Fault Tolerant Controller design for T-S fuzzy system using descriptor approach with application to a three tank system

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***Abstract*— This paper studies sensor fault detection and the associated Fault-Tolerant-Control (FTC) algorithm for Takagi-Sugeno (TS) fuzzy system. Fault detection scheme based on a descriptor approach is considered. A descriptor fuzzy observer is adopted which estimate both the system state and the occurring sensor fault. The convergence of the proposed observer is performed by the search of suitable Lyapunov matrices. Stability conditions of the closed loop system via static output feedback control are also established and expressed in the form of Linear-Matrix-Inequalities (LMI). Furthermore, pole placement technique is used to locate the system pole in a desired region in order to guarantee a satisfactory behavior of the response of closed loop system.A simulated tree tank system is used to demonstrate the effectiveness of the proposed strategy.**

***Keywords*: TS Fuzzy model, observer, descriptor approach, LMI, sensor fault, fault-tolerant control, three tank system.**

I. INTRODUCTION

Over recent year, the control technique based on the TS fuzzy model have attracted extensive attention from researchers in the control community since they are able to approximate some nonlinear systems based on fuzzy logic paradigm [1][2][3]. Indeed, the idea of the fuzzy model approach is to apprehend the global behavior of a system by a set of local models. An interpolation of all these sub-models with non linear function satisfying the convex sum property allows to find out the global behavior of the system described in a large operating scale [4]. Therefore, TS fuzzy model has proven to be a powerful tool in the analysis and synthesis to a nonlinear control system.

Recently, Fault Tolerant Control (FTC) problem for TS systems has been widely treated [5][6] [7][8][9]. This is because of the increasing complexity of modern control systems and the need of more reliable control techniques since unexpected faults or failures may result in substantial damage, and can even be hazardous to plant personnel and the environment. Very interesting approach were done on fault tolerant control for TS fuzzy system subject to sensor and actuator faults such as Lyapunov stability, H_2 optimal performance and/or H_∞ performance [10], robustness and fuzzy control with time delay and uncertainties [11]. An online estimation approach was proposed in [12] to detect and reconstruct the actuator fault for TS fuzzy systems with interval time varying delay by reconstructing an adaptive observer. H_-/H_∞ proposed by [13] to design faults detection observer for TS fuzzy model subject to faults and unknown bounded disturbances. In [14], a TS fuzzy model is employed to approximate a non linear uncertain system, and in order to compensate some component failures, sufficient conditions are derived for robust stabilization of the closed loop system in the sense of Lyapunov asymptotic stability. Authors in [15] have also been interested in the control of TS fuzzy disturbed system with uncertainties subject to actuators faults using the L_2 optimization and Lyapunov theory to guarantee stability and a proportional integral observer to estimate both the faults and the faulty states. Especially, descriptor based techniques has received considerable attention in the design and the analysis of TS fuzzy fault tolerant control systems [16][17][8].

In this paper, motivated by the work in [18], we use a fuzzy descriptor approach to investigate sensor fault and state estimation for the considered TS fuzzy system. The contribution of this paper is to develop a simple algorithm to determinate the gains of the fuzzy observer using a suitable Lyapunov matrices without recourse to the hard calculation of block matrix inversion which gives a non-linear and complex terms like in [16][13]. A static output feedback control is also established and expressed in the LMI form. Furthermore, pole placement technique is used to locate the system pole in a desired region in order to guarantee a satisfactory behavior of the response of closed loop system. The observer and the control gains are obtained by solving a set of LMI. Finally, the validity of the proposed methodology is illustrated by a three tank system.

This paper is organized as follows: In section 2, the class of studied TS fuzzy models is given. In section 3, the design of the fuzzy observer using descriptor approach is investigated to estimate simultaneously state variable and sensor fault. Then, Fault-Tolerant-Control (FTC) algorithm and pole placement technique are addressed in section 4. Section 5 presents an application to three tank system to illustrate the effectiveness of the proposed method.

Notation : X^T and X^{-1} are the transpose and the inverse of matrix X, respectively. $Sym(X) = X + X^T$. I is the identity matrix with appropriate dimension. The symbol $\otimes$ denotes the Kronecker product of matrices.

