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Nonlinear Predictive Control of an Industrial Power Plant Boiler

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Abstract: This paper proposes an efficient nonlinear predictive controller (NMPC) for application to a power plant drum-boiler. The control objectives are to maintain the water and pressure levels in the drum within a desired range. First, the nonlinear model of the drum-boiler is transformed to LTV state-dependent nonlinear form to provide global nonlinear behavior. Next, state-dependent nonlinear Kalman filter is used to estimate the system states. Then, a supervisory NMPC algorithm is used as a second level controller to generate optimal set points to the lower level regulating PID loops while maintaing output constraints. Simulation results are presented to demonstrate the excellent tracking and disturbance rejection performance compared with a stand-alone multi loop PID controllers.

I. INTRODUCTION

During the last decades, the ever-growing demand for electric power, deregulation of power industry and its associated competition and more strict environmental legislation have forced the power generation industry to develop more efficient and clean ways of generating electricity.

The power plant boiler is a key component for generating steam and heat in industrial processing plants as well as to generate electricity. Industrial boilers are highly complex, nonlinear, and time varying systems. The boiler drum water-level control is considered to be a more challenging problem due to shrink and swell non-minimum phase behavior and instability due to the integrating nature of drum-level dynamics. A coordinated control strategy is normally required to optimize plant operations without violating thermal constraints.

Boiler control is of a multi-loop nature with interactions between different loops. The most common control strategy is to use decentralized multi-loop controller to stabilize the boiler outputs. The well-known PID regulatory controllers have been developed and implemented in each loop due to their simplicity and ease of tuning. However, to improve the economic operation through constrained optimization, an advanced control strategy is needed. Model based Predictive Control (MPC) has received wide acceptance in process industries because of its ability to handle constraints and its optimization based formulation [1].

In power plant applications, MPC are usually implemented as a supervisory controller. The application of this strategy to control Combined Cycle Power Plant (CCPP) and Combined Heat and Power Plant (CHPP) have been widely presented and discussed in literature [2], [3]. In this strategy the MPC provides the regulatory level with optimum set-points, based on an objective function employed to optimize boiler performance and a set of inequality constraints to ensure safety and stability of the system.

The dynamics of the boiler system contains severe nonlinearity. In addition, the demands for rapid changes in power generation require frequent changes from one operation point to another and often near the boundary of the admissible region. Under these conditions, the results obtained by the predictive control design based on the linearized model are poor in term of performance and often not sufficient to cope with the process requirements [4]. This suggests the need for nonlinear control strategies, which are based directly on the nonlinear model and which explicitly take account of the nonlinearities during the system synthesis process.

Nonlinear Model Predictive Control (NMPC) techniques involve solving nonlinear differential equations and a nonlinear dynamic optimization problem online. This computational effort is one of the main obstacles to the adoption of non-linear predictive controller in a wider context. In addition, using a nonlinear model changes the control problem from a convex Quadratic Program (QP) to a non-convex non-linear program, for which global optimum solution cannot be guaranteed. This has motivated the study of alternative MPC approaches, requiring the solution of simpler optimization problems in real-time. Most of these approaches are based on linear time-varying (LTV) prediction through local Jacobian linearization [5] or state dependent description of the nonlinear system [6].

State dependent representation of a system model avoids model linearization. The non-linearity is handled by the replacement of the original nonlinear system by a sequence of linear time-varying systems, whose solutions will converge to the solution of the nonlinear problem. A nonlinear regulator technique called the State Dependent Riccati Equation (SDRE) was developed based on state dependent state-space model [6]. This technique has been described as a nonlinear extension of the well-known LQR formulation for linear systems. The NMPC approach based on state-dependent model has been demonstrated for real time applications [7], for helicopter control application [8] and for flight control [9].

In this paper, a supervisory constrained nonlinear model predictive (NMPC) control strategy based on the LTV state- dependent model is formulated and applied for the regulation of a nonlinear boiler model of a power plant. The main difference with the previous work in this area is that the complete algorithm using an optimization procedure, a nonlinear estimator and constraints are employed to design a 2nd level controller to generate optimal set points for 1st level regulating PID loops. Nonlinear estimator based on state-dependent Kalman filter is used to estimate the unmeasured states.

