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Sliding Mode Based On-line Drag Estimation of a Simulated Aircraft Over a Wireless Network

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Abstract— **On-line parameter estimation over a wireless network based on sliding modes for a high fidelity simulation of a four engine transport aircraft is described in this paper. An approach using a sliding mode observer involving a second-order sliding mode 'super-twisting' injection signal to estimate changes in the drag coefficient is proposed. A peer-to-peer wireless network is considered in which one of the computers generates data from a high fidelity four engine transport aircraft model in real-time, which represents a 'virtual transport aircraft', and the other computer processes each data package to carry out on-line estimation of the drag coefficient.**

I. INTRODUCTION

The state vector required for implementing a state-feedback control law may be unavailable in practice. This lack of measurable system states led to the problem of reconstructing state variables. An observer (also called estimator or filter) is a dynamical system, generally model-based, which was initially intended for estimating states using only input and output signals. Classical estimators such as Kalman filters [11] and Luenberger observer [17], high-gain observers [9], adaptive observers [21] and sliding mode observers [1] [5] have been the main techniques applied to the state estimation problem. A sliding mode observer (SMO) is a nonlinear observer employing a discontinuous injection signal which guarantees convergence of the output error vector to zero in finite time, and is insensitive with respect to a particular class of uncertainties.

The potential of using observers for monitoring systems, *e.g.* for fault-diagnosis [10], and estimation of parameters [6] [8] [13], has also been exploited. State-estimation over wireless networks using Kalman filters has been studied, see for example [22] [23] [24] [26]. Typical wireless applications have focussed on sensor networks and actuators for data acquisition and control. For example, monitoring and control of systems in process and manufacturing industries, structural health monitoring, traffic control, health care, military applications, agricultural monitoring, pipeline monitoring, and so on [4]. The main advantages of wireless networks rely mainly on their functionality and economic features [16]. In other words, a wireless network can be implemented in cases were conventional access of wired networks is not possible, extra nodes can be included in a relatively easy way, and low cost implementation since a large number of sensors and actuators can be integrated with no need of wiring each node of the network.

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In this paper, an on-line parameter estimation approach based on sliding modes for a high fidelity simulation of a four engine transport aircraft is proposed. This approach uses a sliding mode observer involving a second-order sliding mode 'super-twisting' injection signal to estimate any change in the drag coefficient. Parametric estimation is carried out over a wireless network architecture involving two computers. A peer-to-peer wireless network is considered in which one of the computers generates data from a high fidelity four engine transport aircraft model in real-time, which represents a 'virtual transport aircraft', and the other computer processes each data package to carry out on-line estimation of the drag coefficient. A conceptual description of the hardware and software implementation of the on-line drag estimation approach is also presented.

This paper is structured as follows: a mathematical model of the longitudinal motion of a four engine transport aircraft is described in Section II. The hardware and software configuration implemented in this work is presented in Section III. Then, the sliding mode schemes applied in this paper are described in Section IV. This comprises the following parts: (i) theoretical fundamentals regarding sliding mode differentiation, and (ii) drag estimation via a sliding mode observer. Computer simulation results obtained over a wireless network are presented in Section V. Concluding remarks are drawn at the end of the paper in Section VI.

II. AIRCRAFT MATHEMATICAL MODEL: LONGITUDINAL MOTION

In a moving aircraft four forces and three moments are present. The forces are thrust, drag, lift and weight, whilst the moments are related to the roll, yaw and pitching axes. The description of these forces and moments depends upon the axes system adopted when dealing with the construction of a mathematical model of an aircraft [3]. In this work, the body-axes system is considered.

Throughout this paper, the following assumptions, describing the flight conditions, are made:

A.1 Cruise, steady-state, straight ($\gamma = 0$) and level flight at an altitude of 10058 m (33000 ft) and speed of 220 ms^{-1} (428 kn).

A.2 The weight of the aircraft is 263 tons and the centre of gravity is 25% MAC.