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II. TS Fuzzy Model

The TS fuzzy model is described by fuzzy IF-THEN rules, whose collection represent the approximation of the nonlinear system. The i^{th} rule of the TS fuzzy model is expressed as follows:

Rule i:
IF $\xi_1(t)$ is M_{1i} and,…, and $\xi_g(t)$ is M_{si}, THEN

$$\begin{aligned}\dot{x}(t) &= A_i x(t) + B_i u(t) \\ y(t) &= Cx(t) + D_s f_s(t)\end{aligned} \tag{1}$$

Where $x(t) \in \Re^n$ is the state vector, $u(t) \in \Re^m$ is the input vector, $y(t) \in \Re^p$ is the output vector and $f_s(t) \in \Re^s$ is the sensor fault vectors. A_i, B_i, C and D_s are constant real matrices with appropriate dimension. It is supposed that the matrix D_s is full column rank. $\xi_j (j = 1, \ldots, g)$ are the premise variables assumed measurable. $M_{ij}(i = 1, \ldots, r; j = 1, \ldots, s)$ are the fuzzy sets and r is the number of rules.
The overall fuzzy model achieved by fuzzy-blending of each local model is given by:

$$\begin{aligned}\dot{x}(t) &= \sum_{i=1}^{r} \mu_i\big(\xi(t)\big)\Big(A_i x(t) + B_i u(t)\Big) \\ y(t) &= Cx(t) + D_s f_s(t)\end{aligned} \tag{2}$$

where

$$\begin{aligned}\mu_i(\xi(t)) &= \omega_i(\xi(t))/\sum_{i=1}^{r}\omega_i(\xi(t)) \\ \omega_i(\xi(t)) &= \prod_{j=1}^{r} M_{ij}(\xi_j(t))\end{aligned}$$

Hence, $\mu_i(\xi(t))$ satisfies

$$\sum_{i=1}^{r}\mu_i(\xi(t)) = 1 \quad and \quad \mu_i(\xi) > 0 \quad for \quad i = 1, \ldots, r$$

In next section, we develop an observer to estimate the system state $x(t)$ and the sensor fault signal $f_s(t)$, simultaneously, by introducing the fault as an auxiliary state.

III. SENSOR FAULT ESTIMATION

In order to estimate state vector $x(t)$ and sensor fault $f_s(t)$, an augmented system is constructed using the descriptor technique. Then the TS fuzzy system (2) can be written as follows:

$$\begin{aligned}\bar{E}\dot{\bar{x}}(t) &= \sum_{i=1}^{r}\mu_i\big(\xi(t)\big)\Big(\bar{A}_i\bar{x}(t) + \bar{B}_i u(t)) + Nx_s(t)\Big) \\ y(t) &= \bar{C}\bar{x}(t) = C_0\bar{x}(t) + x_s(t)\end{aligned} \tag{3}$$

where

$$x_s(t) = D_s f_s(t), \bar{x}(t) = \begin{bmatrix} x(t) \\ x_s(t)\end{bmatrix}$$

$$\begin{aligned}\bar{E} &= \begin{bmatrix} I_n & 0 \\ 0 & 0\end{bmatrix}, \bar{A}_i = \begin{bmatrix} A_i & 0 \\ 0 & -I_p\end{bmatrix}, \bar{B}_i = \begin{bmatrix} B_i \\ 0\end{bmatrix} \\ N &= \begin{bmatrix} 0 \\ I_p\end{bmatrix}, C_0 = \begin{bmatrix} C & 0\end{bmatrix} et\ \bar{C} = \begin{bmatrix} C & I_p\end{bmatrix}\end{aligned}$$