The paper has been organised as follows: Section 2 describes two-level control structure. State-dependent Kalman filter is presented in section 3. NMPC control is described in Section 4. Section 5 describes the nonlinear boiler model. The simulation results are presented in section 6. Conclusions are in section 7.

II. TWO-LEVEL CONTROL ARCHITECTURE

The proposed hierarchical structure control strategy consists of two levels, a conventional PID regulatory level and a supervisory NMPC optimization level as shown in fig. 1. In this two layer architecture, the regulator level is assumed to be the existing plant PID controllers. The supervisory NMPC algorithm is used as a second level controller to generate optimal set points to the lower level regulating PID loops. The advantage of this structure is that the NMPC algorithm is sitting on top of the existing PID control structure and does not interfere with the closed loop control system. In addition, the model used in NMPC design is therefore open-loop stable.

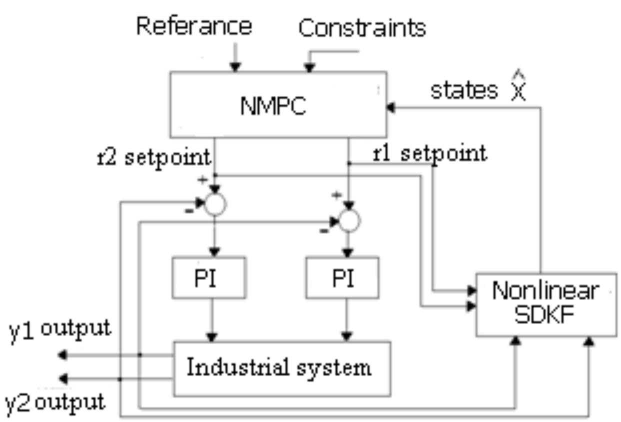

Fig.1. Supervisory control strategy

Let the system under control described by the state-dependent nonlinear model:

$$\dot{x}_p = A_p(x_p)x_p + B_p(x_p)u_p, y_p = C_p(x_p)x_p \quad (1)$$

where: matrices $A_p(x)$, $B_p(x)$ and $C_p(x)$ are functions of state. The state space equation describing the PID controllers can be represented as follows:

$$\dot{x}_c = A_c x_c + B_c e,\ \ u_p = C_c x_c + D_c e \quad (2)$$

where x_c is the state of PID controllers; e is the error signal defined as $e = r - y_p$.

After appropriate substitutions, the state space equations for both the boiler model and the PID controller can be written as follow:

$$\begin{aligned} \dot{x} &= A_1(x)x + B_1(x)u \\ y &= C_1(x)x \end{aligned} \quad (3)$$

where: $x = \begin{bmatrix} x_c \\ x_p \end{bmatrix}, \qquad u = r$

$$A_1(x) = \begin{bmatrix} A_c & -B_c C_p(x) \\ B_p(x) C_c & A_p(x) \end{bmatrix}, \qquad B_1(x) = \begin{bmatrix} B_c \\ 0 \end{bmatrix},$$

$$C_1(x) = \begin{bmatrix} 0 & C_p(x) \end{bmatrix}$$

The state dependent model (3) can be discretised using Euler integration method and converted to a state-dependent nonlinear discrete-time model:

$$\begin{aligned} x_{k+1} &= A_k(x)x_k + B_k(x)u_k \\ y_k &= C_k(x)x_k \end{aligned} \quad (4)$$

where the state-dependent matrices, $A_k(x)$, $B_k(x)$ and $C_k(x)$ are constructed such that the resulting LTV system is locally observable and controllable.