A nonlinear mathematical model of a four engine transport aircraft corresponding to the body-axes longitudinal motion (not including flexible modes or wind effects) has been considered in this paper [18] [19]. Longitudinal motion control involves a movable horizontal stabiliser together with two inboard and two outboard elevators (these elevators move together in normal operation) and four engines for producing the thrust force. As stated in [12], the altitude state h is weakly coupled with the other state variables through its effect on the aerodynamic coefficients due to the air mass density and sound speed. If the air mass density and speed of sound are assumed constant, in the neighbourhood of a specific altitude under consideration, a fourth order nonlinear model, describing the longitudinal dynamics, can be established by neglecting the state equation corresponding to the altitude. The longitudinal dynamics of the transport aircraft are governed by the following nonlinear differential equations [19]:

$$\dot{V}_{TAS} = -\frac{pS_{ref}}{m}C_D + g\sin(\alpha-\theta) + \frac{\tau_n}{m}\cos(\alpha+\sigma_T) \tag{1}$$

$$\dot{\alpha} = \frac{1}{m+\frac{\rho C_{L\dot{\alpha}} S_{ref}\bar{c}}{2}}\Big(-\frac{\rho V_{TAS} S_{ref}}{2}C_L + \frac{mg\cos(\alpha-\theta)}{V_{TAS}} - \frac{1}{V_{TAS}}\tau_n \sin(\alpha+\sigma_T) + mq\Big) \tag{2}$$

$$\dot{\theta} = q \tag{3}$$

$$\dot{q} = \frac{pS_{ref}}{I_{yy}}\Big(\bar{c}C_m - \big(C_D\sin\alpha + C_L\cos\alpha\big)\bar{x}_{CG} + \frac{\bar{c}^2\dot{\alpha}}{V_{TAS}}\big(C_{m\dot{\alpha}} - \frac{\bar{x}_{CG}}{\bar{c}}C_{L\dot{\alpha}}\cos\alpha\big)\Big) + \frac{\tau_{nz_{eng}}}{I_{yy}} \tag{4}$$

where

$$p \triangleq \frac{\rho V_{TAS}^2}{2} \tag{5}$$

and

$$\tau_n \triangleq \sum_{j=1}^{4} T_{nj} \quad , \quad \tau_{nz_{eng}} \triangleq \sum_{j=1}^{4} T_{nj} z_{eng_j} \; . \tag{6}$$

The true air speed, the angle of attack, the pitch angle, and the pitch rate are denoted by V_{TAS}, α, θ, and q respectively. Furthermore, the terms $\bar{x}_{CG} = x_{CGref} - x_{CG}$ where x_{CGref} and x_{CG} are the reference and actual position of the centre of gravity along the datum axis x [25].

Note that the above equations are functions of the dimensionless aerodynamic coefficients C_D, C_L and C_m, as well as the reference area S_{ref}, the wing chord $\bar{c}$, and the position of the j-th engine w.r.t. the z axis z_{eng_j}. The aerodynamic coefficients for the longitudinal motion can be found in [20] and further details are offered in [19].

The following assumptions have been made:

A.3 The total thrust force τ_n is known or can be computed/estimated.

A.4 Each engine produces identical thrust, *i.e.*

$$T_{n1} = T_{n2} = T_{n3} = T_{n4} \; . \tag{7}$$

III. ON-LINE DRAG ESTIMATION OVER A WIRELESS NETWORK

A communication network requires hardware and software elements. These have to be integrated and configured in order to establish data transmission between each component. Here a peer-to-peer wireless network is considered in which one of the computers generates data from a high fidelity aircraft model (to create a virtual aircraft) and the other processes each data package to carry out on-line estimation of the drag coefficient. In this section, a brief description of the hardware and software components are given, including the data-flow of the entire system.

