Robust Fuzzy Observer-Based Tracking Feedback Control Design for Three Tank Systems

Hana Ghorbel[1], Mansour Souissi[1], Mohamed Chaabane[1] and Ahmed El Hajjaji[2]

Abstract— **This paper presents an observer based fuzzy control design method applied to an hydraulic system with a guaranteed H_∞ tracking performance. Our approach is based on Takagi-Sugeno (TS) fuzzy modeling method. For that, a fuzzy observer based tracking control with unmeasurable premise variables is developed to estimate the system states. The parallel distributed compensation (PDC) fuzzy controller is related to the tracking error. Stability conditions are derived from robust control techniques such as quadratic stabilization, H_∞ control theory and Linear Matrix Inequalities (LMIs), to obtain the gains of fuzzy control and fuzzy observers. Finally, simulation results will show the effectiveness of the fuzzy observer-based controller.**

I. INTRODUCTION

The study of tracking control problems has a long-standing history (see for example [8], [10]). For nonlinear system design, various control schemes are introduced including exact feedback linearization [17], [20], [25], sliding mode control [29], adaptive control [5], [6], fuzzy control [26] and neural control [21]. To design fuzzy regulators and fuzzy observers, nonlinear systems are represented by TS fuzzy models. However, Takagi-Sugeno (TS) type fuzzy controllers have been succefully applied to the control design of nonlinear systems [6], [16], [22]. The TS fuzzy model is considered as a popular and powerful tool in approximating a complex nonlinear systems [7]. The basic idea for the TS model structure is the decomposition of the operating space of a nonlinear system into a finite number of operating zones. The behavior of the system in each zone is represented by a local linear model [4]. Each local model is then quantified by means of weighting function. Then, fuzzy tracking controller was developed to stabilize trajectory error. So, Tseng *and al.* [11] have proposed a fuzzy tracking control design for nonlinear system through TS fuzzy model. In [15], the authors have studied an output tracking control problem for nonlinear system of both parameter variations and external disturbances. An H_∞ output tracking control for nonlinear time delays systems is presented in [9].
The authors in [27] and [12] proposed a new parallel distributed compensation (PDC) fuzzy controller in which an integrator action is added. So, the design controller supposed that all the states are available for feedback. In [23], a fuzzy observer based fuzzy tracking controller for induction motor in which an integrator action is added, is designed. However, in the works cited above, the authors suppose that the weighting functions depend on measurable premise variables (like the input or the output of the system). The main contribution of this paper is to synthesize a fuzzy observer-based tracking control for TS fuzzy model which the weighting functions depending on unmeasurable premise variables (like the system state) [3], [4], [13], [14], [19]. The fuzzy tracking control design is parameterized in terms of linear matrix inequalities (LMI) problem [24] which can be solved very efficiently using the convex optimization techniques.

This paper is organized as follows. Section II presents the hydraulic process which includes the nonlinear model. The latter is represented by an equivalent TS type fuzzy model. In section III, a fuzzy observer is developed and the main results are given under LMI formulation. The synthesis of the fuzzy controller with H_∞ performance is formulated in section IV. In section V, the proposal method is applied to a three tank system. Finally, section VI gives some conclusions.
Notation. X^T and X^{-1} are the transpose and the inverse of matrix, respectively. I is the identity matrix with appropriate dimension.

II. THREE TANK SYSTEMS

A. Nonlinear model

The system presented in this paper is a multi-variable process which consists of a three interconnected water tanks as shown in Fig. 1. [18]. This hydraulic process has a great importance in agriculture, irrigation and chemical engineering laboratories. The three tank systems is considered as an adapted model to study performances of the nonlinear system using fuzzy approach.