III. STATE DEPENDENT KALMAN FILTER

The state-dependent Kalman filter (SDKF) method [13] combines the concept of linear time-varying models with the well-known Kalman theory for linear systems. It has been used in nonlinear filter development and control designs for some nonlinear benchmark problems for state estimation. The nonlinearities of the system are fully captured by the state-dependent representation that reduces the nonlinear system to a linear structure with state dependent coefficients. For simplicity the process model in state-dependent coefficient form (4) can be written as follows:

$$x_{k+1} = A_k x_k + B_k u_k + w_k, y_k = C_k x_k + v_k \quad (5)$$

where: matrices $A_k = A_k(x_k), B_k = B_k(x_k)$ and $C_k = C_k(x_k)$ are functions of state. The process noise $\boldsymbol{w}_k$ and measurement noise $\boldsymbol{v}_k$ are independent white Gaussian signals. Q and R are diagonal semi-positive and positive definite matrices, respectively.

The discrete version of the state-dependent Kalman filter can be calculated as follows:
Prediction:

$$\hat{x}_{k+1}^{-} = \hat{A}_k \hat{x}_k + \hat{B}_k u_k$$
(7)

$$P_k^{-} = \hat{A}_k P_{k-1} \hat{A}_k^T + Q \quad (8)$$

Correction:

$$K_k = P_k^{-} \hat{C}_k^T (\hat{C}_k P_k^{-} \hat{C}_k^T + R)^{-1}$$
(9)

$$\hat{x}_{k+1} = \hat{x}_{k+1}^{-} + K_k (y_k - \hat{C}_k \hat{x}_{k+1}^{-}) \quad (10)$$

$$P_k = (I - K_k \hat{C}_k) P_k^{-} \quad (11)$$

IV. NON-LINEAR PREDICTIVECONTROL

In this section, a nonlinear generalised model predictive control formulation based on state dependent state-space models will be described. The state-space NMPC multivariable formulation presented below is based on paper [14]. This algorithm is based on both the SDRE technique and the idea of extending the previous optimal trajectory to the current time instant, which is referred as the tail trajectory [5]. The discrete state dependent system given in (5), current estimated state $\boldsymbol{x_k}$ and past input U_{k-1} are used to compute the future state prediction and the future state dependent matrices, within the prediction horizon. The prediction equation for the states can be represented in a vector form as follows:

$$\hat{X} = \hat{\Lambda}\, \hat{x}_0 + \hat{\Phi}\hat{U} \quad (12)$$

where: $\hat{\Phi} = \hat{L}\hat{B}$

$$\hat{\Lambda} = \begin{bmatrix} \phi_{k_0}^{k_0} \\ \vdots \\ \phi_{k_0}^{k_0+N-1} \end{bmatrix}, \qquad \hat{L} = \begin{bmatrix} I & 0 & \cdots & 0 \\ \phi_{k_0+1}^{k_0+1} & I & 0 & \vdots \\ \vdots & \ddots & I & 0 \\ \phi_{k_0+1}^{k_0+N-1} & \cdots & \phi_{k_0+N-1}^{k_0+N-1} & I \end{bmatrix}$$

$$\hat{B} = \begin{bmatrix} B_{k_0} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & B_{k_0+N-1} \end{bmatrix}$$

$$\phi_i^k = A_k\, A_{k-1} \ldots\ldots A_i$$

The future values of state $\hat{X}$, control U and output $\hat{Y}$ are defined as follows:

$$\hat{X} = \begin{bmatrix} x_{k_0+1}^T & \cdots & x_{k_0+N}^T \end{bmatrix}^T, \; U = \begin{bmatrix} u_{k_0}^T & \cdots & u_{k_0+N-1}^T \end{bmatrix}^T$$

$$\hat{Y} = \begin{bmatrix} y_{k_0+1}^T & \cdots & y_{k_0+N}^T \end{bmatrix}^T$$

The cost function to be minimised is of the following quadratic form:

$$J = \frac{1}{2}\sum_{j=1}^{N} \| \hat{y}_{k+j|k} - r_{k+j} \|_Q^2 + \| u_{k+j|k} - u_{k+j-1|k} \|_S^2 \quad (13)$$

where: N is the maximum output horizon, Q and S are the weighting on the tracking error and the control increments respectively. The objective function was represented in vector form as in paper [15] where terminal weighting obtained from solving the algebraic Riccati equation can be added to guarantee the closed loop stability. The MATLAB QP function was used to solve the optimization problem.

