Nonlinear Observer Design of a Diesel Engine model

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***Abstract*— This paper addresses the problem of designing a nonlinear observer for a general class of nonlinear systems in the presence of disturbances. Two criteria are combined in this approach : the modified H_∞ criterion and the modified mean value theorem. The first criterion is used to solve the problem of nonlinear observer design in the presence of disturbances. While the second allows to express the nonlinear error dynamics as a convex combination of known matrices with time varying coefficients. The sufficient conditions are derived and given in terms of linear matrix inequality. The advantage of the proposed method is that it can be applied to a wider class of nonlinear systems. Performances of the proposed approach are shown through the application to a diesel engine model.**

***Index Terms*— Diesel engine, LPV, H_∞, Nonlinear observers, LMI.**

I. Introduction

The problem of designing nonlinear observers has received great attention in literature. This problem is motivated by certain applications such as fault diagnosis and control system design. The challenge in the observer design is to construct an observer such that it can estimate the states of the considered systems asymptotically despite the presence of perturbations. In the continuous-time case, various state observation methods for nonlinear Lipschitz systems have been proposed, see for example [1], [2], [3], [4], [5], [6], [7], to name only a few. Other methods dedicated to the discrete-time case are presented in ([8], [9], [10], [11], [12]). In all these approaches, a small value of the Lipschitz constant of nonlinearities is required for ensuring the convergence of the observer. Otherwise, no observer gain exists. To overcome this problem, in [13] and [14] the authors have proposed a novel observation strategy for nonlinear systems whose nonlinearities have positives growths. In [15], the mean value theorem (MVT) is used to deal with the problem of nonlinear system having large Lipschitz constant. In [16], this result is extended to a more general nonlinear systems. However, this method is computationally demanding since the number of LMIs that should be solved depends on the number of nonlinearities and the dimension of the system. Recently, a solution is proposed in [17] for reducing the number of LMIs to be solved. This solution is based on the modified mean value theorem (MMVT) which is used to express the nonlinear error dynamics as a convex combination of known matrices with time varying coefficients.

Most of the proposed works in the observer design problem for a turbo-charged diesel engine with an Exhaust Gas Recirculation (EGR) and Variable Nozzle Turbine (VNT) were in the diagnosis framework. Model-based diagnosis of automotive engines has been considered in earlier papers (see e.g. [18] and [19]). However, the engines investigated in these works were all gasoline-fuelled and did not include EGR and VNT. Both these components make the diagnosis problem significantly more difficult since the air flow through the EGR-valve, and also the exhaust side of the engine have to be taken into account. An interesting approach to model-based air-path faults detection for an engine which includes EGR and VNT can be found in [20] and [21]. In [21] the authors propose an extended adaptive Kalman filter to find which faulty model best matches with measured data, then a structured hypothesis allows going back to the faults. Other approaches to detect intake leakages in diesel engines based on adaptive observers are proposed in [22] and [23] and recently in [24]. However, few results are addressed in the observation framework ([25], [26], [27], [28]). Indeed, most of them are based on the linearized model knowing that this model is a stiff system. Therefore, the obtained linearized model's A-matrix of this kind of systems is often ill-conditioned. In [29], an adaptive air charge observer is proposed for a turbocharged diesel engines using intake manifold pressure, temperature and engine speed measurements. These results are applied only with diesel engines without EGR. Two observers to estimate both the EGR flow and the burned gas rate (BGR) are proposed in [30]. Both observers are based on the output injection technique.

This work deals with the problem of nonlinear observer design for a general class of nonlinear systems. The considered class contains a linear parameter-varying (LPV) part and a nonlinear part having large Lipschitz constant. In order to solve the problem of observer design in the presence of disturbances, the known

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H_∞ criterion is used. Besides, the modified mean value theorem (MMVT) presented in [17] is used to express the nonlinear error dynamics as a convex combination of known matrices with time varying coefficients. The sufficient conditions are formulated in terms of linear matrix inequalities. Indeed, using the MMVT approach will drastically reduce the number of LMIs that should be solved. The proposed method is applicable to a wide class of class of nonlinear systems.

The notations used in this paper are quite standard. Let $\Re$ denotes the set of real numbers. The set of p by q real matrices is denoted as $\Re^{p\times q}$. A^T and A^{-1} represent the transpose of A and its left inverse (assuming A has full column rank), respectively. I_r represents the identity matrix of dimension r. $(\star)$ is used for the blocks induced by symmetry. $\|.\|$ represents the usual Euclidean norm.

II. Nonlinear observer design

In this section, inspired by [17] we will present a nonlinear observer (NO) design for the class of Lipschitz nonlinear systems given by (1). Indeed, the considered system represents a large class of nonlinear systems with a large or small Lipschitz constant. It is well known that most of the existing nonlinear observer design approaches for this class of systems fail to provide a solution when the Lipschitz constant is large. Recently, in [17] the authors have proposed an approach to overcome this problem based on a MMVT. The observer gain matrices are formulated in terms of LMIs.