The hardware and software components involved in the on-line parameter estimation of the drag coefficient of a simulated high fidelity model of a transport aircraft are sumarised in the sequel. Two computers equipped with intel core duo processors and a joystick have been used (see Figure 1). Both computers have a 32-bit Windows operating system. Matlab/Simulink and the FlightGear Flight Simulator (FGFS) [2] software are used in the real time computer simulations via wireless communication. The aircraft simulation comprises a 6-DOF dynamic model including sensors and actuators, and PID controllers deployed under the Matlab/Simulink software platform [19]. FGFS can be configured to receive input signals generated by the model running in Matlab/Simulink to create a visualization of the scenario.

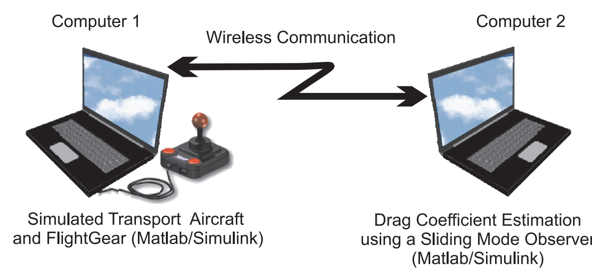


Fig. 1. Wireless Network Components

The aircraft Matlab/Simulink model runs on Computer 1 along with FGFS. A joystick is connected via USB to this computer in order to change the pitch angle and also to incite changes in the drag coefficient. A sliding mode observer, employed for estimating the change in the drag coefficient, runs on Computer 2 in real time under Matlab/Simulink. Furthermore, Computer 2 runs a Matlab/Simulink graphical user interface (GUI) comprising a dial for showing the drag reduction (in percentage terms) and two displays to show the pitch rate and the altitude of the aircraft.

Blocks from the Instrumentation Toolbox of Matlab/Simulink are used in order to transmit data from Computer 1 to Computer 2 via a wireless network. Transmission of data from the aircraft dynamic model to FGFS is achieved by using blocks from the aerospace toolbox of Matlab/Simulink. The model is configured to run in real time to provide real time transmission of the 'sensor' data for the estimation scheme in Computer 2. Figure 2 illustrates the flow of data between the software components and the computers. The data vectors d_1, d_2 and d_3 are given by

$$d_1 = \begin{bmatrix} q & V_{TAS} & \alpha & \theta & h & m & \rho & g & \delta_e \end{bmatrix}^{\mathrm{T}} \quad (8)$$

$$d_2 = \begin{bmatrix} l & \mu & h & \phi & \theta & \psi \end{bmatrix}^{\mathrm{T}} \quad (9)$$

$$d_3 = \begin{bmatrix} D_{reduction} & q & h \end{bmatrix}^{\mathrm{T}} \quad (10)$$

where q is the pitch rate [deg/sec], V_{TAS} is the speed [m/seg], α is the angle of attack [deg], θ is the pitch angle [deg], h is the aircraft altitude [m], m is the aircraft mass [kg], ρ is the air mass density [kg/m^3], g is the gravity constant [m/sec^2], δ_e is the stabiliser deflection [deg], l is the longitude, μ is the latitude associated with the current position, ϕ is the roll angle [deg], ψ is the yaw angle [deg], and $D_{reduction}$ is the percentage of drag reduction [%].