In order to achieve offset-free performance the process model (5) was augmented to include constant step output disturbances [1]. The augmented state-space system is given by:

$$\begin{bmatrix} x_{k+1} \\ d_{k+1} \end{bmatrix} = \begin{bmatrix} A(x_k) & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} x_k \\ d_k \end{bmatrix} + \begin{bmatrix} B(x_k) \\ 0 \end{bmatrix} u_k + \omega_k \quad (14)$$

$$y_k = \begin{bmatrix} C(x_k) & I \end{bmatrix} \begin{bmatrix} x_k \\ d_k \end{bmatrix} + v_k \quad (15)$$

The output of a process can be limited by introducing upper and lower output constraints y_{max} and y_{min}. Then the output constraints are specified as:

$y_{\min} \le y_{k+j} \le y_{\max}$ over the horizon j=1: N

The output constraints are expressed in terms of U_k and using vector notations:

$$Y_{\min} \le \Lambda\hat{x}(k) + \Phi U_k \le Y_{\max}$$

$$Y_{\max} = [Y_{1\max}^T \cdots Y_{m\max}^T]^T \quad (16)$$

$$Y_{\min} = [Y_{1\min}^T \cdots Y_{m\min}^T]^T \quad (17)$$

Finally, the model predictive control in the presence of hard constraints is proposed as finding the parameter vector U_k that minimizes (13) subject to the inequality constraints:

$$AU_k \le B \quad (18)$$

where: $A = \begin{bmatrix} -\Phi \\ \Phi \end{bmatrix}, \qquad B = \begin{bmatrix} -Y_{\min} + \Lambda\hat{x}(k) \\ Y_{\max} - \Lambda\hat{x}(k) \end{bmatrix}$

A. The Algorithm

The following steps summarize the NMPC control technique presented in this paper:

1. Estimate the current state vector $\hat{x}_k$ using the state-dependent Kalman filter.
2. At k=0, the initial control trajectory $U_{k,N}$ can be assumed as step control signal with amplitude from normal operating range for the control.
3. For k > 0, the vector $U_{k,N}$ is calculated in previous iteration and removes the first element, which has already been used in the previous iteration for control and repeating the last element of the vector once again, i.e., $u_{k+N-1} = u_{k+N-2}$.
4. Substitute the calculated $U_{k,N}$ into the state equation and calculate iteratively the state prediction and associated matrices. $\hat{x}_{k+n}$, A_{k+n}, B_{k+n} and C_{k+n} for n=0....N-1.
5. Calculate the output predictions and the control vector $U_{k,N}$.
6. Check the difference between the control vectors: $\left|U_{k,N}(new) - U_{k,N}(old)\right| < \sigma$. If this condition is not satisfied put $U_{k,N}(old) = U_{k,N}(new)$ then go to step (3). If it is satisfied apply the first element of

$U_{k,N}$ to the plant and put k=k+1, then go to step (1).

Convergence: The minimum requirement for the convergence of the iterative solutions of the sequence of LTV equations is that the system model (3) $\dot{x} = f(x)$ is locally Lipschitz-continuous [16]. This can be satisfied if there is a constant $L > 0$, known as the Lipschitz constant, such that

$$\|f(y) - f(z)\| = L\|y - z\| \ , \qquad \forall y, z \in B \tag{19}$$

where: the open set $B \subseteq \mathbb{R}^m$.

In this paper, simulation analysis is used to verify this condition instead of analytical analysis due to the complexity of the system model.