Let us consider a general class of nonlinear systems described by the following equations

$$\dot{x} = \sum_{j=1}^{n_\rho} \rho_j A_j x + B_g g(\upsilon,y,u) + B_f f(x,u) + B_w w \quad (1a)$$

$$y = Cx + D_w w \quad (1b)$$

where $x \in \Re^{n_x}$ is the state vector, $u \in \Re^{n_u}$ is the control input vector, $y \in \Re^{n_y}$ is the output vector, $w \in \Re^{n_w}$ is the vector of disturbances and $\upsilon \in \Re^{n_\upsilon}$ is the vector of measurable signals. A_j, with $j = 1,\cdots,n_\rho$, B_g, B_f, B_w, D_w and C are constant matrices with appropriate dimensions. The functions $g(\upsilon,y,u)$ and $f(x,u)$ are nonlinear. The function $f(x,u)$ is assumed to be differentiable. The weighting functions ρ_j are assumed known and depend on measurable variables, and verify

$$\sum_{j=1}^{n_\rho} \rho_j = 1, \rho_j \geq 0, \quad \forall j \in \{1,\cdots,n_\rho\} \quad (2a)$$

The aim is to design a nonlinear observer which can reconstruct the state behavior despite the presence of the disturbances.

The considered observer for the system (1) is given by:

$$\dot{\hat{x}} = A(\rho)\hat{x} + B_g g(\upsilon,y,u) + B_f f(\hat{x},u) + L(\rho)(y-\hat{y}) \quad (3a)$$

$$\hat{y} = C\hat{x} \quad (3b)$$

with

$$A(\rho) = \sum_{j=1}^{n_\rho} \rho_j A_j, \quad L(\rho) = \sum_{j=1}^{n_\rho} \rho_j L_j \quad (4)$$

$\hat{x}$ represents the state estimation vector of x. The matrices $L(\rho)$ are the observer gains which must be determined such that $\hat{x}$ converges asymptotically to x. Notice that the index ρ is omitted where not necessary in order to keep the notation clear and easy to read.

By defining the state estimation error as $e(t) = \hat{x}(t) - x(t)$, the error dynamics can be expressed as

$$\dot{e} = (A - LC)e + B_f\tilde{f} + (LD_w - B_w)w \quad (5)$$

with $\tilde{f} = f(\hat{x},u) - f(x,u)$.

In order to minimize the effect of perturbations on the observer error, the H_∞ performance is used. Indeed, for nonlinear systems, the $\mathscr{L}_2$-norm can be used since it is considered as an extension of the H_∞-norm of linear systems.

The H_∞ estimation problem consists to compute the gain matrices L such that

$$\lim_{t\to\infty} e(t) = 0 \quad \text{for} \quad w(t) = 0 \quad (6a)$$

$$\|e\|_2 \leq \gamma\|w\|_2 \quad \text{for} \quad w(t) \neq 0;\ e(0) = 0 \quad (6b)$$

where $\gamma = \sup_{w(t)\in\mathscr{L}_2} \frac{\|e\|_2}{\|w\|_2}$.

Then to satisfy (6a)-(6b), it is sufficient to propose a Lyapunov function Υ, where Γ holds

$$\Gamma = \dot{\Upsilon} + e^T e - \gamma^2 w^T w < 0 \quad (7)$$

where $\Upsilon = e^T P e$ with P is a positive definite symmetric matrix. From (5), Γ is given by

$$\begin{aligned}\Gamma = {} & e^T\big((A-LC)^T P + P(A-LC)\big)e + e^T P B_f\tilde{f} + (B_f\tilde{f})^T Pe \\ & + e^T P(LD_w - B_w)w + w^T(LD_w - B_w)^T Pe - e^T e - \gamma^2 w^T w\end{aligned} \quad (8)$$

Before introducing our main result, let us present the MMVT [17] for a vector function :

Theorem 1: [17] Let the canonical basis of the vectorial spase $\Re^s$ for all $s \geq 1$ be defined by

$$E_s = \{e_s(i) \mid e_s(i) = (\underbrace{0,...,0,\overbrace{1}^{i\text{ th}},0,...,0}_{s\text{ components}})^T,\ i = 1\cdots s\} \quad (9)$$

Let $f(x) : \Re^n \to \Re^n$ be a vector function continuous on $[a, b] \in \Re^n$ and differentiable on convex hull of the set (a, b). For s_1, $s_2 \in [a, b]$, there exist δ_{ij}^{max} and δ_{ij}^{min} for $i = 1\cdots n$ and $j = 1\cdots n$ such that

$$\begin{aligned}f(s_2) - f(s_1) = {} & \left[\left(\sum_{i,j=1}^{n,n} H_{ij}^{\max}\delta_{ij}^{\max}\right) + \left(\sum_{i,j=1}^{n,n} H_{ij}^{\min}\delta_{ij}^{\min}\right)\right](s_2 - s_1) \\ & \delta_{ij}^{\max},\ \delta_{ij}^{\min} \geq 0, \delta_{ij}^{\max} + \delta_{ij}^{\min} = 1\end{aligned} \quad (10)$$

where

- $h_{ij}^{\max} \geq \max(\frac{\partial f_i}{\partial x_j})$ and $h_{ij}^{\min} \leq \min(\frac{\partial f_i}{\partial x_j})$
- $H_{ij}^{\max} = e_n(i) e_n^T(j) h_{ij}^{\max}$ and $H_{ij}^{\min} = e_n(i) e_n^T(j) h_{ij}^{\min}$.

