

Bond Graph Model based for structural component diagnosability analysis

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Abstract— The paper presents a method for a component fault detectability and isolability analysis based on bond graph model properties. The used in the literature analytical and graphical approaches need models under state space or set of constraints for this task. The innovative interest of proposed method is based only on the architectural representation of bond graph model deduced from physical system to be monitored before real design of online monitoring system. A developed methodology is illustrated by an application to a two tanks hydraulic system.

I. INTRODUCTION

A safe and reliable plant depends on Fault Detection and Isolation (FDI) procedures. There has been a significant research in the area of model based fault detection [1]-[4]. The analytical redundancy approach consists in finding the over constrained subsystem (representing the monitorable part of the overall system) since it is the only one to exhibit some redundancies. The first step consists in generating a set of residuals (relations between the known variables of the system) used for alarm generation in presence of faults. The second step consists in fault isolation using the generated fault indicators (called Analytical Redundancy Relations (ARRs)) from which is deduced the fault signature matrix using specific logic.

A key factor determining the performance of FDI techniques is the model accuracy. Once this is established, the FDI performance in terms of diagnosability (the ability to detect and isolate faults) can be established by formal analysis of the analytical fault models derived from the system models. FDI methods based on quantitative analysis suffer from a number of problems. Their behaviors are typically defined by high order differential equations with complex nonlinearities that are hard to model and analyze using analytical or numerical schemes. However, graphical methods present a big interest because their models capture system structure by representing the system variables and set of behavior equations just as nodes and arc. Furthermore, the graph structure is independent of the numerical values of the

system parameters. This is why graphical methods are well suited for defining qualitative diagnosis methods.

A graph theory was enormously developed and widely used to study the structural analysis problem [2]-[5]. Determination of diagnosability properties (which components can be monitored and how to make them monitorable based on optimal sensor placement) could be very useful before industrial implementation. Most of developed graphical models are based on directed and bipartite graphs which are powerful and efficient to study many system properties, such as observability, controllability and diagnosability. However, bipartite graphs and digraphs use analytical mathematical models for generating the structural model. The digraph [2] is usually generated and formed from linear state equations and the bipartite graph is deduced from the set of constraints (derived from state equations or first principle methods) [1], [3], [4].

For the digraph [2], the diagnosability analysis concerns only actuators and sensors while the graph is deduced from state equation (nodes are inputs and outputs). Lack of the cited methods consist in that the architecture of the system is not explicitly displayed, the considered faults are generally input and output vertices of the graph and are not associated with a physical component while dynamic model is given under analytical equations or state space format (often linear [2]).

Among qualitative graphical approaches, the signed digraph (SDG) method for fault diagnosis is one of the model-based methods that have been widely applied [6] especially to chemical processes. SDGs use a directed graph representation to capture causal relations among system variables. Their structure is similar to the digraphs, but the difference is that the nodes (system variables) assume qualitative values: “0” “+” and “-” in relation to the variable’s reference value. The qualitative output states (fault signatures) are stacked to form a truth table. The truth table is similar to the fault signature matrix and its structure determines online which faults can be detected and which faults can be isolated. A comparison of this tool with Bond Graph (BG) for FDI is given in [7].

This paper focuses specifically on component fault detectability and isolability analysis from the bond graph model. The aim purpose is to show how the bond graph can be easily used for detectability and isolability analysis of the fault which may affect any system with any need of complex calculation. Furthermore, it will be shown how the well-known Dulmage Mensorshon canonical decomposition of any system under over, just and under determined

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subsystems [8] from analytical model can be graphically displayed in a modular fashion from the BG model.

The outline of this paper is as follows. Section 2 presents a brief introduction to the structural analysis. In section 3, the decomposition of Dulmage Mendelshon from bond graph model is given and the different types of subsystems are defined. The fourth section concerns the detectability and isolability fault properties. In the fifth part; the developed theory is illustrated by an application to a hydraulic system. Finally, a conclusion is given.

II. BOND GRAPH STRUCTURAL ANALYSIS BASIS

Because of limited place, bond graph theory is not developed here (for more details see [9]). The structural behavior model of any physical system can be defined by a pair (C, Z) , where $Z = \{z_1, z_2, \dots, z_n\}$ is a set of variables and parameters, and $C = \{c_1, c_2, \dots, c_n\}$ is a set of constraints. The constraints may be expressed in several forms like algebraic, difference equations, rules, etc.