V. THE NONLINEAR BOILER MODEL

In this paper, a nonlinear boiler model [10] based on first principles physics was used to describe the dynamics of drum, downcomer and riser system. The parameters used in this boiler model are taken from plant data [11] . In this simulation, simplified PID controllers for the boiler without a super heater were considered. The controlled variables are drum level (l_d) and drum pressure (P). The manipulated variables are feedwater flow (w_e) and delivered power (Q). The model equations can be summarized as:

$$\begin{aligned}
&e_{11}\frac{dV_{wt}}{dt} + e_{12}\frac{dp}{dt} = q_f - q_s \\
&e_{21}\frac{dV_{wt}}{dt} + e_{22}\frac{dp}{dt} = Q + q_f h_f - q_s h_s \\
&e_{32}\frac{dp}{dt} + e_{33}\frac{d\alpha_r}{dt} = Q - \alpha_r h_c q_{dc} \\
&e_{42}\frac{dp}{dt} + e_{43}\frac{d\alpha_r}{dt} + e_{44}\frac{dV_{sd}}{dt} = \frac{\rho_s}{T_d}\left(V_{sd}^0 - V_{sd}\right) + \frac{h_f - h_w}{h_c} q_f \\
&l_d = \frac{V_{wd} + V_{sd}}{A_d}
\end{aligned} \tag{20}$$

where $e_{11}, e_{12}, e_{21}, e_{22}, e_{32}, e_{33}, e_{42}, e_{43}$ and e_{44} are nonlinear functions of the states and the thermodynamic properties of water and steam. These coefficients and the quadratic functions of steam tables are taken from [10].

By letting the states, input and outputs are defined as:

$$\begin{aligned}
&u_P^T = [q_f, Q] = [u_1, u_2] \ , \\
&x_p^T = [V_w, p, \alpha_r, V_{sd}] = [x_1, x_2, x_3, x_4], \\
&y_p = [l_d, p]^T = [y_1, y_2]^T
\end{aligned} \tag{21}$$

where q_f *is* the feed water flow; Q is the heat in combustion room; V_w *is* total water volume; P is the drum pressure; α_r is the steam quality; V_{sd} is volume of steam in drum; l_d is the water level in drum.

Using the above model the state-dependent space model of the boiler can be defined as follows:

$$\begin{aligned}
&\dot{x}_p = A_p(x_p)x_p + B_p(x_p)u_p \\
&y_p = C_p(x_p)x_p
\end{aligned} \tag{22}$$

where:

$$A_p(x_p) = \begin{bmatrix} a_{12} & 0 & 0 & 0 \\ 0 & a_{22} & 0 & 0 \\ a_{32} & 0 & a_{34} & 0 \\ a_{45} & a_{42} & a_{44} & a_{46} \end{bmatrix}$$

$$B_p(x_p) = \begin{bmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \\ a_{31} & a_{33} \\ a_{41} & a_{43} \end{bmatrix}; \ C_p(x_p) = \begin{bmatrix} a_{51} & 0 & 0 & a_{52} \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

The a_{ij} functions as defined in the appendix.

The state space equations for both the boiler model and the PID controllers can be represented as described in section 2 as follow:

$$\begin{aligned}
&x_{k+1} = A_k(x)x_k + B_k(x)u_k \\
&y_k = C_k(x)x_k
\end{aligned} \tag{23}$$

where: matrices $A_k(x)$, $B_k(x)$ and $C_k(x)$ are defined in the appendix . These matrices may be formulated in an infinite number of ways, but they should be constructed such that the resulting LTV system is locally observable and controllable.

VI. SIMUALTIONS AND RESULTS

In this simulation, the PID set points are manipulated using nonlinear model-based predictive control to achieve a better performance. Hence PIDs are mainly used to regulate the systems and NMPC is used to improve tracking and disturbance rejection as well as to minimise an economic performance index.

The following NMPC tuning parameters were chosen in the simulation, and were determined by trial and error to achieve suitable performance. The best values for the PID gains are found using a number of Multivariable PID tuning methods as described in [12].

Table 2. NMPC Parameters

Time	Q	S	N
T=0.05	180, 150	15, 20	80

Fig. 2 shows the plot of Lipschitz constant L versus the pressure state x_4 for any $y, z \in (x,\ x + 0.05)$, where the state vector x is chosen in the domain of the neighbourhood of the operating point x_0. From this figure it's clearly shown that L

is bounded which demonstrate that the local Lipschitz condition is satisfied.

The performance of SDKF is demonstrated in Fig. 3 which shows a comparison between boiler true states and their estimates to step change in drum pressure set points.