■

The proof of this theorem is given in [17].

In our case, the nonlinear function f depends on the state vector x and also on the known input u. Although the previous theorem is applicable in our case without changes or modifications since u is bounded.

The following theorem gives sufficient conditions for the existence of matrices L.

Theorem 2: The observer error $e(t)$ converges asymptotically towards zero if there exists matrices $\bar{L}_k$, with $k = 1, \cdots, n_\rho$, a positive definite symmetric matrix P and a positif scalar μ such that the following LMIs are satisfied

$$P > 0 \tag{11a}$$

$$\begin{bmatrix} \Xi_{ijk}^{\max} & \bar{L}_k D_w - P B_w \\ (*) & -\mu I \end{bmatrix} < 0 \tag{11b}$$

$$\begin{bmatrix} \Xi_{ijk}^{\min} & \bar{L}_k D_w - P B_w \\ (*) & -\mu I \end{bmatrix} < 0 \tag{11c}$$

$\forall i = 1, \cdots, n,\ j = 1, \cdots, n$ and $k = 1, \cdots, n_\rho$, where

$$\Xi_{ijk}^{\max} = \left(A_k + B_f \bar{H}_{ij}^{\max}\right)^T P + P\left(A_k + + B_f \bar{H}_{ij}^{\max}\right) - C^T \bar{L}_k^T - \bar{L}_k C + I$$

$$\Xi_{ijk}^{\min} = \left(A_k + + B_f \bar{H}_{ij}^{\min}\right)^T P + P\left(A_k + + B_f \bar{H}_{ij}^{\min}\right) - C^T \bar{L}_k^T - \bar{L}_k C + I$$

$$\bar{H}_{ij}^{\max} = Z_H H_{ij}^{\max}, \quad \bar{H}_{ij}^{\min} = Z_H H_{ij}^{\min}$$

with $Z_H = n \times n$. Solving LMIs (11a) -(11c) leads to determinate matrices P and $\bar{L}_k$. The matrices L_k, with $k = 1, \cdots, n_\rho$, can be obtained from $L_k = P^{-1} \bar{L}_k$.

■

Proof: Using theorem 1, the nonlinear term $\tilde{f}$ can be written as

$$\tilde{f} = \left[\left(\sum_{i,j=1}^{n,n} H_{ij}^{\max} \delta_{ij}^{\max} \right) + \left(\sum_{i,j=1}^{n,n} H_{ij}^{\min} \delta_{ij}^{\min} \right) \right] e \tag{12}$$

$$\delta_{ij}^{\max},\ \delta_{ij}^{\min} \geq 0, \delta_{ij}^{\max} + \delta_{ij}^{\min} = 1 \tag{13}$$

To simplify the form of the final result as in [17], the term $\sum_{i=1}^{n,n} (\delta_{ij}^{\max} + \delta_{ij}^{\min})$ must be scaled to one. If all the terms in $\frac{\partial f_i}{\partial x_j}$ are not zero, then the scaling factor Z_H can be computed by

$$\sum_{i=1}^{n,n} \left(\delta_{ij}^{\max} + \delta_{ij}^{\min} \right) = n \times n = Z_H, \frac{\sum_{i=1}^{n,n} \left(\delta_{ij}^{\max} + \delta_{ij}^{\min} \right)}{Z_H} = 1$$

Then, (12) is rewritten as

$$\tilde{f} = \left[\left(\sum_{i,j=1}^{n,n} \bar{H}_{ij}^{\max} \bar{\delta}_{ij}^{\max} \right) + \left(\sum_{i,j=1}^{n,n} \bar{H}_{ij}^{\min} \bar{\delta}_{ij}^{\min} \right) \right] e \tag{14}$$

$$\bar{\delta}_{ij}^{\max},\ \bar{\delta}_{ij}^{\min} \geq 0, \bar{\delta}_{ij}^{\max} + \bar{\delta}_{ij}^{\min} = \frac{1}{Z_H}, \sum_{i=1}^{n,n} \left(\bar{\delta}_{ij}^{\max} + \bar{\delta}_{ij}^{\min} \right) = 1$$

where

- $\bar{H}_{ij}^{\max} = Z_H H_{ij}^{\max}$ and $\bar{H}_{ij}^{\min} = Z_H H_{ij}^{\min}$;
- $\bar{\delta}_{ij}^{\max} = \delta_{ij}^{\max} / Z_H$ and $\bar{\delta}_{ij}^{\min} = \delta_{ij}^{\min} / Z_H$.