A bipartite graph $G = (C, Z, A)$ is separated into two disjoint sets C and Z . The set of variables Z can be partitioned as $Z = K \cup X$, where K is the set of known variables (measured variables y and the set of input control u), and X is the set of unknown variables. An edge (c, z) , for $c \in C$ and $z \in Z$ stands for variable z is involved in equation c in such a way that every edge has end point in C and the other one in Z . A bipartite graph can be represented by an adjacency matrix (named incidence matrix). This is a Boolean matrix where each row corresponds to a constraint c_i and each column to a variable z_j . A "1" at position (i, j) indicates that there is an edge connecting the constraint c_i and the variable z_j . This matrix is used in diagnosis to find the redundancies (ARR) based on different matching [1].

A. Bipartite graphs and Bond graphs

Comparing with other graphical models (SDG, digraphs, bipartite graphs, . . .) where vertices are variables and the edges represent mutual influence between those variables, BG is also a graph, but the vertices are BG elements (physical components and junctions) labeled by power variables, and the edges represent the exchanged power. Among all graphical models, BG is well recognized as a powerful tool for modeling. Furthermore only Bond graphs allow not only generating dynamic state space equations but also control and diagnosis analysis using dedicated software (CAMP-G, Twente sim, Symbols2000...) [10], [11]. Based on consulted literature, only in [12] is presented dedicated software (using Matlab platform) for ARR generation from state space equation using bipartite graph theory. Consider first the link of BG with bipartite graph. The constraints C from the bond graph model

$$C = \{C_s\} \cup \{C_b\} \cup \{C_m\}$$

can be seen as any relation which is connected to the variables and to the parameters of the system. They are represented by the structural constraints C_s , deduced from

the set of junction equations C_j which represent the mass and energy conservation laws. The number of junction equations is then equal to the number of equations in 0-junction (common effort), 1-junction (common flow) and 2-ports elements (transformer TF, gyrator GY):

$$C_s = \{C_{J0}\} \cup \{C_{J1}\} \cup \{C_{TF}\} \cup \{C_{GY}\}$$

Behavior equations (C_b) describe the physical phenomena in occurred in passive bond graph elements (Resistive R , Capacitive C and Inertial I):

$$C_b = \{C_C\} \cup \{C_I\} \cup \{C_R\}$$

Measurement (C_m) equations represent the sensor equations

$$C_m = \{C_{De}\} \cup \{C_{Df}\}$$

where De and Df are effort and flow detectors respectively. The set of variables $Z = K \cup X$ consists of known (K) and unknown (X) variables. The known variables set K contains the effort (Se) and flow (Sf) source variables: $\{MSe\} \cup \{MSf\} \cup \{Se\} \cup \{Sf\}$ and measurement variables $\{De\} \cup \{Df\}$. Sources can be controlled (modulated sources MSe and MSf). Unknown variables X are the pair of conjugated power variables (flow and effort):

$$X = \{e_1, f_1\} \cup \{e_2, f_2\} \cup \dots \cup \{e_n, f_n\}$$

Finally, set of known and unknown variables is :

$$Z = K \cup X = \{MSe\} \cup \{MSf\} \cup \{Se\} \cup \{Sf\} \cup \{e_1, f_1\} \cup \{e_2, f_2\} \cup \dots \cup \{e_n, f_n\}$$

B. Dulmage Mendelsohn Decomposition

This section presents the concept of Dulmage–Mendelsohn decomposition. These formal definitions are primarily used in [8]. The structural approach is based on determining a complete matching between C and X in the bipartite graph G . Matching is defined as a set of edges from graph G where no two edges have a common end vertex and a perfect matching is a matching in a graph G that covers all its vertices.

Definition 1

In [8], it is shown that any system to be monitored can be decomposed into three main parts:

- *The structurally over-constrained* subsystem M^+ , where the matching is complete with respect to unknown variables X but incomplete with respect to the constraints C . Otherwise, a set C of equations is structurally over determined if C has more equations than unknowns variable X : this set of equations should be regarded, in the generic case, as the sets of equations containing redundancy. To localize the faults in the model and to obtain good fault isolation small over determined sets are important.

- *The structurally just-constrained* subsystem M^0 : there is a complete matching with respect to variables X and to the

constraints C . Then, the number of equations in the system is equal to the number of variables.

• *The structurally under-constrained M* , where the matching is complete on the constraints C but not on the variables X : the number of variables is greater than the number of equations.