Implementing the iteration algorithm described in section 4.1 where the required error norm for the control input to be satisfied is chosen as $\sigma = 0.001$. The maximum iteration limit is defined as i_{max}=60. Fig. 4 shows the response of the boiler drum pressure to set point change. It shows also the number of iterations required for convergence. It is clear that the norm of the error is satisfied during the whole trajectory. It can also be seen that during the transition region more iterations are required for convergence due to the changes of the system states.

To maintain a high level of boiler safety output constraints on boiler drum level are introduced in this simulation. The introduced maximum and minimum output constraints are 4.4m and 2.4m respectively. Fig. 5 shows the responses of the boiler drum level and boiler pressure to set points changes using constrained and unconstrained NMPC. It can be observed that the imposed output constrained are satisfied using the constrained MPC.

Simulation results comparing the performance of supervisory NMPC and the classical PID controllers are shown in Fig. 6 which shows that NMPC is able to reach the set point faster than the PID which has an oscillatory response around the set point with a larger overshoot. This is because the boiler drum is an integration process and can easily become unstable if the controller's integral gain is set too high. It also shows the ability of NMPC to compensate the non-minimum phase shrink and swell effects.

VII- CONCLUSIONS

In this paper, an efficient supervisory NMPC algorithm based on state-dependent approach is presented for drum pressure and water level control of boiler power plant. The non-linear boiler system was represented in state-dependent form to provide global nonlinear behavior. The nonlinearity of the model and the lack of measurement were handled by nonlinear state estimation using state-dependent Kalman filter. The NMPC was used in the second layer to tune the performance of PID controllers. The simulation results showed that the NMPC controller has better performance than the classical PID control schemes and allows the inclusion of output constraints.

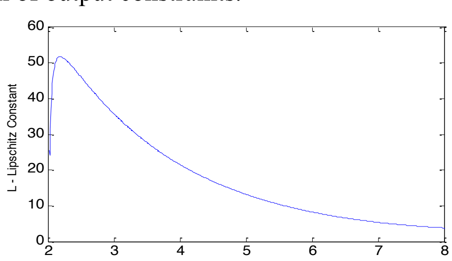

Fig.2. Lipschitz constant

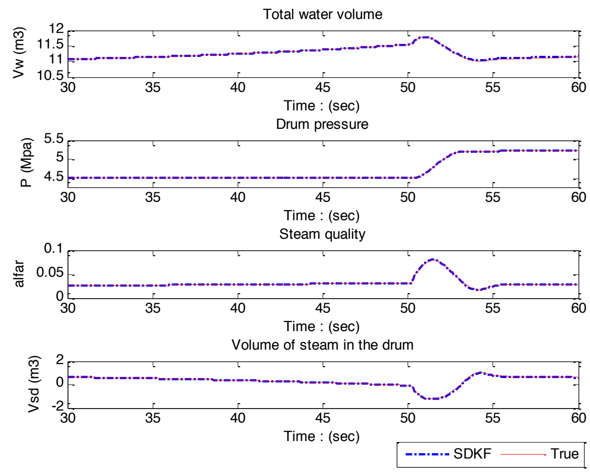

Fig.3. Boiler true states and their estimates

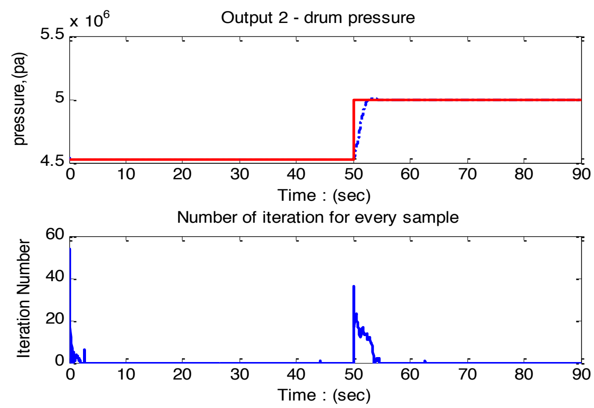

Fig.4. Number of iteration and max-norm of the error.

