \begin{bmatrix} 2.4142 & 2.4142 & 1.0000 \\ 2.4142 & 2.4142 & 2.4142 \\ 1.0000 & 2.4142 & 2.4142 \end{bmatrix}$$

$$C_0 = 49.62, K_m = 6.38, K_M = 46.77, t_F = 5s$$

$$z(0) = [-2 \quad -2 \quad 0]^T,\ \delta_1(0) = 0.7615,$$
$$\delta_2(0) = 1.5907,\ \delta_3(0) = 0.9399$$

Finally, the integral sliding mode control proposed by Defoort et al. [4] is applied to the model of the car. It's given by

$$w(z) = w_{nom}(z) + w_{disc}(z, z_{aux})$$
$$\dot{z}_{aux} = -w_{nom}(z)$$
$$w_{nom}(z) = -sign(z_1)|z_1|^{\frac{1}{2}} - 1.5 sign(z_2)|z_2|^{\frac{3}{5}} - 1.5 sign(z_3)|z_3|^{\frac{3}{4}}$$

$$w_{disc}(z, z_{aux}) = -10\left(\frac{2}{\pi}\arctan\left(\frac{z_3 + z_{aux}}{0.001}\right)\right)$$

with z_0, z_1 and z_2 are the estimation of s, $\dot{s}$ and $\ddot{s}$.

If $y_{ref}^{(3)}$ is independent of u, the expressions of $\overline{\varphi}$ and $\overline{\phi}$ are given by

$$\overline{\varphi} = \left(\frac{v^2}{L}\left(1+\tan^2(\psi)\right)\cos(\theta)\right),$$

$$\overline{\phi} = \left(\frac{v^3}{L^2}\tan^2(\psi)\sin(\theta) + y_{ref}^{(3)}\right)$$

The control objective of en forcing the car trajectory to track a reference trajectory given by:

$$y_{ref} = 10\sin(0.05x) + 5 \quad (22)$$

So, one has

$$\overline{\phi} = \left[\frac{1}{800}\cos\left(\frac{x}{20}\right)(\cos(\theta))^2\right]v^3\cos(\theta)\tan(\psi) -$$
$$\left[\frac{1}{40L}\sin\left(\frac{x}{20}\right)\sin(\theta)\tan(\psi)\right]v^3\cos(\theta)\tan(\psi) +$$
$$\left[-\frac{1}{20}\sin\left(\frac{x}{20}\right)\cos(\theta)\sin(\theta)\right]v^3\frac{\tan(\psi)}{L} +$$
$$\left[\frac{\left(\frac{1}{2}\cos\left(\frac{x}{20}\right)\cos(\theta) - \sin(\theta)\right)\tan(\psi)}{L}\right]v^3\frac{\tan(\psi)}{L}$$
$$\overline{\varphi} = \frac{v^2}{L}\left[\frac{1}{2}\cos\left(\frac{x}{20}\right)\sin(\theta) + \cos(\theta)\right]\left[1+\tan^2(\psi)\right] \quad (23)$$

First, we consider the case where noises are absents. Simulation results plotted on figure 3 show that control objective is fulfilled using different algorithms. Morover, one can show that the control law proposed by Laghrouche converges exactly after 5s to the sliding surface which is the convergence time chosen a priori. Besides, we note that the controller proposed by Defoort et al. [4] has very low amplitude compared with other techniques of HOSMC.

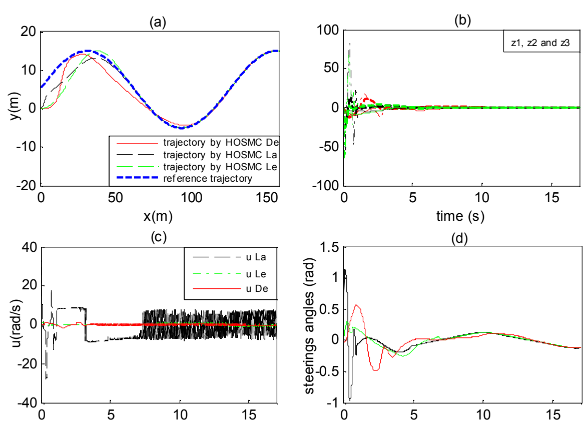

Fig. 3. Tracking performances without noises, (a) y and yref, (b) z_i $0 \le i \le 2$, (c) Controls signals, (d) Steering angle.

Now, the measurement is assumed noisy. So, the output y is affected by a noise of expression 0.5sin(2t), t is the time, and the longitudinal velocity is assumed variable in time according to the law v(t)=10+0.5sin(t).

Simulations results depicted on figure 4 show that the controller of Defoort et al. is more robust against parametric variation and external disturbances.

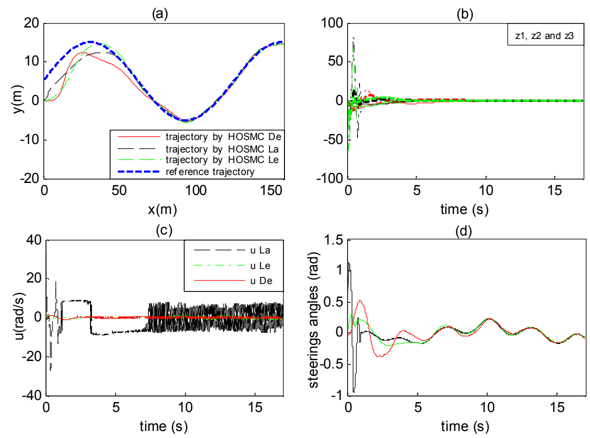

Fig. 4. Tracking performances under disturbances, (a) y and yref, (b) z_i $0 \le i \le 2$, (c) Controls signals, (d) Steering angle.