This canonical decomposition first proposed by Dulmage–Mendelsohn [8] is represented by the adjacent matrix (Fig. 1) of a bipartite graph with C and X as node sets. In figure 1, white areas are zeros, grey areas contain ones and the oblique line represents a maximal matching in the graph defined by this adjacent matrix.

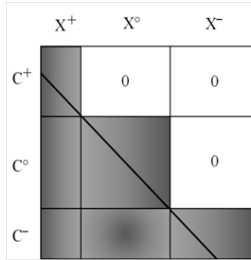


Fig. 1. Dulmage-Mendelsohn decomposition

The over constrained subsystems may contain several smaller over constrained or just constrained ones and no under constrained sub system can be found in the over constrained. Similarly, the just constrained sub system can contain several smaller just constrained. In the under constrained subsystem several under constrained may exist.

C. Dulmage-Mendelsohn Decomposition from BG Model

A method based graph theory loses sometimes certain information when writing the structure of a graph representing system structure. This is due to the fact that construction is generally from state equation, which does not explain all relations constitutive of the system studied. Furthermore, Dulmage-Mendelsohn decomposition based on structural model needs determination of the whole of analytical equations (Constraints) and corresponding variables: this task is not trivial for complex systems. In the next subsection, it will be shown how the BG representation can be used in order to deduce different subsystems directly from bond graph. The method is based on cardinality of the set of constraint and the set of the variables from BG model.

D. Cardinality from BG model

The fundamental building blocks for a junction structure are bonds and the nodes represented by BG junctions: 0, 1, TF, and GY. The power-preserving node is called a 0-junction and 1-junction. Additional power preserving nodes are TF (TransFormer) and GY (Gyrator) used to models phenomena transformation from one energy to another. The nodes 0, 1, TF and GY will be generally noted as junction structure nodes. Two conjugate variables will be associated with each bond in every junction structure (JS). These are

called “effort” variable and “flow” variable, denoted by e and f , respectively. The E-nodes represents the set of components which consists of sources (Se, Sf), bond graph passive elements (I, C, and R) and the set of detectors (De, Df). The junction structure can be represented by Figure 2.

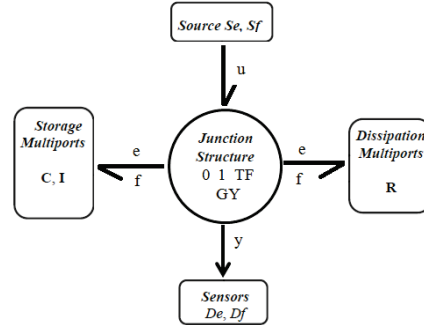


Fig.2. Junction structure representation

• **Cardinal of constraints**

Consider (Fig.3) the j^{th} junction structure (JS) where occur several phenomena represented by set of n bond graph elements E_1, \dots, E_n . To this junctions are connected m sensors S_1, \dots, S_m .

Where: $E \in \{I \ C \ R \ De \ Df \ Se \ Sf\}$

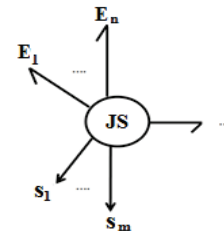


Fig.3. Subsystem, models bond graph

This junction is completely defined by one structural equation (energy conservation), n behavioral equations (how this energy is transformed) and m measurement equations.

$$\begin{aligned} \text{card}(C_b) &= |C_b^j| = \sum_{i=1}^n E_i \\ \text{card}(C_s) &= 1 \\ \text{card}(C_m) &= |C_m^j| = \sum_{j=1}^m S_j \end{aligned}$$

The cardinal of constraints $|C^j|$ on the j^{th} junction is equal to the number of all the constraints given above.

$$\text{card}(C^j) = |C^j| = \sum_{i=1}^n E_i + 1 + \sum_{j=1}^m S_j = |C_b^j| + |C_m^j| + 1 \quad (1)$$