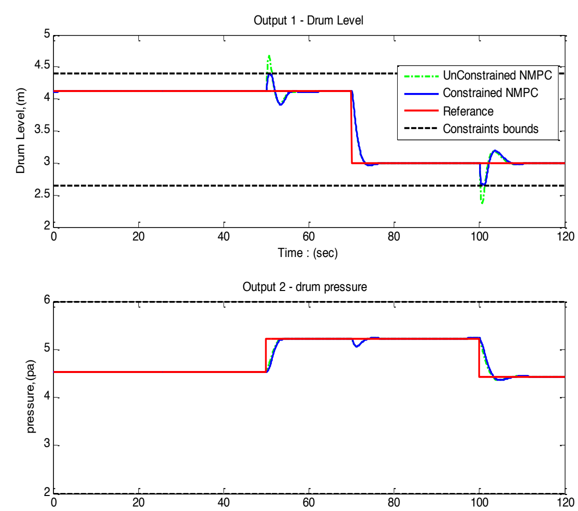

Fig.5. Boiler drum level and drum pressure responses set points changes using constrained and unconstrained NMPC.

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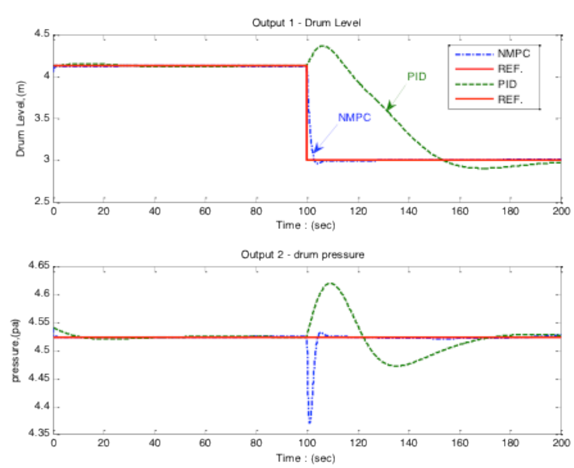

Fig.6. Comparison of boiler control performance using supervisory NMPC and PID controllers.

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APPENDIX

- State-dependent matrices of the discrete boiler model

$$A_k(x)=\begin{bmatrix}1 & 0 & A_{13} & 0 & 0 & A_{16}\\ 0 & 1 & 0 & A_{24} & 0 & 0\\ A_{31} & A_{32} & A_{33} & A_{34} & 0 & A_{36}\\ A_{41} & A_{42} & A_{43} & A_{44} & 0 & A_{46}\\ A_{51} & A_{52} & A_{53} & A_{54} & A_{55} & A_{56}\\ A_{61} & A_{62} & A_{63} & A_{64} & A_{65} & A_{66}\end{bmatrix},\qquad B_k(x)=\begin{bmatrix}B_{11} & 0\\ 0 & B_{22}\\ B_{31} & B_{32}\\ B_{41} & B_{42}\\ B_{51} & B_{52}\\ B_{61} & B_{62}\end{bmatrix},$$

$$C_k(x)=\begin{bmatrix}0 & 0 & a_{51} & 0 & 0 & a_{52}\\ 0 & 0 & 0 & 1 & 0 & 0\end{bmatrix}$$

where:

$A_{13}=-Ta_{51},\ A_{16}=-Ta_{52}, A_{24}=-T,\ \ A_{31}=Ta_{11}ki1,$

$A_{32}=Ta_{13}ki2,\ A_{33}=T\left(a_{12}-a_{11}a_{51}kp1\right)+1, A_{34}=-Ta_{13}kp2,$

$A_{36}=-Ta_{11}a_{52}kp1,\ A_{41}=Ta_{21}ki1,\ \ A_{42}=Ta_{23}ki2,$

$A_{43}=-Ta_{21}a_{51}kp1,\ \ \ A_{44}=\ T\left(a_{22}-a_{23}kp2\right)+1, A_{46}=-Ta_{21}a_{52}kp1,$