The vector $x_s(t)$ is considered as an auxiliary state of the augmented system (3). So if the state estimation of the augmented system (3) exists then the state estimation of the original system (2) and the fault estimation exist too.
In this section, a fuzzy observer is designed to estimate the system state and the sensor fault at the same time. Therefore, we consider the following observer structure:

$$\begin{aligned}E\dot{z}(t) &= \sum_{i=1}^{r}\mu_i\big(\xi(t)\big)\Big(F_i z(t) + \bar{B}_i u(t)\Big) \\ \hat{\bar{x}}(t) &= z(t) + Ly(t)\end{aligned} \tag{4}$$

where $z(t) \in \Re^{n+p}$ is an auxiliary state vector of the observer and $\hat{\bar{x}}(t) \in \Re^{n+p}$ is the state estimation of (3).
By substituting $z(t) = \hat{\bar{x}}(t) - L\bar{C}\bar{x}(t) = \hat{\bar{x}}(t) - LC_0\bar{x}(t) - Lx_s(t)$ into the differential algebraic equation of (4), we obtain:

$$\begin{aligned}E\dot{\hat{\bar{x}}}(t) - EL\bar{C}\dot{\bar{x}}(t) &= \sum_{i=1}^{r}\mu_i\big(\xi(t)\big)\Big(F_i\big(\hat{\bar{x}}(t) - LC_0\bar{x}(t) \\ &- Lx_s(t)\big) + \bar{B}_i u(t)\Big)\end{aligned} \tag{5}$$

Subtracting (5) from (3) yields:

$$\begin{aligned}\big(\bar{E} + EL\bar{C}\big)\dot{\bar{x}}(t) - E\dot{\hat{\bar{x}}}(t) = \sum_{i=1}^{r}\mu_i\big(\xi(t)\big)\Big(\\ \big(\bar{A}_i + F_i LC_0\big)\bar{x}(t) - F_i\hat{\bar{x}}(t) + \big(N + F_i L\big)x_s(t)\Big)\end{aligned} \tag{6}$$

Let the error estimation defined by $\bar{e}(t) = \bar{x}(t) - \hat{\bar{x}}(t)$, and suppose that:

$$\begin{aligned}E &= \bar{E} + EL\bar{C} \\ F_i &= \bar{A}_i + F_i LC_0 \\ N &= -F_i L\end{aligned} \tag{7}$$

the error dynamic can be written as follow:

$$E\dot{\bar{e}}(t) = \sum_{i=1}^{r}\mu_i\big(\xi(t)\big)F_i\bar{e}(t) \tag{8}$$

In order to guarantee the constraints (7), the observer parameters are chosen as below:

$$E = E_1 + KE_2, F_i = \begin{bmatrix} A_i & 0 \\ -C & -I_p\end{bmatrix} et\ L = \begin{bmatrix} 0 \\ I_p\end{bmatrix} \tag{9}$$

where:

$$E_1 = \begin{bmatrix} I_n & 0 \\ 0 & 0\end{bmatrix},\ K = \begin{bmatrix} RM \\ M\end{bmatrix} et\ E_2 = \begin{bmatrix} C & I\end{bmatrix} \tag{10}$$

M and R are free parameters to be determined. Choosing the matrix M non singular, the error dynamic can be rewritten as follow:

$$\dot{\bar{e}}(t) = \sum_{i=1}^{r}\mu_i\big(\xi(t)\big)E^{-1}F_i\bar{e}(t) \tag{11}$$

This implies that the error dynamic (11) converges asymptotically to zeros if there exist a positive definite symmetric matrix X such that:

$$sym(F_i^T XE) < 0 \tag{12}$$

To summarize, we have the following theorem:

Theorem 1: If there exist a positive definite symmetric matrix X and non singular matrix K such that:

$$sym(F_i^T XE_1 + F_i^T SE_2) < 0 \tag{13}$$

where

$$S = XK \tag{14}$$

then there exist a descriptor observer in the form (4) to asymptotically estimate the state and sensor faults for system (3).