• **The cardinal of unknown variables**

The cardinal of unknown variables X related to the j^{th} junction structure JS $|X^j|$ represents the number of unknown variables on JS. The number of unknown variables in 0-junction is equal to the set of flow variables $\sum f_i = 0$ plus the common effort variable ($e_1 = e_i, e_2 = e_i, \dots, e_n = e_i$) which links all elements. Similarly on the 1-junction, the number of unknown variables is the sum of effort variables labeling the components bond graph $\sum e_i = 0$ plus the common flow variable ($f_1 = f_i, f_2 = f_i, \dots, f_n = f_i$). In a more General case, the unknown variables cardinal $|X^j|$ can be written by the relation:

$$\text{card}(X^j) = |X^j| = \left(\sum_{i=1}^n E_i \right) + 1 \quad (2)$$

Consider now the global bond graph model of the system to be monitored which consists of ℓ junctions. The cardinal of the unknown variables $|X|$ and the cardinal of constraints $|C|$ can be given through the following relations:

$$\begin{aligned} \text{card}(C) = |C| &= \sum_{j=1}^{\ell} |C^j| = \mathfrak{n} + \sum_{i=1}^{\mathfrak{n}} \left(|C_b^j| + |C_m^j| \right) \\ \text{card}(X) = |X| &= \sum_{j=1}^{\ell} |X^j| = \mathfrak{n} + \sum_{i=1}^{\mathfrak{n}} \left(|C_b^j| \right) \end{aligned} \quad (3)$$

III. FDI CONDITIONS BASED ON BG MODEL

A. Structural detectability from bond graph model

Proposition 1

A fault F which may affect a component E_i belonging to a subsystem modeled by a bond graph model in derivative causality is detectable $C_{jE_i} \in C^+$ if and only if there is at least one sensor (detector of effort or flow: $|C_m| \geq 1$).

Proof

Consider a bond graph model in derivative causality. From equation (3), the cardinality of constraints is:

$$\text{card}(C) = |C| = |X| + |C| = \text{card}(X) + \text{card}(C_m) \quad (4)$$

The condition of over determination is: $\text{card}(C_m) \neq 0$.

This means that there is at least one sensor.

Recall that the Analytical Redundancy Relations (RRAs) (which are relations where all variables are known *i.e.* measured ones) exist only for over determined subsystems. They correspond to the relations that are not involved in the complete matching and, consequently are not needed to

eliminate unknown variables. From causal point of view, RRAs are no matched constraints (all variables are known) (Fig.4a). In a bond graph model in derivative causality [9] candidate ARR is generated from a structural equation (0 or 1 junction) connected to a sensor: unknown variables (efforts and flows) are eliminated using covering causal paths from unknown to known ones (sensors). (Fig. 4b)

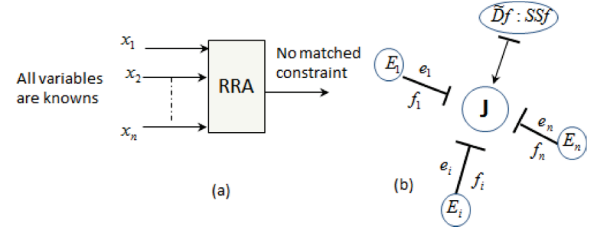


Fig.4: ARR as no matched constraint from bipartite graph (a) and from bond graph point of view (b)

• Particular case: causal conflict

Consider for illustration a RLC electrical system with current sensor (detector of flow) given by its bond graph (Fig.5

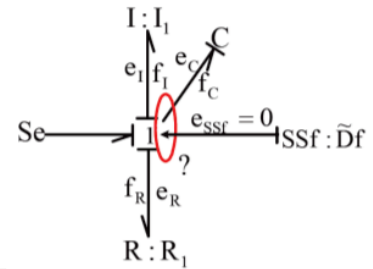


Fig.5: Bond graph causality conflict

From acausal bond graph model it's easy to see that $\text{card}(C)=5$ and $\text{card}(X)=4$ the systems seems to be monitorable. But, putting the bond graph in derivative causality and dualizing the flow sensor, a causal conflict appears on bond graph when trying to put both dynamic elements in derivative causality: the system is then not over determined but just determined. Consequently, one of dynamic element (I or C) has to be assigned with integral causality: in this case a new unknown variable appears: the initial condition. We have then in fact: $\text{card}(C)=5$ and $\text{card}(X)=4+1$.