Based on (14), the function Γ becomes

$$\Gamma = \begin{bmatrix} e^T \\ w^T \end{bmatrix} \begin{bmatrix} \Lambda & P(LD_w - B_w) \\ (*) & -\mu I \end{bmatrix} \begin{bmatrix} e^T \\ w^T \end{bmatrix}$$

where $\mu = \gamma^2$ and

$$\Lambda = \sum_{i=1}^{n,n} \bar{\delta}_{ij}^{\max} \left(\left(A - LC + B_f \bar{H}_{ij}^{\max}\right)^T P + P\left(A - LC + B_f \bar{H}_{ij}^{\max}\right) + I \right) + \sum_{i=1}^{n,n} \bar{\delta}_{ij}^{\min} \left(\left(A - LC + B_f \bar{H}_{ij}^{\min}\right)^T P + P\left(A - LC + B_f \bar{H}_{ij}^{\min}\right) + I \right)$$

the negativity of Γ is ensured if

$$\sum_{i,j=1}^{n,n} \bar{\delta}_{ij}^{\max} \underbrace{\begin{bmatrix} \Xi_{ij}^{\max} & P(LD_w - B_w) \\ (*) & -\mu I \end{bmatrix}}_{M_{ij}^{\max}} + \sum_{i,j=1}^{n,n} \bar{\delta}_{ij}^{\min} \underbrace{\begin{bmatrix} \Xi_{ij}^{\min} & P(LD_w - B_w) \\ (*) & -\mu I \end{bmatrix}}_{M_{ij}^{\min}} < 0 \tag{15}$$

where

$$\Xi_{ij}^{\max} = \left(A - LC + B_f \bar{H}_{ij}^{\max}\right)^T P + P\left(A - LC + B_f \bar{H}_{ij}^{\max}\right) + I$$

$$\Xi_{ij}^{\min} = \left(A - LC + B_f \bar{H}_{ij}^{\min}\right)^T P + P\left(A - LC + B_f \bar{H}_{ij}^{\min}\right) + I$$

Since we have $\bar{\delta}_{ij}^{\max},\ \bar{\delta}_{ij}^{\min} \geq 0$, the previous equation is equivalent to

$$M_{ij}^{\max} < 0 \quad \text{and} \quad M_{ij}^{\min} < 0 \tag{16}$$

By substituting the matrices A and L by their expressions, (16) can be written as follows

$$\sum_{k=1}^{n_\rho} \rho_k M_{ijk}^{\max} < 0 \quad \text{and} \quad \sum_{k=1}^{n_\rho} \rho_k M_{ijk}^{\min} < 0 \tag{17}$$

where

$$M_{ijk}^{\max} = \begin{bmatrix} \Xi_{ijk}^{\max} & P(L_k D_w - B_w) \\ (*) & -\mu I \end{bmatrix}$$

$$M_{ijk}^{\min} = \begin{bmatrix} \Xi_{ijk}^{\min} & P(L_k D_w - B_w) \\ (*) & -\mu I \end{bmatrix}$$

with

$$\Xi_{ijk}^{\max} = \left(A_k - L_k C + B_f \bar{H}_{ij}^{\max}\right)^T P + P\left(A_k - L_k C + B_f \bar{H}_{ij}^{\max}\right) + I$$

$$\Xi_{ijk}^{\min} = \left(A_k - L_k C + B_f \bar{H}_{ij}^{\min}\right)^T P + P\left(A_k - L_k C + B_f \bar{H}_{ij}^{\min}\right) + I$$

Since $\rho_k > 0, \quad \forall k = 1, \cdots, n_\rho$, the condition (17) holds if

$$M_{ijk}^{\max} < 0 \quad \text{and} \quad M_{ijk}^{\min} < 0 \tag{18}$$

$\forall i = 1, \cdots, n,\ j = 1, \cdots, n$ and $k = 1, \cdots, n_\rho$.

For obtaining the LMIs (11a)-(11c), it is sufficient to take $\bar{L}_k = PL_k$, and this ends the proof. □

Notice that if there exist terms of $\frac{\partial f_i}{\partial x_j} = 0$, then the scaling factor $\sum_{i=1}^{n,n}(\delta_{ij}^{\max} + \delta_{ij}^{\min})$ is less than 1. Consequently, the scaling factor $\bar{Z}_H$ must be redefined as follows

$$\bar{Z}_H = \sum_{i=1}^{n,n}\left(\delta_{ij}^{\max} + \delta_{ij}^{\min}\right) = n \times n - n_0, \frac{\sum_{i=1}^{n,n}\left(\delta_{ij}^{\max} + \delta_{ij}^{\min}\right)}{\bar{Z}_H} = 1$$

where n_0 is number of terms in $\frac{\partial f_i}{\partial x_j}$ that equals zero.