The sensor faults estimation is

$$\widehat{f}_s(t) = (D_s^T D_s)^{-1} D_s^T x_s(t) \tag{15}$$

IV. FAULT TOLERANT CONTROL

The considered faults here may be unbounded, constants even time-varying. thus the stability of the closed loop system may even be damaged in the presence of the fault. In order to guarantee that the system operates normally as sensors fault occur, a fault tolerant control law is designed in this section. Consider the following fuzzy system:

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{r} \mu_i\big(\xi(t)\big)(A_i x(t) + B_i u(t) \\ y(t) &= Cx(t) + D_s f_s(t) \end{aligned} \tag{16}$$

The proposed static output feedback controller takes the form:

$$u(t) = \sum_{i=1}^{r} \mu_i\big(\xi(t)\big) K_i y(t) \tag{17}$$

where K_i is the output feedback controller gain. Define the compensated measurement $y_c(t)$ as follow:

$$y_c(t) = y(t) - \hat{x}_s(t) = Cx(t) - [0 \dots I_p]\bar{e}(t) \tag{18}$$

Using the compensated measurement $y_c(t)$ to replace the actual measurement $y(t)$ in (17), the resulting closed-loop system becomes:

$$\begin{aligned} \dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(\xi(t))\mu_j(\xi(t))((A_i \\ +B_i K_j C)x(t) - B_i K_j [0 \dots I_p]\bar{e}(t)) \end{aligned} \tag{19}$$

Our objective is to compute an output feedback matrix K_i which guarantees the asymptotic stability of the closed loop system.

Theorem2 [19]: The closed-loop TS system (19) is asymptotically stable if there exist symmetric definite positive matrix X, matrices N_i, S_{ij} and M satisfying the following conditions for $i, j = 1, 2, \dots, r$

$$Sym(A_i X) + Sym(B_i N_i C) + S_{ii} < 0 \tag{20}$$

$$\begin{aligned} &Sym\big((A_i + A_j)X\big) + Sym\big((B_i N_j + B_j N_i)C\big) \\ &+Sym(S_{ij}) \leq 0 \end{aligned} \tag{21}$$

$$\begin{bmatrix} S_{11} & \dots & S_{1r} \\ \vdots & \ddots & \vdots \\ S_{r1} & \dots & S_{rr} \end{bmatrix} > 0 \tag{22}$$

where $CX = MC$

The feedback controller gain K_i is defined by:

$$K_i = N_i M \tag{23}$$

In the synthesis of control system, meeting some desired performances should be considered in addition to stability. Indeed, a satisfactory behavior of the response of closed loop system can be guaranteed by locating the poles of each subsystem in a prescribed sub-region in the complex left half plane [20].

Definition [21]: A subset D of the complex plane is called an LMI region if there exist a symmetric matrix α and a matrix β such that:

$$D = \{z \in \mathcal{C} / f_D = \alpha + \beta z + \beta^T \overline{z} < 0\}$$

A dynamical system is called D stable if all its poles lie in D (that is, all eigenvalues of the matrix A lie in D).

lemma [22]: A is D stable if and only if there exists a symmetric matrix $X > 0$ such that

$$\alpha \otimes X + \beta \otimes (AX) + \beta^T \otimes (XA^T) < 0$$

where $\otimes$ denotes the Kronecker product of matrices.

Theorem3 [19]: The closed loop system (19) is D-stable (all the complex poles of each subsystem lying in LMI region D) If there exist symmetric positive definite matrices X, M and N_i matrices such that the following LMI condition hold:

$$\alpha \otimes X + \beta \otimes (A_i X + BN_i C) + \beta^T \otimes (A_i X + BN_i C)^T < 0$$

V. ILLUSTRATIVE EXAMPLE

The three tank system model is written using the well known mass balance equations.

The dynamics equation can be conveniently represented as in [23] by:

$$\begin{aligned} \dot{x}_1(t) &= u_1(t) - R_1\sqrt{x_1(t)} \\ \dot{x}_2(t) &= u_2(t) - R_2\sqrt{x_2(t)} \\ \dot{x}_3(t) &= R_1\sqrt{x_1(t)} + R_2\sqrt{x_2(t)} - R_3\sqrt{x_3(t)} \end{aligned} \tag{24}$$

where $R_1 = R_2 = 0.95$, $R_3 = 1.3$ and $x_1(t), x_2(t), x_3(t)$ are the liquid level in the three tanks.