B. . Structural isolability

The equations of power balance on the junctions constitute the candidate ARR [9]. Consider m candidates ARR. During the covering causal paths for RRA generation (which leads to an oriented graph where nodes are constraints represented by bond graph elements), a set of components (noted $COMP(ARR)$) will be covered during this sequence:

$$\begin{cases} COMP(ARR_1) = \{E_{11} \dots E_{1i} \dots E_{1n}\} \\ \dots \\ COMP(ARR_m) = \{E_{m1} \dots E_{mi} \dots E_{mn}\} \end{cases} \quad (5)$$

Because of functional and graphical aspect of bond graph, it's important to note that generated ARR from bond graph should be considered with their component support:

$$COMP(ARR) \subset \{C, R, I, TF, GY, De, Df, Se, Sf\} \\ = \bigcup_i COMP(ARR_i)$$

Proposition 2

Two component faults F_j and F_i are said strongly isolable (or discriminable) if and only if for given two over determined subsystems M_i^+ and M_j^+ such that

$$F_j \in M_j^+, F_i \in M_i^+ : COMP(ARR_j) \cap COMP(ARR_i) = \emptyset$$

Corollary, any pair (F_j, F_i) where $F_j \neq F_i$ are said not isolable) if and only if for given two over determined subsystems M_i^+ and M_j^+ such that $F_j \in M_j^+, F_i \in M_i^+ :$

$$COMP(ARR_j) \cap COMP(ARR_i) = \{Is\} \neq \emptyset$$

Remark: $Card(Is)$ can be used to evaluate the degree of isolability (Strong, weak)

Proof

Let r the evaluation of ARR ($r = Eval(ARR)$), if r is near to zero, then the ARR is satisfied. If any fault on E component occurs, the ARR is violated then r generates an alarm (this is the detection step for which the condition of structural detectability is given above). The problem is how to isolate fault (to identify which component is faulty). In FDI model based is used the fault Boolean signature matrix (FSM) which crosses ARR in rows and set of faults F (which may affect component E) in columns:

The Boolean value s_{ij} equals 1 if the i^{th} residual is affected by the j^{th} fault. The signature vector of each component fault E_j is given by the row vector

$$V_{E_j} \quad j=1, \dots, m = [s_{j1}, s_{j2}, \dots, s_{jn}]$$

A pair of component faults (E_j, E_i) are isolable if their signature vector are different :

$$\forall \ell (\ell=1, \dots, m), V_{E_j} \neq V_{E_i} (j \neq i)$$

From the bond graph model as developed above, the FSM can be formed as given figure 6.

COMP/E	E_{F1}	...	E_{Fj}	...	E_{Fk}	...	E_{Fn}
C_1^+ { $COMP(ARR_i)$	E_i	...	E_j				
.
$COMP(ARR_j)$			E_{ij}				E_{in}
.
.
C_m^+ { $COMP(ARR_m)$					E_k	...	E_n

Fig.6: FSM from Bond graph model

E_{Fj} represents the fault which may affect the j^{th} component. Any element of the FSM matrix E_{ij} represent BG elements covered by the causal paths during the unknown variable elimination procedure. The grey areas represent zero (*i.e.* the fault doesn't affect the corresponding ARR).

$$E_{ij} = \begin{cases} E_{ij} & \text{if } E_{Fj} \subset COMP(ARR_i) \\ 0 & \text{else} \end{cases}$$

Consider two over determined subsystems C_1^+ and C_2^+ . From the FSM, we have:

$COMP(ARR_1) \cap COMP(ARR_m) = \emptyset$, and the signature vectors of each component in subsystem C_1^+ are different from those in C_m^+ :

$$V_{E_{\ell}(\ell=1, \dots, j)} = [1 \ 0]^T \neq V_{E_{r}(r=k, \dots, n)} = [0 \ 1]^T$$

I

V Application

Consider for illustration the two coupled tanks depicted in figure 7a. The process consists of two tanks T_1 and T_2 connected by a valve V_1 . T_1 is filled by a pump modeled as source of flow $Sf:F_1$. The water flow Q_1 between tanks is measured by the sensor F_{1m} . The aim of the two tanks is to provide a continuous water flow to a consumer via the valve V_2 . P_0 represents atmospheric pressure. The pressures at the bottom of each tank are measured by sensors P_{1m} and P_{2m} respectively. The bond graph model in derivative causality is given by the figure 7b. The sensors are dualized into source of signal (SS). This imposed signal is the starting point for the elimination of unknown variables. Three ARRs