III. Engine description

The diesel engine considered in this paper is a four-cylinder engine with a high-pressure exhaust gas recirculation circuit (EGR) and a Variable Geometry Turbocharger (VGT) as described in [31]. A principle illustration scheme is shown in Fig .1. The air path

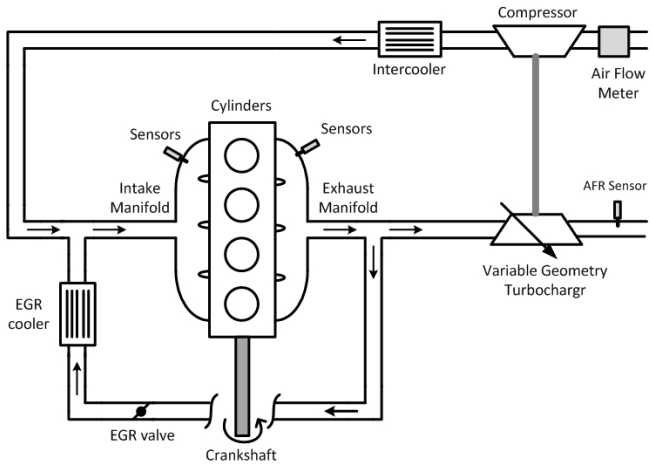

Fig. 1. A schematic picture of air-intake system

system consists of two parts : The turbocharger and Exhaust Gas Recirculation (EGR). The turbocharger is a turbine driven by the exhaust gas and connected via a common shaft to the compressor, which compresses the air in the intake. The exhaust gas recirculation (EGR) allows to recirculating gas from the exhaust manifold to the intake manifold. The recirculation of the exhaust gas through an EGR valve into the intake manifold where it dilutes the incoming fresh air is a well-established and efficient means of reducing in-cylinder NO_x emissions.

The mean value engine modelling approach is the most considered approach in the literature [32]. It uses temporal and spatial averages of relevant temperatures, pressures and mass flow rates. In this work, the used model is based on principles described in ([33], [19]). Since the mass-flow and the temperature after the CAC are already known variables because they are measured by the production sensors, it is not necessary to include the compressor and the CAC in the model as shown in [19].

The engine is equipped with sensors measuring in-flowing air W_{HFM}, the temperature after CAC T_{CAC}, inlet-manifold pressure P_{Inlet}, exhaust-pressure P_{Exh} and exhaust-temperature T_{Exh}. The control inputs are the injected fuel W_{Fuel}, the turbine vane position X_{VNT}, and the EGR-valve position determining the valve opening-area A_{EGR}. Notice that the following variables : T_{CAC}, W_{HFM}, N_{Eng} and T_{Exh} are considered as measurable signals.

The considered model is expressed as follows:

$$\begin{aligned}
\dot{P}_{Inlet} &= \frac{1}{V_{Inlet}}\left(\frac{R_{Air}c_{p,Air}}{c_{v,Air}}W_{HFM}T_{CAC} + \frac{R_{Exh}c_{p,Exh}}{c_{v,Exh}}W_{EGR}T_{EGR} - \frac{R_{Inlet}c_{p,Inlet}}{c_{v,Inlet}}W_{Inlet}T_{Inlet}\right) && (19)\\
\dot{m}_{Air} &= W_{HFM} - \frac{m_{Air}}{m_{Air}+m_{EGR}}W_{Inlet} && (20)\\
\dot{m}_{EGR} &= W_{EGR} - \frac{m_{EGR}}{m_{Air}+m_{EGR}}W_{Inlet} && (21)\\
\dot{m}_{Exh} &= W_{Exh} - W_{Turb} - W_{EGR} && (22)
\end{aligned}$$