The full TS representation of the adopted system (25) is then obtained as follow [23]:

$$\begin{aligned} \dot{x}(t) &= \sum_{r=1}^{8} \mu_r(\xi(t)) A_r x(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \tag{25}$$

When the state matrices are given by the following expressions:

$$A_1 = \begin{bmatrix} -0.3004 & 0 & 0 \\ 0 & -0.2372 & 0 \\ 0.3004 & 0.2372 & -0.3162 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -0.3004 & 0 & 0 \\ 0 & -0.2372 & 0 \\ 0.3004 & 0.2372 & -0.2236 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} -0.3004 & 0 & 0 \\ 0 & -0.1677 & 0 \\ 0.3004 & 0.1677 & -0.3162 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} -0.3004 & 0 & 0 \\ 0 & -0.1677 & 0 \\ 0.3004 & 0.1677 & -0.2236 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} -0.2124 & 0 & 0 \\ 0 & -0.2372 & 0 \\ 0.2124 & 0.2372 & -0.3162 \end{bmatrix}$$

$$A_6 = \begin{bmatrix} -0.2124 & 0 & 0 \\ 0 & -0.2372 & 0 \\ 0.2124 & 0.2372 & -0.2236 \end{bmatrix}$$

$$A_7 = \begin{bmatrix} -0.2124 & 0 & 0 \\ 0 & -0.1677 & 0 \\ 0.2124 & 0.1677 & -0.3162 \end{bmatrix}$$

$$A_8 = \begin{bmatrix} -0.2124 & 0 & 0 \\ 0 & -0.1677 & 0 \\ 0.2124 & 0.1677 & -0.2236 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

We suppose the sensor fault in the following form:

$$f_s(t) = \begin{bmatrix} f_{s1}(t) \\ f_{s2}(t) \\ f_{s3}(t) \end{bmatrix} D_s = \begin{bmatrix} 0.3 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.1 \end{bmatrix} \tag{26}$$

where:

$$f_{s1}(t) = \begin{cases} 0 & 0 \leq t < 50 \\ 15 & 50 \leq t < 65 \\ 0 & t \geq 65 \end{cases}$$

$$f_{s2}(t) = \begin{cases} 0 & 0 \leq t < 40 \\ 20 & 40 \leq t < 60 \\ 0 & t \geq 60 \end{cases}$$

$$f_{s3}(t) = \begin{cases} 0 & 0 \leq t < 55 \\ 10 & 55 \leq t < 70 \\ 0 & t \geq 70 \end{cases}$$

From (9), we get the observer gains:

$$F_1 = \begin{bmatrix} -0.3004 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.2372 & 0 & 0 & 0 & 0 \\ 0.3004 & 0.2372 & -0.3162 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 & -1 \end{bmatrix}$$

$$F_2 = \begin{bmatrix} -0.3004 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.2372 & 0 & 0 & 0 & 0 \\ 0.3004 & 0.2372 & -0.2236 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 & -1 \end{bmatrix}$$

$$F_3 = \begin{bmatrix} -0.3004 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.1677 & 0 & 0 & 0 & 0 \\ 0.3004 & 0.1677 & -0.3162 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 & -1 \end{bmatrix}$$

$$F_4 = \begin{bmatrix} -0.3004 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.1677 & 0 & 0 & 0 & 0 \\ 0.3004 & 0.1677 & -0.2236 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 & -1 \end{bmatrix}$$

$$F_5 = \begin{bmatrix} -0.2124 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.2372 & 0 & 0 & 0 & 0 \\ 0.2124 & 0.2372 & -0.3162 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 & -1 \end{bmatrix}$$

$$F_6 = \begin{bmatrix} -0.2124 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.2372 & 0 & 0 & 0 & 0 \\ 0.2124 & 0.2372 & -0.2236 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 & -1 \end{bmatrix}$$

$$F_7 = \begin{bmatrix} -0.2124 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.1677 & 0 & 0 & 0 & 0 \\ 0.2124 & 0.1677 & -0.3162 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 & -1 \end{bmatrix}$$

$$F_8 = \begin{bmatrix} -0.2124 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.1677 & 0 & 0 & 0 & 0 \\ 0.2124 & 0.1677 & -0.2236 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 & -1 \end{bmatrix}$$