To give the laws of the conservation of the fluid, we can describe the operating mode of each tank, we then obtain a nonlinear model defined as :

$$\begin{cases} \dot{n}_1(t) = -p_1\sqrt{n_1(t)} + p_{11}u_1(t) \\ \dot{n}_2(t) = -p_2\sqrt{n_2(t)} + p_{22}u_2(t) \\ \dot{n}_3(t) = p_1\sqrt{n_1(t)} + p_2\sqrt{n_2(t)} - p_3\sqrt{n_3(t)} \end{cases} \quad (1)$$

where p_1, p_2, p_3, p_{11} and p_{22} are constants.
The nonlinear model of the three tank system (1) can be rewritten in the following form :

$$\begin{cases} \dot{x}(t) = A(x(t))x(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \quad (2)$$

*This work was not supported by any organization

[1]H. Ghorbel, M. Souissi and M. Chaabane are with the Electrical Engineering School, University of Sfax, Rue Soukra, BP 1173, 3038,Tunisia hana.ghorbel @yahoo.fr

[1]H. Ghorbel and [2]A. El Hajjaji are with University of Picardie Jules Vernes (UPJV), 7, Rue Moulin Neuf 8000 Amiens, France. ahmed.alhajjaji@u-picardie

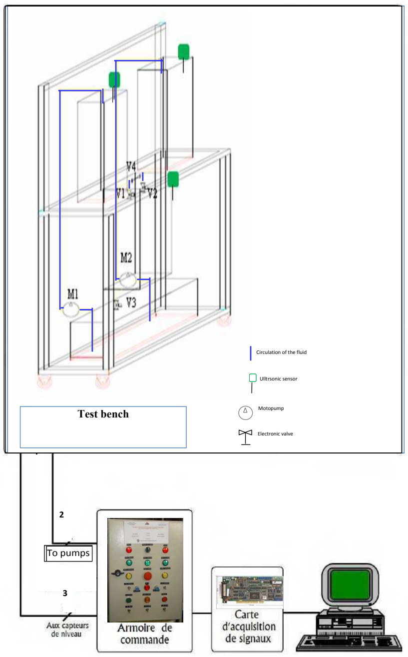

Fig. 1. Three tanks system

where the system states represent the levels in each tank :

$$x(t) = \begin{pmatrix} n_1(t) & n_2(t) & n_3(t) \end{pmatrix}^T$$

$$A(x(t)) = \begin{pmatrix} -\frac{p_1}{\sqrt{n_1(t)}} & 0 & 0 \\ 0 & -\frac{p_2}{\sqrt{n_2(t)}} & 0 \\ \frac{p_1}{\sqrt{n_1(t)}} & \frac{p_2}{\sqrt{n_2(t)}} & -\frac{p_3}{\sqrt{n_3(t)}} \end{pmatrix}$$

$$B = \begin{pmatrix} p_{11} & 0 \\ 0 & p_{22} \\ 0 & 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

B. *Takagi-Sugeno fuzzy model*

By considering the sector of nonlinearities of the terms $\xi_j(t) = \frac{1}{\sqrt{n_j(t)}} \in [\xi_{j,min}, \xi_{j,max}] (j = 1,2,3)$ of the matrix $A(x(t))$, we can transform the nonlinear terms under the following shape :

$$\xi_j(t) = M_{j,min}(\xi_j)\xi_{j,max} + M_{j,max}(\xi_j)\xi_{j,min} \tag{3}$$

where

$$\begin{cases} M_{j,min}(\xi_j) = \frac{\xi_j - \xi_{j,min}}{\xi_{j,max} - \xi_{j,min}} \\ M_{j,max}(\xi_j) = \frac{\xi_{j,max} - \xi_j}{\xi_{j,max} - \xi_{j,min}} \end{cases} \tag{4}$$

The fuzzy model is then described by a set of $r = 8$ fuzzy *If-Then* rules and will be employed here to deal with the control design problem for the hydraulic system. The i^{th} rule is defined as follows :

Rule i
If ($\xi_1(t)$ is M_{i1}) and ($\xi_2(t)$ is M_{i2}) and ($\xi_3(t)$ is $M_{i3}(k)$) **Then**

$$\dot{x}(t) = A_i x(t) + Bu(t), i = 1,2,...,8 \tag{5}$$

The global TS model is then inferred as follows :