with

$$\begin{aligned}
W_{EGR} &= \frac{A_{EGR}P_{Exh}}{\sqrt{R_{Exh}T_{Exh}}}\Psi_{\kappa_{Exh}}\left(\frac{P_{Inlet}}{P_{Exh}}\right) && (23)\\
\Psi\left(\frac{p_1}{p_0}\right) &= \begin{cases} \sqrt{\frac{2\kappa}{\kappa-1}\left\{\left(\left(\frac{p_1}{p_0}\right)^{\frac{2}{\kappa}} - \left(\frac{p_1}{p_0}\right)^{\frac{\kappa+1}{\kappa}}\right)\right\}} \\ if\ \left(\frac{p_1}{p_0}\right) \geq \left(\frac{2}{\kappa+1}\right)^{\frac{\kappa}{\kappa-1}} \\ \sqrt{\kappa\left(\frac{2}{\kappa+1}\right)^{\frac{\kappa+1}{\kappa-1}}}\ otherwise, \end{cases} && (24)\\
W_{Inlet} &= f_{vol}\left(N_{eng}, \frac{P_{Inlet}}{T_{Inlet}R_{Inlet}}\right)\frac{N_{eng}P_{Inlet}}{T_{Inlet}R_{Inlet}}\frac{V_{Eng}}{120} && (25)\\
T_{Inlet} &= \frac{P_{Inlet}V_{Inlet}}{(m_{Air}+m_{EGR})R_{Inlet}} && (26)\\
T_{EGR} &= \left(\frac{P_{Inlet}}{P_{Exh}}\right)^{\frac{\kappa_{Exh}-1}{\kappa_{Exh}}}T_{Exh} && (27)\\
W_{Exh} &= W_{Inlet} + W_{Fuel} && (28)\\
T_{Exh} &= T_{Inlet} + \frac{Q_{LHV}h(W_{Fuel}, N_{Eng})}{c_{p,Exh}(W_{Inlet}+W_{Fuel})} && (29)\\
P_{Exh} &= \frac{m_{Exh}T_{Exh}}{V_{Exh}} && (30)\\
W_{Turb} &= \frac{P_{Exh}}{\sqrt{T_{Exh}}}\tau\left(\frac{P_{Exh}}{P_{Atm}}, X_{VNT}\right) && (31)\\
R_{Inlet} &= \frac{R_{Air}m_{Air} + R_{Exh}m_{EGR}}{m_{Air}+m_{EGR}} && (32)\\
c_{v,Inlet} &= \frac{c_{v,Air}m_{Air} + c_{v,Exh}m_{EGR}}{m_{Air}+m_{EGR}} && (33)\\
c_{p,Inlet} &= c_{v,Inlet} + R_{Inlet} && (34)
\end{aligned}$$

where, P_{Inlet} is the pressure in intake manifold. m_{Air} and m_{EGR} are respectively the mass of air and EGR−gaz in

intake manifold. m_{Exh} represents the mass of exhaust gas in exhaust manifold. The other variables and parameters used in this model are listed in Table I in the appendix. The temperature T_{EGR} is assumed to be constant and equal to 329.436K. The static functions : f_{vol}, h and τ are represented as interpolation in lookup tables.

The system (19)-(22) can be written in state space form as (1), where the state, known input and output vectors and the variables ρ and υ are defined as

$$x = \begin{bmatrix} P_{Inlet} & m_{Air} & m_{EGR} & m_{Exh} \end{bmatrix}^T \quad (35)$$

$$u = \begin{bmatrix} A_{EGR} & X_{VNT} & W_{Fuel} \end{bmatrix}^T \quad (36)$$

$$y = \begin{bmatrix} P_{Inlet} & P_{Exh} \end{bmatrix}^T \quad (37)$$

$$\rho = N_{Eng} \quad (38)$$

$$\upsilon = \begin{bmatrix} T_{CAC} & W_{HFM} \end{bmatrix}^T \quad (39)$$

The matrices A_1 and A_2 are given by :

$$A_1 = \begin{bmatrix} -\frac{f_{vol}\underline{N}_{Eng}V_{Eng}}{120V_{Inlet}} & 0 & 0 & 0 \\ 0 & -\frac{f_{vol}\underline{N}_{Eng}V_{Eng}}{120V_{Inlet}} & 0 & 0 \\ 0 & 0 & -\frac{f_{vol}\underline{N}_{Eng}V_{Eng}}{120V_{Inlet}} & 0 \\ 0 & \frac{f_{vol}\underline{N}_{Eng}V_{Eng}}{120V_{Inlet}} & \frac{f_{vol}\underline{N}_{eng}V_{Eng}}{120V_{Inlet}} & 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -\frac{f_{vol}\overline{N}_{Eng}V_{Eng}}{120V_{Inlet}} & 0 & 0 & 0 \\ 0 & -\frac{f_{vol}\overline{N}_{Eng}V_{Eng}}{120V_{Inlet}} & 0 & 0 \\ 0 & 0 & -\frac{f_{vol}\overline{N}_{Eng}V_{Eng}}{120V_{Inlet}} & 0 \\ 0 & \frac{f_{vol}\overline{N}_{Eng}V_{Eng}}{120V_{Inlet}} & \frac{f_{vol}\overline{N}_{eng}V_{Eng}}{120V_{Inlet}} & 0 \end{bmatrix}$$

where $\underline{N}_{Eng}$ and $\overline{N}_{Eng}$ are, respectively, the minimum and maximum value of the measurable variable N_{Eng}. ρ_1 and ρ_2 are defined as :

$$\rho_1 = \frac{\overline{N}_{Eng} - N_{Eng}}{\overline{N}_{Eng} - \underline{N}_{Eng}} \quad \text{and} \quad \rho_2 = \frac{N_{Eng} - \underline{N}_{Eng}}{\overline{N}_{Eng} - \underline{N}_{Eng}} \quad (40)$$