$$L = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} K = \begin{bmatrix} -0.5244 & 0.3541 & -0.4730 \\ 0.4486 & -0.6188 & -0.4730 \\ 0.4486 & 0.3541 & -0.9730 \\ 0.2296 & 0 & -142.6882 \\ 0 & 0.2168 & -108.4385 \\ 142.6276 & 108.3907 & 0.2328 \end{bmatrix}$$

$$E = \begin{bmatrix} 0.4756 & 0.3541 & -0.4730 & -0.5244 & 0.3541 & -0.4730 \\ 0.4486 & 0.3812 & -0.4730 & 0.4486 & -0.6188 & -0.4730 \\ 0.4486 & 0.3541 & 0.0270 & 0.4486 & 0.3541 & -0.9730 \\ 0.2296 & 0.0000 & -142.6882 & 0.2296 & 0.0000 & -142.6882 \\ 0 & 0.2168 & -108.4385 & 0 & 0.2168 & -108.4385 \\ 142.6276 & 108.3907 & 0.2328 & 142.6276 & 108.3907 & 0.2328 \end{bmatrix}$$

The gains of output feedback control are obtained with solving the conditions of theorem (2):

$$K_1 = \begin{bmatrix} -0.2734 & -0.1199 & -0.3496 \\ -0.1050 & -0.3146 & -0.3722 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} -0.2719 & -0.1177 & -0.3391 \\ -0.1023 & -0.3105 & -0.3535 \end{bmatrix}$$

$$K_3 = \begin{bmatrix} -0.2739 & -0.1180 & -0.3550 \\ -0.0984 & -0.3695 & -0.3041 \end{bmatrix}$$

$$K_4 = \begin{bmatrix} -0.2724 & -0.1157 & -0.3446 \\ -0.0956 & -0.3655 & -0.2854 \end{bmatrix}$$

$$K_5 = \begin{bmatrix} -0.3504 & -0.1070 & -0.2721 \\ -0.1017 & -0.3157 & -0.3794 \end{bmatrix}$$

$$K_6 = \begin{bmatrix} -0.3488 & -0.1048 & -0.2617 \\ -0.0989 & -0.3117 & -0.3607 \end{bmatrix}$$

$$K_7 = \begin{bmatrix} -0.3508 & -0.1051 & -0.2776 \\ -0.0951 & -0.3707 & -0.3113 \end{bmatrix}$$

$$K_8 = \begin{bmatrix} -0.3493 & -0.1028 & -0.2671 \\ -0.0923 & -0.3666 & -0.2926 \end{bmatrix}$$

Figure 1 shows the evolution of the state variables and their estimated. The figures 3 to 5 represent the sensor fault estimation. It follows that the proposed fuzzy observer has good performance to estimate the sensor fault and state variables and provide a satisfying result. The trajectory of the output response in figure 6 confirm that the performances of the FTC strategy are very satisfactory and allowing normal functioning of the system even in the occurrence of fault.

VI. CONCLUSION

This paper presents a design of static output feedback tolerant control of TS fuzzy systems subject to sensor fault. In order to decouple the sensor faults, an augmented descriptor model has been constructed by supposing the sensor faults as an auxiliary state variable. A descriptor observer is next designed for the augmented model and the simultaneous estimates of the original state and sensor faults are thus obtained. The convergence of the proposed observer has been performed by the search of suitable Lyapunov matrices. A pole placement in LMI form has been considered to guaranteed the stabilization and good performance of the response for the closed loop system using static output feedback control. The observer and control gains are determined using LMI formulation. Simulation results using a three tank system show the effectiveness and the feasibility of the given results. The proposed observer can be extended to develop a technique of actuator fault estimation.