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{8} \mu_i(\xi(t)) A_i x(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \tag{6}$$

where

$$\mu_i(\xi(k)) = \frac{\lambda_i(\xi(k))}{\sum_{i=1}^{8} \lambda_i(\xi(k))} \tag{7}$$

$$\lambda_i(\xi(k)) = \prod_{j=1}^{3} M_{ji}(\xi(k)) \tag{8}$$

Hence, the weighting functions verify the following properties :

$$\begin{cases} \sum_{i=1}^{8} \mu_i(\xi(k)) = 1 \\ 0 \le \mu_i(\xi(k)) \le 1 \qquad \forall i \in \{1,2,...,8\} \end{cases} \tag{9}$$

The matrices A_i are :

$$A_1 = \begin{pmatrix} -p_1\beta_1 & 0 & 0 \\ 0 & -p_2\beta_2 & 0 \\ p_1\beta_1 & p_2\beta_2 & -p_3\beta_3 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} -p_1\beta_1 & 0 & 0 \\ 0 & -p_1\beta_2 & 0 \\ p_1\beta_1 & p_2\beta_2 & -p_3\alpha_3 \end{pmatrix}$$

$$A_3 = \begin{pmatrix} -p_1\beta_1 & 0 & 0 \\ 0 & -p_2\alpha_2 & 0 \\ p_1\beta_1 & p_2\alpha_2 & -p_3\beta_3 \end{pmatrix}$$

$$A_4 = \begin{pmatrix} -p_1\beta_1 & 0 & 0 \\ 0 & -p_2\alpha_2 & 0 \\ p_1\beta_1 & p_2\alpha_2 & -p_3\alpha_3 \end{pmatrix}$$

$$A_5 = \begin{pmatrix} -p_1\alpha_1 & 0 & 0 \\ 0 & -p_2\beta_2 & 0 \\ p_1\alpha_1 & p_2\beta_2 & -p_3\beta_3 \end{pmatrix}$$

$$A_6 = \begin{pmatrix} -p_1\alpha_1 & 0 & 0 \\ 0 & -p_2\beta_2 & 0 \\ p_1\alpha_1 & p_2\beta_2 & -p_3\alpha_3 \end{pmatrix}$$

$$A_7 = \begin{pmatrix} -p_1\alpha_1 & 0 & 0 \\ 0 & -p_2\alpha_2 & 0 \\ p_1\alpha_1 & p_2\alpha_2 & -p_3\beta_3 \end{pmatrix}$$

$$A_8 = \begin{pmatrix} -p_1\alpha_1 & 0 & 0 \\ 0 & -p_2\alpha_2 & 0 \\ p_1\alpha_1 & p_2\alpha_2 & -p_3\alpha_3 \end{pmatrix}$$

III. FUZZY OBSERVER DESIGN

In this paper, we consider the system (6) with weighting functions depending on the system state :

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{8} \mu_i(x(t))A_i x(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \quad (10)$$

The multiple model with unmeasurable variables (10) can be reduced to a perturbed multiple model with measurable variables as follows :

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{8} \mu_i(\hat{x}(t))A_i x(t) + Bu(t) + w(t) \\ y(t) = Cx(t) \end{cases} \quad (11)$$

where

$$w(t) = \sum_{i=1}^{r} \big(\mu_i(x(t)) - \mu_i(\hat{x}(t))\big)A_i x(t) \quad (12)$$

Before designing the fuzzy observer-based fuzzy tracking controller, we can define the tracking error as :

$$e_r(t) = x_r(t) - x(t) \quad (13)$$

where $x_r(t)$ represents the reference signal. It is assumed that $x_r(t)$, for all $t \geq 0$, represents a desired trajectory for $x(t)$ to follow.
By differentiating (13), we get :

$$\dot{e_r}(t) = \sum_{i=1}^{8} \mu_i(\hat{x}(t))\Big(A_i e_r(t) - Bu(t) - A_i x_r(t) - w(t)\Big) \quad (14)$$