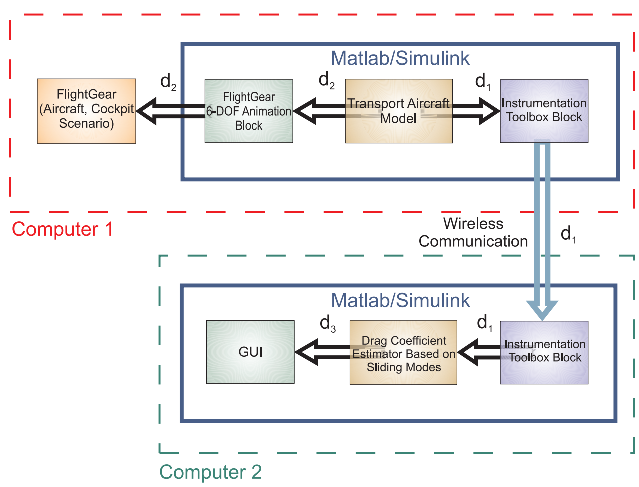


Fig. 2. Data Flow Diagram

IV. Parametric Estimation Using Sliding Modes

The main structural difference between sliding mode and *Luenberger* observers lies in using discontinuous output error injection vectors. This provides insensitivity with respect to a particular class of system uncertainty and/or disturbance signals. The discontinuous injection signal is designed in such a way that the observer's trajectories reach in finite time and remain within a particular domain in the state estimation error space. This domain corresponds to a sliding surface in terms of the state estimation error. In this paper, a second-order sliding mode 'super-twisting' injection signal is considered. A brief description of the 'super-twisting'algorithm is presented in the sequel.

A. Super-twisting algorithm

The 'super-twisting' algorithm was proposed within the robust exact differentiation framework based upon sliding mode concepts presented in [15]. This approach is a modification of the second-order sliding mode (2-sliding mode) method presented earlier in [14].

Let $f(t)$ be an input signal belonging to the space of measurable bounded functions on an interval $[a, b]$. This space is denoted by $\mathcal{M}_{[a,b]}$ and the input signal is such that $\|f\| = \sup |f(t)|$. In the context of a practical real-time differentiator, it is assumed that the input signal $f(t)$ is a measurable locally bounded function defined on the interval $[0, \infty)$ and consists of an unknown base signal involving a derivative with a *Lipschitz* constant $L > 0$, and noise [15]. The aim is to construct a differentiator using sliding mode concepts. To this end, consider an auxiliary differential equation of the form

$$\dot{x} = \nu \ . \quad (11)$$

In order to guarantee $x(t) - f(t) = 0$, the following 2-sliding mode algorithm, the so-called 'super-twisting' is applied [15]:

$$\nu = \nu_1 - \lambda |x(t) - f(t)|^{1/2} \mathrm{sgn}(x(t) - f(t)) \quad (12)$$

$$\dot{\nu}_1 = -\kappa \mathrm{sgn}(x(t) - f(t)) \quad (13)$$

where λ and κ are positive real numbers. Note that due to the discontinuous nature of the system (11)-(13), the solution of this system must be interpreted by means of Filippov's theory [7].

In [15] the following sufficient conditions for the convergence of the 'super-twisting' algorithm are given

$$\kappa > L \quad (14)$$

$$\lambda_{\mathcal{D}}^2 \geq 4L \frac{\kappa + L}{\kappa - L} \ . \quad (15)$$

The above ideas can be applied to construct a 2-sliding mode observer. With no loss of generality, consider the uncertain scalar dynamical system

$$\dot{x}(t) = f(t, x) + \eta(t) \quad (16)$$

where $x \in \Re \subseteq \mathcal{X}$ is the state variable, f is a smooth function in $\mathcal{X}$, and $\eta(t)$ is a bounded uncertain term. A 2-order sliding mode observer for (16) is defined by

$$\dot{\hat{x}} = \hat{f}(t, \hat{x}) + \nu_1 - \lambda |e(t)|^{1/2} \mathrm{sgn}(e(t)) \quad (17)$$

$$\dot{\nu}_1 = -\kappa \mathrm{sgn}(e(t)) \quad (18)$$

where $e(t) = x(t) - \hat{x}(t)$.