Previously, we have compared three techniques for high-order sliding mode control. The simulation results show that the HOSMC proposed by Defoort et al [4] is the most efficient algorithm. So we used the fuzzy logic to improve such technique of control.

Now, using our approach, the control law is reformulated as follows

$$w(z) = w_{nom}(z) + w_{disc}(z, z_{aux})$$
$$w_{nom}(z) = -sign(z_1)|z_1|^{\frac{1}{2}} - 1.5sign(z_2)|z_2|^{\frac{3}{5}} - 1.5sign(z_3)|z_3|^{\frac{3}{4}}$$
$$w_{disc}(z, z_{aux}) = -k_{flou}\left(\frac{2}{\pi}\arctan\left(\frac{z_3 + z_{aux}}{0.001}\right)\right),$$
$$\dot{z}_{aux} = -w_{nom}(z)$$

The gain of the discontinuous control is calculated using the sliding variable, its first and second derivatives. Their fuzzy membership functions are given by

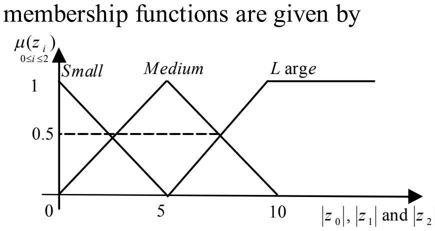

Fig. 5. The membership function of $|z_0|$, $|z_1|$, $|z_2|$.

Using the membership functions $\mu(z_i)_{0\le i\le 2}$ seen above (figure 5), those allows to give a fuzzy representation of each inputs variables. The membership degrees of each variables used to apply the rules fuzzy that were globally defined. So the degrees of appartenece output variable

$\mu(k_{flou})$ is obtained using the method of Mamdani.

The fuzzy membership function of output k_{flou} is given by

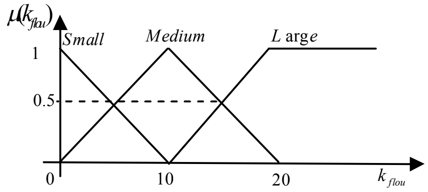

Fig. 6. The membership function of k_{flou}.

To transform the fuzzy representation of output to the gain of the discontinuous control we used the Barycenter Defuzzification approach.

The functions $\bar{\phi}$ and $\bar{\varphi}$ are given by (23)

First, we consider the case where noises are absents. simulation results plotted on figures 7 show the effectiveness of the proposed controller.

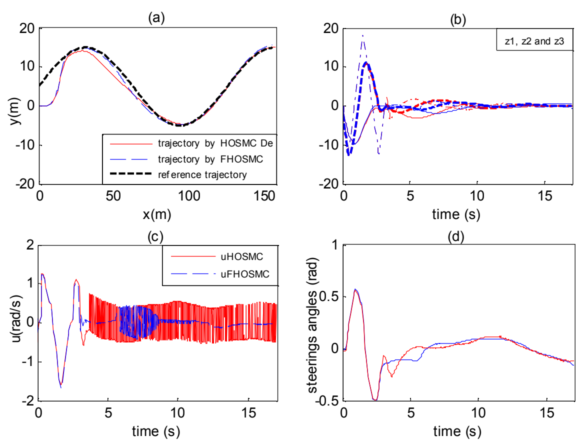

Fig. 7. Tracking performances without noises, (a) y and yref, (b) $z_i \; 0 \le i \le 2$, (c) Controls signals, (d) Steering angle.

Now, we assume that a disturbance d(t)=0.5sin(2t), t is the time, affects the system output .

Results depicted on figure 8 show the robustness of the proposed control strategy against external disturbances.

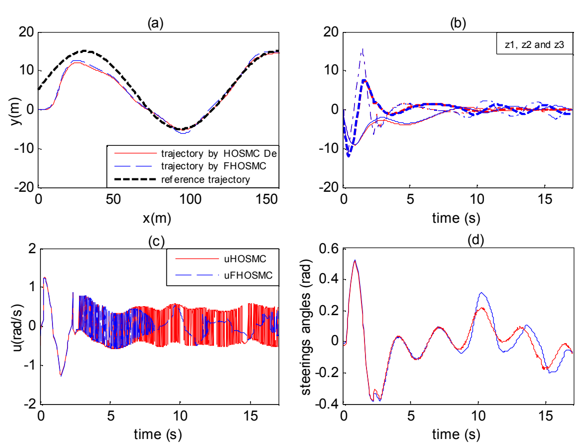

Fig. 8. Tracking performances under disturbances, (a) y and yref, (b) $z_i \; 0 \le i \le 2$, (c) Controls signals, (d) Steering angle.

V. CONCLUSION

In this work we have compared three techniques for high-order sliding mode control. Furthermore, we have evaluated the robustness of such controllers against parametric uncertainties and output noises. Applied to the model of a car, the simulation results show that the integral HOSMC is the most efficient. In addition, a fuzzy HOSMC has been proposed for a class of nonlinear systems. The proposed approach uses the fuzzy logic to compute the gain of the controller. It preserves the properties of the HOSMC and diminished the chattering phenomenon.

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