Next, we will design the fuzzy observer to estimate the inaccessible states of the fuzzy model (11) which the weighting functions depend on the estimated states. The structure of the fuzzy observer is given as follows :

Observer Rule i
If ($\xi_1(t)$ is M_{i1}) and ($\xi_2(t)$ is M_{i2}) and ($\xi_3(t)$ is $M_{i3}(k)$) **Then**

$$\dot{\hat{x}}(t) = A_i\hat{x}(t) + Bu(t) + L_i(y(t) - \hat{y}(t)), i = 1,2,...,8 \quad (15)$$

where, $\hat{x}(t)$ and $\hat{y}(t)$ denotes the estimation of $x(t)$ and $y(t)$ respectively. L_i is the observer gain of the i^{th} observer rule to be determined.
The overall fuzzy observer is represented as follows :

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{8} \mu_i(\hat{x}(t))\Big(A_i\hat{x}(t) + Bu(t) + L_i(y(t) - \hat{y}(t))\Big) \\ \hat{y}(t) = C\hat{x}(t) \end{cases} \quad (16)$$

Let us consider the estimation error state as :

$$e_0(t) = x(t) - \hat{x}(t) \quad (17)$$

To derive (17), we obtain the error dynamics :

$$\dot{e}_0(t) = \sum_{i=1}^{8} \mu_i(\hat{x}(t))\Big((A_i - L_iC)e_0(t)\Big) + w(t) \quad (18)$$

In the next section, we will propose a feedback controller-based observer and rejection disturbance.

IV. FUZZY CONTROLLER DESIGN

In this section, a PDC fuzzy controller related to the tracking error is presented. In this case the fuzzy controller is designed as [28] :

Control Rule i
If ($\xi_1(t)$ is M_{i1}) and ($\xi_2(t)$ is M_{i2}) and ($\xi_3(t)$ is $M_{i3}(k)$) **Then,**

$$u(t) = K_i(x_r(t) - \hat{x}(t)) \quad (19)$$

However, the overall fuzzy controller is given by :

$$u(t) = \sum_{i=1}^{8} \mu_i(\hat{x}(t))K_i(x_r(t) - \hat{x}(t)) \quad (20)$$

From (13), we get :

$$x_r(t) = x(t) + e_r(t) \quad (21)$$

Equation (17) gives :

$$\hat{x}(t) = x(t) - e_0(t) \quad (22)$$

Subtracting (23) from (21), we get :

$$x_r(t) - \hat{x}(t) = e_r(t) + e_0(t) \quad (23)$$

Then, the equation (20) can be rewritten as follows :

$$u(t) = \sum_{i=1}^{8} \mu_i(\hat{x}(t))K_i(e_r(t) + e_0(t)) \quad (24)$$

Substituting (24) into (14) yields :

$$\begin{aligned} \dot{e}_r(t) = & \sum_{i=1}^{8} \mu_i(\hat{x}(t))\Big((A_i - BK_i)e_r(t) - BK_ie_0(t) - \\ & A_ix_r(t) - w(t)\Big) \end{aligned} \quad (25)$$

After manipulation, we can express the augmented system in the following form :

$$\dot{\bar{e}}(t) = \sum_{i=1}^{8} \mu_i(\hat{x}(t))\Big(\bar{A}_i\bar{e}(t) + \bar{D}_i\bar{w}(t)\Big) \quad (26)$$

where :

$$\bar{A}_i = \begin{bmatrix} A_i - L_iC & 0 \\ -BK_i & A_i - BK_i \end{bmatrix}, \bar{D}_i = \begin{bmatrix} 0 & I \\ -A_i & -I \end{bmatrix},$$

$$\bar{e}(t) = \begin{bmatrix} e_0(t) \\ e_r(t) \end{bmatrix} \quad and \quad \bar{w}(t) = \begin{bmatrix} x_r(t) \\ w(t) \end{bmatrix}$$