B. Drag reduction estimation using a second-order sliding mode observer

In this section it is assumed that the nominal values of the aerodynamic coefficients C_L, C_m and C_D are known. Moreover, there is no change in the pitch moment and lift coefficients. Only changes in drag, due to drag reduction, are considered to occur. Hence, the drag coefficient C_D is decomposed as follows

$$C_D(t) = C_{D_0} + \delta C_D(t) \quad (19)$$

where δC_D denotes any change in the drag force which is to be estimated. By substituting for $C_D(t)$ in (1), it follows that

$$\dot{V}_{TAS} = -\frac{\rho V_{TAS}^2 S_{ref}}{2m}(C_{D_0} + \delta C_D(t)) + g\sin(\alpha - \theta) + \frac{\tau_n}{m}\cos(\alpha + \sigma_T) \ . \tag{20}$$

A sliding mode observer for (20) is given by

$$\dot{\hat{V}}_{TAS} = -\frac{\rho \hat{V}_{TAS}^2 S_{ref}}{2m} C_{D_0} + g\sin(\alpha - \theta) + \frac{\tau_n}{m}\cos(\alpha + \sigma_T) + \nu \tag{21}$$

where $\hat{V}_{TAS}$ is the estimated true air speed, and ν is a second-order sliding mode 'super-twisting' injection signal. This discontinuous signal, as introduced previously, is given by

$$\nu = \nu_1 - k_1|e(t)|^{1/2}\mathrm{sign}(e(t)) \tag{22}$$

$$\dot{\nu_1} = -k_2 \mathrm{sign}(e(t)) \tag{23}$$

where $e(t) = V_{TAS}(t) - \hat{V}_{TAS}(t)$, and the scalars k_1 and k_2 are gains to be designed. Note that equations (22)-(23) define a second-order sliding mode and constitute the so-called 'super-twisting algorithm'. Since $e(t) = V_{TAS}(t) - \hat{V}_{TAS}(t)$, by using (20) and (21), it follows that the error system is given by

$$\dot{e} = -\frac{0.5\rho S_{ref}}{m} e^2(t) - \frac{0.5\rho V_{TAS}^2 S_{ref}}{m}\delta C_D(t) - \nu \ . \tag{24}$$

In the sliding mode the error signal $e(t)$ and its time derivative $\dot{e}(t)$ are both forced to zero in finite time; hence, the change in drag is governed by

$$\delta C_D = -\nu \frac{m}{0.5\rho V_{TAS}^2 S_{ref}} \ . \tag{25}$$

V. Simulation Results

In what follows, computer simulation results considering a sliding mode observer are shown. Two experiments were carried out. Firstly, the drag force in the simulated transport aircraft is modified from 0% up to 2%. Secondly, modification of both drag and pitch rate are performed.

Figure 3 shows the estimated drag reduction (as a percentage) obtained from the sliding mode observer. The drag reduction estimation is achieved accurately. The state variables (along with the other data defined in (8)) of the simulated transport aircraft are transmitted over the wireless network. The state variables received by Computer 2 are presented in Figures 4(a)-4(c). Note that the drag reduction slightly affects both angle of attack and pitch angle as shown in Fig. 4(b). Also notice that since there is no change of altitude (the aircraft is in straight and level flight at 33000 ft), the angle of attack and pitch angle, in steady-state are equal. The aircraft's altitude is presented in Fig. 4(d).

The second experiment, besides the drag reduction, also involves changes in the pitch rate as shown in Fig. 6(c). The true and estimated drag reduction values are depicted in Fig. 5. The estimate is affected by changes in the pitch rate. The true air speed is affected, as expected, by both changes as shown in Fig. 6(a). The time evolution of the angle of attack and the pitch angle are depicted in Fig. 6(b). The altitude of the aircraft varies accordingly with the pitch rate change as shown in Fig. 6(d).

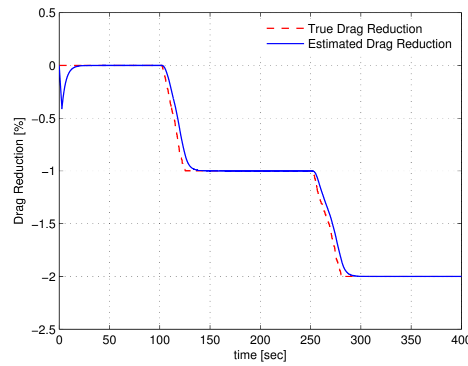
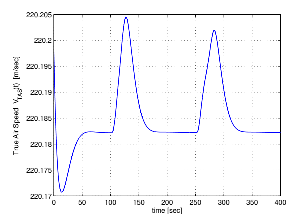
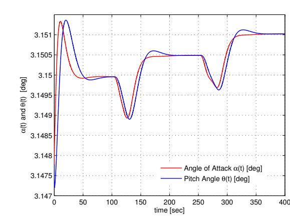