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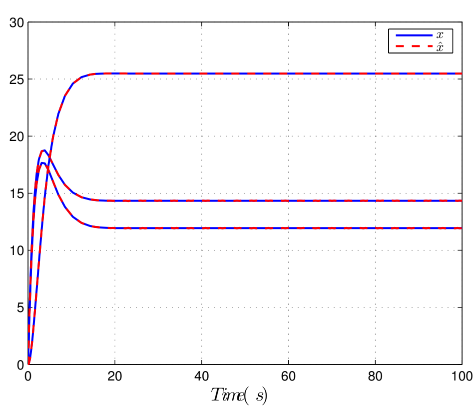

Fig. 1. Variables state and their estimated (x_1, x_2, x_3)

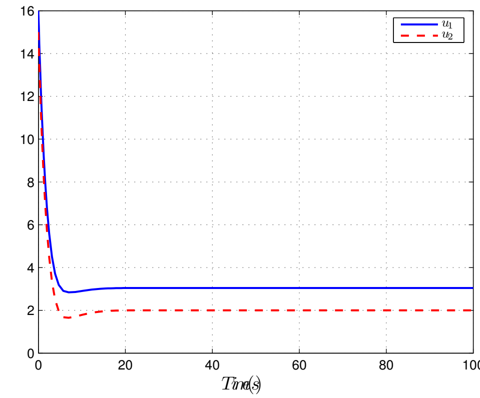

Fig. 2. Control signal (u_1, u_2)

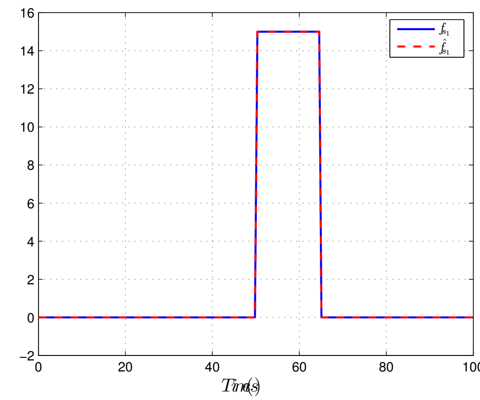

Fig. 3. Sensor faults signal and their estimated f_{s_1}

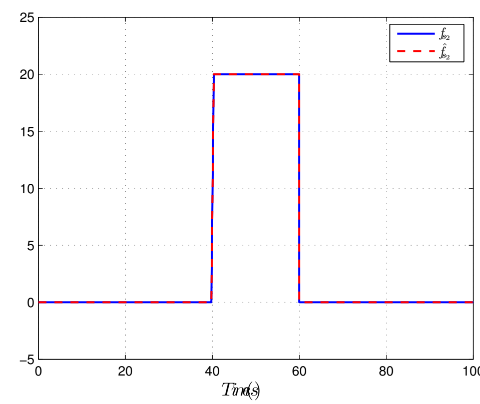

Fig. 4. Sensor faults signal and their estimated f_{s_2}

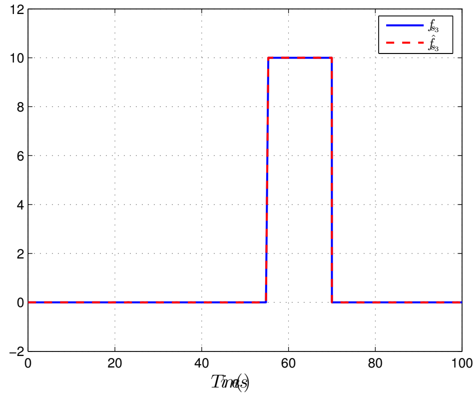

Fig. 5. Sensor faults signal and their estimated f_{s_3}

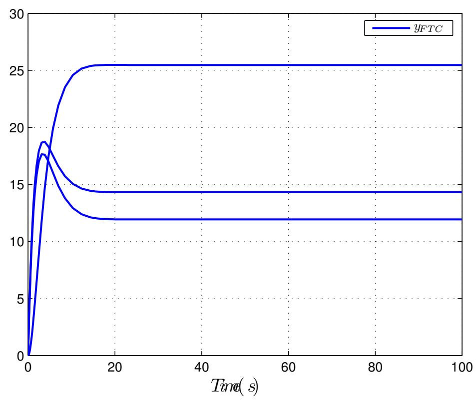

Fig. 6. Output Signal with Fault Tolerant Control y_1, y_2, y_3

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