Let us consider the H_∞ performance which allows to eliminate the effect of $\bar{w}(t)$ on the control system. If we consider the initial condition, the H_∞ performance is defined as follows [1], [11] :

$$\begin{aligned} & \int_0^{t_f} e_0^T(t)Qe_0(t)dt + \int_0^{t_f} e_r^T(t)Qe_r(t)dt \\ & \leq \bar{e}^T(0)\bar{P}\bar{e}(0) + \gamma^2 \int_0^{t_f} \bar{w}^T(t)\bar{w}(t)dt \end{aligned} \quad (27)$$

The condition (27) can be rewritten in the following form :

$$\int_0^{t_f} \bar{e}^T(t)\bar{Q}\bar{e}(t)dt \le \bar{e}^T(0)\bar{P}\bar{e}(0) + \gamma^2 \int_0^{t_f} \bar{w}^T(t)\bar{w}(t)dt \quad (28)$$

where :

t_f terminal time
γ prescribed attenuation level
$\bar{P}$ symmetric positive definite matrix

and

$$\bar{Q} = \begin{bmatrix} Q & 0 \\ 0 & Q \end{bmatrix}$$

The objective of this study is to determine feedback gains K_i and observer gains L_i for the augmented model (26) such as :

- The closed loop system with fuzzy observer and fuzzy controller is asymptotically stable.
- The H_∞ tracking control performance is guaranteed and the attenuation level γ^2 is minimized.

Theorem 1: The system (26) is stable and the $\pounds_2$ gain of the transfer from $\bar{w}(t)$ to the state estimation error $\bar{e}(t)$ is bounded by γ, if there exists symmetric and positive definite matrices $\bar{P} = \bar{P}^T > 0$, a matrix $\bar{Q}$ and a prescribed positive scalar γ^2 such that the following inequalities are satisfied :

$$\begin{bmatrix} N_{11} & P_1 & N_{13} \\ P_1 & -\gamma^2 I & 0 \\ N_{13}^T & 0 & N_{33} \end{bmatrix} < 0 \quad \forall i \in \{1,2,...,8\} \quad (29)$$

where :

$$\begin{aligned} N_{11} = & \; A_i^T P_1 + P_1 A_i - C^T Z_i^T - Z_i C + Q \\ N_{13} = & \; -K_i{}^T B^T P_2 - \tfrac{1}{\gamma^2} P_1 P_2 \\ N_{33} = & \; X A_i^T + A_i X - BYi - Yi^T B + \tfrac{1}{\gamma^2}(A_i A_i^T + I) \end{aligned}$$

where : $Z_i = P_1 L_i$

Proof: To prove that the augmented model (26) is quadratically stable, let us consider the Lyapunov function :

$$V(\bar{e}(t)) = \bar{e}^T(t)\bar{P}\bar{e}(t) \quad (30)$$

By differentiating (30), we obtain :

$$\begin{aligned} \dot{V}(t) = & \; \dot{\bar{e}}^T(t)\bar{P}\bar{e}(t) - \bar{e}^T(t)\bar{P}\dot{\bar{e}}(t) \\ = & \; \sum_{i=1}^{8} \mu_i(\hat{x}(t)) \Big(\bar{e}^T(t)(\bar{A}_i^T\bar{P} + \bar{P}\bar{A}_i)\bar{e}(t) + \\ & \; \bar{w}^T(t)\bar{D}_i^T\bar{P}\bar{e}(t) + \bar{e}^T(t)\bar{P}\bar{D}_i\bar{w}(t) \Big) \end{aligned} \quad (31)$$

The closed loop system with controller based observer is stable and has H_∞ norm is limited by γ if and only if :

$$\dot{V}(t) + \bar{e}^T\bar{Q}\bar{e} - \gamma^2 \bar{w}^T\bar{w} < 0 \quad (32)$$