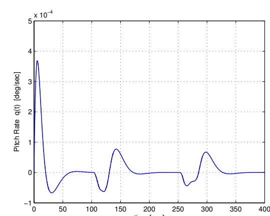
Fig. 3. Drag Reduction - Experiment 1



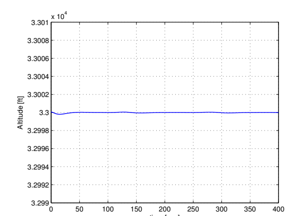
(a) True air speed V_{TAS}



(b) Angle of attack $\alpha(t)$ and pitch angle $\theta(t)$



(c) Pitch rate $q(t)$



(d) Altitude

Fig. 4. State variables and altitude - Experiment 1

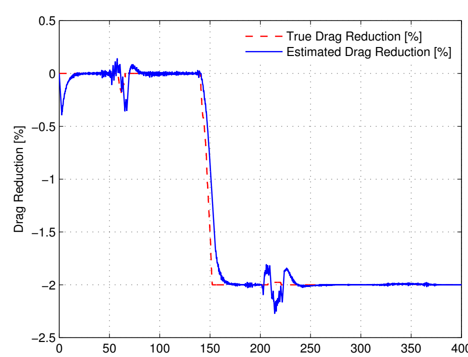
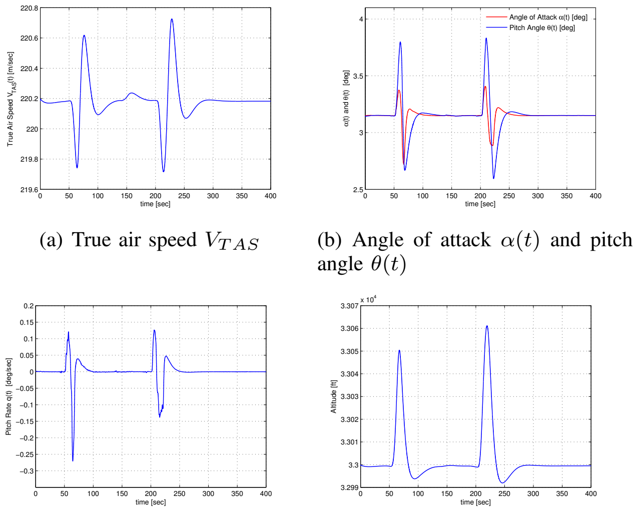


Fig. 5. Drag Reduction - Experiment 2



(a) True air speed V_{TAS} (b) Angle of attack $\alpha(t)$ and pitch angle $\theta(t)$

(c) Pitch rate $q(t)$ (d) Altitude

Fig. 6. State variables and altitude - Experiment 2

VI. CONCLUSIONS

A four engine transport aircraft and on-line drag reduction estimation algorithm based on sliding mode ideas have been simulated over a wireless platform involving data transmission between two computers. A peer-to-peer wireless network has been considered in which data from a high fidelity transport aircraft model is generated in one computer, representing a virtual transport aircraft, and the on-line estimation of the drag coefficient (changes in the drag coefficient) is performed in another computer. A sliding mode observer, whose injection signal is the so-called 'super-twisting algorithm', has been proposed to estimate changes in the drag coefficient. Two experiments were carried out in which a transport aircraft was simulated and data was transmitted over a wireless network. On-line estimation using a sliding mode scheme was performed by processing data received from the simulated aircraft via wireless communication. Results show that good estimation of drag (drag reduction) is achieved.

VII. ACKNOWLEDGMENTS

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