The matrices B_g, B_f, B_w and C are expressed as :

$$B_g = \begin{bmatrix} \frac{R_{Air}c_{p,Air}}{c_{v,Air}V_{Inlet}} & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, B_f = \begin{bmatrix} -\frac{f_{vol}V_{Eng}N_{Eng}}{120V_{Inlet}} & \frac{R_{Exh}^2 T_{EGR}c_{p,Exh}}{V_{Inlet}c_{v,Exh}V_{Exh}} & 0 \\ 0 & 0 & 0 \\ 0 & \frac{R_{Exh}}{V_{Exh}} & 0 \\ 0 & \frac{-R_{Exh}}{V_{Exh}} & 1 \end{bmatrix} \quad (41)$$

$$B_w = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{R_{Exh}T_{Exh}^{moy}}{V_{Exh}} \end{bmatrix} \quad (42)$$

with T_{Exh}^{moy} is the mean value of measurable variable T_{Exh}. This approximation is chosen in order to decrease the number of variables to be used in the observer design for decreasing the number of LMIs to solve, which is important from a practical point of view. Indeed, the general case when the output equation is written by

$$y = \sum_{j=1}^{n_{\bar{\rho}}} \bar{\rho}_j C_j x$$
$$= Cx + \sum_{j=1}^{n_{\bar{\rho}}} \bar{\rho}_j (C_j - C) x = Cx + D_w w$$

with $D_w = \begin{bmatrix} 0 & 1 \end{bmatrix}^T$. Hence, w is considered as perturbation which can be rejected using H_∞ performance.

The functions g and f are given by :

$$g = \begin{bmatrix} W_{HFM}T_{CAC} & W_{HFM} \end{bmatrix}^T$$

$$f = \begin{bmatrix} \frac{R_{Inlet}P_{Inlet}}{c_{v,Inlet}} \\ \frac{A_{EGR}m_{Exh}\Psi_{\kappa_{Exh}}T_{Exh}}{\sqrt{R_{Exh}T_{Exh}}} \\ W_{Fuel} - \frac{R_{Exh}T_{Exh}m_{Exh}\tau\left(\frac{P_{Exh}}{P_{Atm}}, X_{VNT}\right)}{V_{Exh}\sqrt{T_{Exh}}} \end{bmatrix}$$

IV. Simulation results

The proposed approach has been tested on a four-cylinder diesel engine model running on Mathworks Matlab®/Simulink.

From diesel engine model simulations, we have :

$$h_{11}^{max} = 0.3604, h_{12}^{max} = 4.85 \times 10^5, h_{13}^{max} = -3.72 \times 10^6,$$
$$h_{24}^{max} = 3.5 \times 10^{-5}, h_{34}^{max} = -40.5$$
$$h_{11}^{min} = 0.3602, h_{12}^{min} = 4.08 \times 10^5, h_{13}^{min} = -3.74 \times 10^6$$
$$h_{24}^{min} = 2.5 \times 10^{-5}, h_{34}^{min} = -41. \quad (43)$$

the rest of h_{ij}^{max} and h_{ij}^{min} are zeros. The scalar factor Z_H is equal to 5 $(3 \times 4 - 7)$, where 7 is the number of $\frac{\partial f_i}{\partial x_j}$ equal to zero. We obtain, after solving the LMIs (11a)-(11c) using YALMIP, a toolbox for modeling and optimization in Matlab®, the following results :

$$P = \begin{bmatrix} 4 \times 10^{-7} & 0 & 0 & -3.13 \times 10^{-7} \\ 0 & 0.439 & -0.00035 & -0.0046 \\ 0 & -0.00035 & 0.4915 & -2.673 \times 10^{-5} \\ -3.13 \times 10^{-7} & -0.0046 & -2.673 \times 10^{-5} & 9.80 \times 10^{-5} \end{bmatrix}$$

$$L_1 = L_2 = \begin{bmatrix} 1.7019 \times 10^8 & 0.045 \\ 12093.9 & 0.00013 \\ 71.14 & 7.56 \times 10^{-7} \\ 0.0115 \times 10^8 & 0.0122 \end{bmatrix}$$

The value of the attenuation of disturbance term is chosen as $\gamma = 0.1$.

The initial conditions for the system and the observer are chosen as :

$$x_0 = \begin{bmatrix} 1.7 \times 10^5 & 0.7 \times 10^{-3} & 1 \times 10^{-4} & 10^{-4} \end{bmatrix}^T$$
$$\hat{x}_0 = \begin{bmatrix} 1.8 \times 10^5 & 0.5 \times 10^{-3} & 3 \times 10^{-3} & 2 \times 10^{-4} \end{bmatrix}^T$$

The engine speed behavior is illustrated in figure 2. The minimum and maximum value of N_{eng} ($\underline{N}_{eng}$ and $\overline{N}_{eng}$) are : 2500 and 3500rpm respectively.