The inequality can be rewritten as follows :

$$\bar{A}_i^T\bar{P} + \bar{P}\bar{A}_i + \frac{1}{\gamma^2}\bar{P}\bar{D}_i\bar{D}_i^T\bar{P} + \bar{Q} < 0 \quad (33)$$

for $i \in \{1,2,...,8\}$

To facilitate the resolution of controller gains and observer gains, the matrix variable ($\bar{P} > 0$) is chosen diagonal with respect to appropriate matrix blocks [9], [11] :

$$\bar{P} = diag \begin{bmatrix} P_1 & P_2 \end{bmatrix} \quad (34)$$

By substituting (34) into (33), we obtain :

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix} < 0 \quad \forall i \in \{1,2,...,8\} \quad (35)$$

where :

$$\begin{aligned} S_{11} = & \; (A_i - L_i C)^T P_1 + P_1(A_i - L_i C) + \tfrac{1}{\gamma^2} P_1 P_1 + Q \\ S_{12} = & \; -K_i{}^T B^T P_2 - \tfrac{1}{\gamma^2} P_1 P_2 \\ S_{22} = & \; F_i^T P_2 + P_2 F_i + \tfrac{1}{\gamma^2} P_2 (A_i A_i^T + I) P_2 + Q \end{aligned}$$

where : $F_i = A_i - BK_i$ We consider the change of variable $Z_i = P_1 L_i$ and by applying the Shur complement, we obtain the inequality (29).

Since four parameters P_1, P_2, K_i and Z_i should be determined from (29), there are no effective algorithms for solving them simultaneously, until now. So, we can solve them by the following two-step procedures.
Firstly, note that (29) implies that $N_{33} < 0$.

$$(A_i - BK_i)^T P_2 + P_2(A_i - BK_i) + \frac{1}{\gamma^2} P_2 (A_i A_i^T + I) P_2 + Q < 0 \quad (36)$$

It should be noted that the condition (36) is nonlinear with respect to the variables P_2 and K_i. Multiplying (36) of both sides by P_2^{-1} and considering the change of variable $X = P_2^{-1}, Y_i = K_i X$, we obtain :

$$X A_i^T + A_i X - BY_i - Y_i^T B^T + \frac{1}{\gamma^2}(A_i A_i^T + I) + XQX < 0 \quad (37)$$

By the Schur complement, (37) is equivalent to the following LMI :

$$\begin{bmatrix} H_{11} & X \\ X & -Q^{-1} \end{bmatrix} < 0 \quad (38)$$

where :

$$H_{11} = X A_i^T + A_i X - Y_i^T B^T - BY_i + \frac{1}{\gamma^2}(A_i A_i^T + I) \quad (39)$$

The parameters $P_2 = X^{-1}$ and $K_i = Y_i X^{-1}$ are obtained by solving LMI (38).
Secondly, by substituting P_2 and K_i into (29), the last condition becomes a standard LMI and we can easily solve P_1 and L_i (thus $L_i = P_1^{-1} Z_i$).

■

V. SIMULATION RESULTS

To prove the performance of the proposed fuzzy observer-based controller in terms of tracking performance and disturbance rejection, numerical simulations are presented in Fig. 2-3.
Solving LMIs (29) and (38) by LMI optimization algorithm, controller gains and observer gains matrices can be obtained

as :

$$K_1 = \begin{pmatrix} 43.2156 & -1.0383 & 14.9754 \\ -1.0688 & 51.5946 & 14.9012 \end{pmatrix}$$

$$K_2 = \begin{pmatrix} 43.2184 & -1.0356 & 14.9789 \\ -1.0665 & 51.5968 & 14.9040 \end{pmatrix}$$

$$K_3 = \begin{pmatrix} 43.2151 & -1.0373 & 14.9677 \\ -1.1020 & 51.6635 & 14.3050 \end{pmatrix}$$