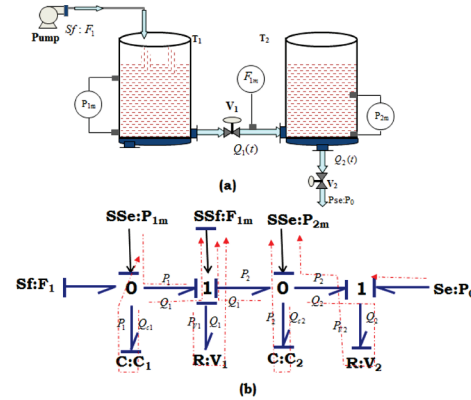


Fig. 7 : Hydraulic system (a) and its bond graph model

candidates can be generated from three over determined subsystems. They are deduced from followings junctions:

$$J0: F_1 - Q_{c1} - Q_1 = 0$$

$$J1: P_1 - P_{V1} - P_2 = 0$$

$$J0: Q_1 - Q_{c2} - Q_2 = 0$$

For example from junction "0" unknown variables Q_{c1} and Q_1 are eliminated from following causal paths shown by dashed lines (Fig.7b) leading to an oriented graph where different covered components can be fixed. Sequences of components that is sensible to given ARR are:

$$\begin{aligned}
COMP(ARR_1) &\subset \{F_1, P_{1m}, F_{1m}, C_1\} \\
COMP(ARR_2) &\subset \{P_{1m}, P_{2m}, F_{1m}, V_1\} \\
COMP(ARR_3) &\subset \{F_{1m}, P_{2m}, V_2, C_2\}
\end{aligned} \tag{6}$$

The set of global components to be monitored (named diagnosis candidates) is:

$$COMP(ARR) \in \{F_1, P_{1m}, C_1, F_{1m}, V_1, P_{2m}, C_2, V_2\}$$

Based on the BG model (as developed before), the hydraulic system can be decomposed into three over determined subsystems and one just determined subsystem (Fig.8) One note $card(C)=11$, $card(X)=8$, $card(C_m)=3$, $card(C_b)=4$.

Diagnosability analysis. From equations (6), can be formed the FSM (fig.9). If all components are diagnosis candidate (fixed by the technical specifications in industry), the degrees of isolability are :

$$\begin{aligned}
Card(COMP(ARR_1) \cap COMP(ARR_2)) &= 2 \\
Card(COMP(ARR_1) \cap COMP(ARR_3)) &= 1 \\
Card(COMP(ARR_2) \cap COMP(ARR_3)) &= 2
\end{aligned}$$

		x_1^+		x_2^+		x_3^+		x_4^0	
C/X		P_1	Q_{c1}	Q_1	P_{v1}	P_1	Q_{c2}	P_{v2}	Q_2
C_1^+	C_{j0}								
	C_{c1}								
	C_{p1m}								
C_2^+	C_{j1}								
	C_{v1}								
	C_{Q3m}								
C_3^+	C_{j0}								
	C_{c2}								
	C_{p2m}								
C_4^0	C_{j1}								
	C_{v2}								

Fig. 8: Canonical decomposition of the system

This degree of isolability can be improved if for instance the flow sensor F_{1m} is not diagnosis candidate (assuming that its reliability is high for instance): in this case (from Fig.10):

$$\begin{aligned}
Card(COMP(ARR_1) \cap COMP(ARR_2)) &= 1 \\
Card(COMP(ARR_1) \cap COMP(ARR_3)) &= 0 \\
Card(COMP(ARR_2) \cap COMP(ARR_3)) &= 1
\end{aligned}$$

The subsystems 3 and 1 become strongly isolable.

C/X	F_1	C_1	F_{1m}	P_{1m}	V_1	P_{2m}	C_2	V_2
$COMP(ARR_1)$	1	1	1	1				
$COMP(ARR_2)$			1	1	1	1		
$COMP(ARR_3)$			1			1	1	1

FIG. 9: FSM if all components are diagnosis candidate

C/X	F_1	C_1	F_{1m}	P_{1m}	V_1	P_{2m}	C_2	V_2
ARR1	1	1	1	1				
ARR2			1	1	1	1		
ARR3			1			1	1	1

FIG. 10 : FSM if F_{1m} is not a diagnosis candidate

IV. CONCLUSION

The detectability and isolability problem from bond graph model has been addressed in this paper. Dulmage-Mendelsohn decomposition can be directly deduced from the bond graph model. The degree of isolability of different subsystems can be then proposed for optimal design of online FDI algorithms. However, this structural decomposition which can help for instrumentation architecture has to be validated online. Further work concerns optimal sensor placement for diagnosability and reconfigurability analysis, where some duality between monitoring and control properties (observability, controllability) could be proposed.

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