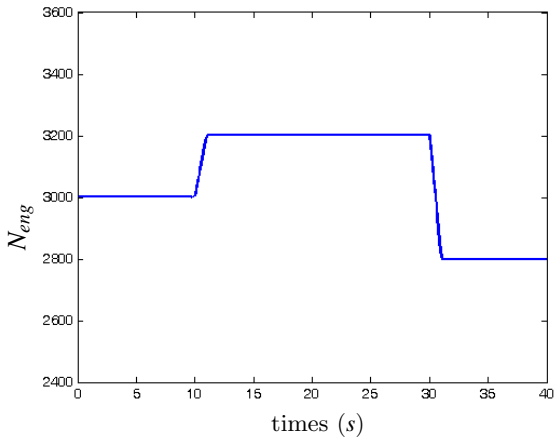

Fig. 2. Engine speed behavior

The output estimation errors are shown in Fig.3. It is clear that the proposed method provides good estimates of the system outputs since the output estimation errors reach rapidly to zero. One can also notice that the observer is insensitive to the presence of perturbations w.

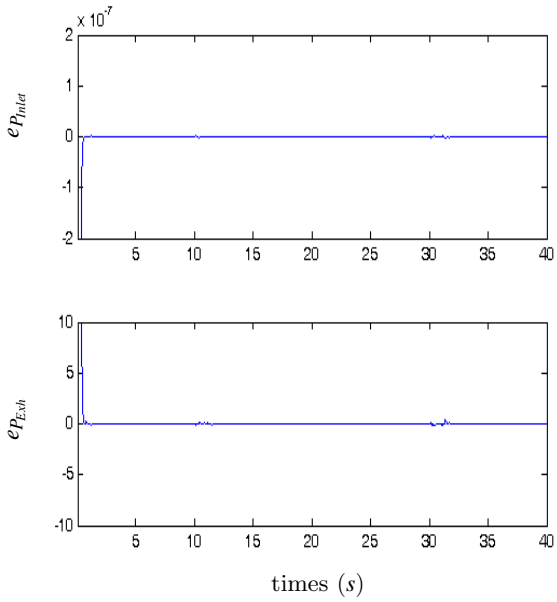

Fig. 3. Output estimation errors

V. Conclusion

A nonlinear observer design of a diesel engine model is presented in this work. This observer is developed for a general class of nonlinear systems with a locally or globally bounded jacobian. Sufficient conditions are given in terms of matrix inequalities formulation. The modified mean value theorem is used to express the nonlinear error dynamics as a convex combination of known matrices with time varying coefficients. The H_∞ performance is used to attenuate the disturbances. The efficiency of the proposed approach is shown through the application to a diesel engine model. An evaluation of the proposed approach using a professional diesel engine simulator AMEsim(LMS) is under work.

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Appendix I
Nomenclature

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Symb.	Quantity	Unit
P_{Inlet}	Pressure in intake manifold	Pa
V_{Inlet}	Volume of intake manifold	m^3
R_{Air}	Gas constant of air	$J.(Kg.K)$
$c_{p,Air}$	Specific heat at const. pres. of air	$J.(Kg.K)$
$c_{v,Air}$	Specific heat at const. vol. of air	$J.(Kg.K)$
R_{Exh}	Gas constant of Exhaust gas of Exhaust gas	$J.(Kg.K)$
$c_{p,Exh}$	Specific heat at const. pres. of Exhaust gas	$J.(Kg.K)$
$c_{v,Exh}$	Specific heat at const. vol. of Exhaust gas	$J.(Kg.K)$
R_{Inlet}	Gas constant in intake manifold	$J.(Kg.K)$
$c_{p,Inlet}$	Specific heat at const. pres. in intake manifold	$J.(Kg.K)$
$c_{v,Inlet}$	Specific heat at const. vol. in intake manifold	$J.(Kg.K)$
κ	Ratio of specific heats	c_p/c_v
W_{HFM}	Air mass-flow past the air mass-flow sensor	$kg.s^{-1}$
T_{CAC}	Temperature of the air after the charge-air cooler	K
W_{EGR}	EGR mass-flow into intake manifold	$Kg.s^{-1}$
T_{EGR}	Temperature of EGR gas-flow into the i.m	K
W_{Inlet}	Mass-flow into engine inlet -ports	$kg.s^{-1}$
T_{Inlet}	Temperature in the intake manifold	K
m_{Air}	Mass of air in intake manifold	Kg
m_{EGR}	Mass of EGR-gas in intake manifold	kg
W_{Exh}	Exhaust mass-flow into the exhaust manifold	$kg.s^{-1}$
m_{Exh}	Mass of exhaust gas in exhaust manifold	kg
P_{Exh}	Pressure in exhaust manifold	Pa
T_{Exh}	Temperature in exhaust manifold	K
A_{EGR}	Effective area of EGR valve	m^2
V_{Eng}	Engine displacement	m^3
N_{Eng}	Engine speed	min^{-1}
W_{Fuel}	Mass-flow of injected fuel	$kg.s^{-1}$
Q_{LHV}	Lower Heating Value	$J.kg^{-1}$
V_{Exh}	Volume of exhaust manifold	m^3
p_{Atm}	Atmospheric pressure	Pa
W_{Turb}	Exhaust mass-flow past the turbine	$kg.s^{-1}$
X_{VNT}	Position of VNT vanes	%

TABLE I

THE VARIABLES USED IN THE ENGINE MODEL

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