$$K_4 = \begin{pmatrix} 43.2179 & -1.0346 & 14.9711 \\ -1.0998 & 51.6658 & 14.3076 \end{pmatrix}$$

$$K_5 = \begin{pmatrix} 43.2802 & -1.0666 & 14.3678 \\ -1.0680 & 51.5940 & 14.8903 \end{pmatrix}$$

$$K_6 = \begin{pmatrix} 43.2829 & -1.0639 & 14.3711 \\ -1.0658 & 51.5963 & 14.8928 \end{pmatrix}$$

$$K_7 = \begin{pmatrix} 43.2797 & -1.0656 & 14.3601 \\ -1.1012 & 51.6630 & 14.2940 \end{pmatrix}$$

$$K_8 = \begin{pmatrix} 43.2825 & -1.0629 & 14.3633 \\ -1.0990 & 51.6652 & 14.2964 \end{pmatrix}$$

$$L_1 = \begin{pmatrix} 89.3133 & -0.0001 \\ 0.0001 & 89.3149 \\ 0.7613 & 0.6387 \end{pmatrix}$$

$$L_2 = \begin{pmatrix} 89.3133 & -0.0000 \\ 0.0001 & 89.3149 \\ 0.7613 & 0.6387 \end{pmatrix}$$

$$L_3 = \begin{pmatrix} 89.3133 & 0.0000 \\ -0.0000 & 89.3196 \\ 0.7612 & 0.6306 \end{pmatrix}$$

$$L_4 = \begin{pmatrix} 89.3133 & 0.0000 \\ 0.0000 & 89.3196 \\ 0.7613 & 0.6306 \end{pmatrix}$$

$$L_5 = \begin{pmatrix} 89.3189 & 0.0000 \\ 0.0000 & 89.3149 \\ 0.7515 & 0.6386 \end{pmatrix}$$

$$L_6 = \begin{pmatrix} 89.3189 & 0.0000 \\ 0.0000 & 89.3149 \\ 0.7516 & 0.6387 \end{pmatrix}$$

$$L_7 = \begin{pmatrix} 89.3189 & 0.0000 \\ 0.0000 & 89.3196 \\ 0.7515 & 0.6305 \end{pmatrix}$$

$$L_8 = \begin{pmatrix} 89.3189 & 0.0000 \\ 0.0000 & 89.3196 \\ 0.7516 & 0.6306 \end{pmatrix}$$

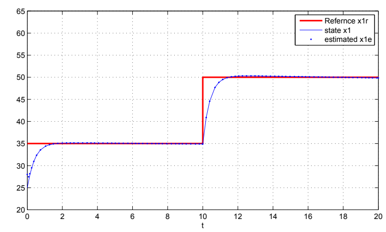

Fig. 2. The trajectories of the output y_1 and his estimated

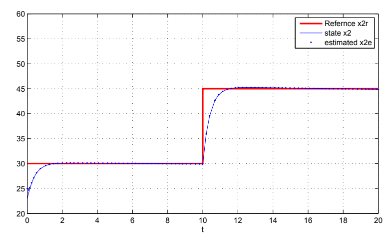

Fig. 3. The trajectories of the output y_2 and his estimated

The simulation results illustrate the performance of the developed approach. Indeed, we remark that the estimated variables converge to the real variables and to the desired trajectory. Then, we can prove the observer's convergence.

VI. CONCLUSION

In this study, a fuzzy feedback control scheme has been proposed for continuous nonlinear model. To design fuzzy observer based tracking control, nonlinear system have been represented by a TS fuzzy model. The concept of parallel distributed compensation has been employed to reduce the tracking error and to guaranty the H_∞ performance. A simple and systematic algorithm based on LMI optimization technique is presented to solve the tracking control problem. The proposed approach is adequate for the hydraulic system. The simulation results show the effectiveness of the proposed method.

ACKNOWLEDGMENT

We thank the ministry of higher education and scientific research of Tunisia and AECID (A/030410/10 and AP/039213/11) for funding this work.

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