$A_{51}=Ta_{31}ki1,\ A_{52}=Ta_{33}ki2,\ A_{53}=T\left(a_{32}-a_{31}a_{51}kp1\right),$

$A_{54}=-Ta_{33}kp2,\ A_{55}=\ Ta_{34}\ +\ 1,\ A_{56}=-Ta_{31}a_{52}kp1,$

$A_{61}=Ta_{41}ki1, A_{62}=Ta_{43}ki2,\ A_{63}=T\left(a_{45}-a_{41}a_{51}kp1\right),$

$A_{64}=T\left(a_{42}-a_{43}kp2\right),\ A_{65}=\ Ta_{44}, A_{66}=\ T\left(a_{46}-a_{41}a_{52}kp1\right)+1,$

$B_{11}=T,\ \ B_{22}=T, B_{31}=\ Ta_{11}kp1, B_{32}=Ta_{13}kp2,\ B_{41}=T_{a21}kp1,$

$B_{42}=Ta_{23}kp2, B_{51}=Ta_{31}kp1, B_{52}=Ta_{33}kp2, B_{61}=Ta_{41}kp1, B_{62}=Ta_{43}kp2$

$$a_{11}=\frac{e_{22}-e_{12}h_f}{e_{11}e_{22}-e_{12}e_{21}},\qquad a_{12}=\frac{(e_{12}h_s-e_{22})q_s}{(e_{11}e_{22}-e_{12}e_{21})x_p(1)}$$

$$a_{13}=\frac{-e_{12}}{e_{11}e_{22}-e_{12}e_{21}},\qquad a_{21}=\frac{e_{11}h_f-e_{21}}{e_{11}e_{22}-e_{12}e_{21}},$$

$$a_{21}=\frac{e_{11}h_f-e_{21}}{e_{11}e_{22}-e_{12}e_{21}},\qquad a_{22}=\frac{(e_{21}-e_{11}h_s)q_s}{(e_{11}e_{22}-e_{12}e_{21})x_p(2)}$$

$$a_{23}=\frac{e_{11}}{e_{11}e_{22}-e_{12}e_{21}},\qquad a_{45}=\frac{\rho_s V_{sd}^0}{T_d e_{44}x_p(1)},$$

$$a_{31}=\frac{e_{21}e_{32}-e_{11}e_{32}h_f}{\left(e_{11}e_{22}-e_{21}e_{12}\right)e_{33}},\qquad a_{32}=\frac{(e_{11}e_{32}h_s-e_{21}e_{32})q_s}{\left(e_{11}e_{22}-e_{21}e_{12}\right)e_{33}x_p(1)},$$

$$a_{41}=\frac{h_f\ e_{11}e_{32}e_{43}-h_f e_{11}e_{33}e_{42}-e_{21}e_{32}e_{43}+e_{21}e_{33}e_{42}}{\left(e_{11}e_{22}-e_{21}e_{12}\right)e_{33}e_{44}}+\frac{h_f-h_w}{h_c e_{44}},$$

$$a_{42}=\frac{(e_{21}e_{32}e_{43}-e_{21}e_{33}e_{42}-e_{11}e_{32}e_{43}h_s+e_{11}e_{33}e_{42}h_s)q_s}{\left(e_{11}e_{22}-e_{21}e_{12}\right)e_{33}e_{44}x_p(2)},$$

$$a_{43}=\frac{e_{11}e_{32}e_{43}-e_{11}e_{33}e_{42}}{\left(e_{11}e_{22}-e_{21}e_{12}\right)e_{33}e_{44}}-\frac{e_{43}}{e_{44}e_{33}},\qquad a_{44}=\frac{e_{43}h_c q_{dc}}{e_{44}e_{33}},$$

$$a_{46}=\frac{-\rho_s}{T_d e_{44}},\qquad a_{51}=\frac{1}{A_d}\left(\frac{x_p(1)-V_{dc}-\left(1-alfav\right)V_r}{x_p(1)}\right),\qquad a_{52}=\frac{1}{A_d}$$

$kp1, ki1, kp2$ and $ki2$ are the PID controllers parameters. T is the sampling time.