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# Power analysis in wind generation with doubly fed induction generator with polynomial optimization tools

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*Abstract*— **In this work, new analysis and prediction strategies for the power control in wind generators that use a doubly fed induction generator are addressed. The wind generator model and control strategies most widely used are first presented, including strategies used to punctually increase the power generation in order to respond to power demands derived from power system frequency deviations. A new on-line prediction strategy is addressed based on a wind observer plus the use of the different norms that the dynamic model presents from the inputs (the wind or the power demand) to the outputs (rotor speed, accelerations, and power generation). For that purpose, the wind observer and the computational derivation of the different input-output performances are developed via optimization over polynomials techniques. The developed analysis and predicting strategies are applied to a wind generator model and controller proposed in the literature, showing the effectiveness of the presented approach.**

## I. INTRODUCTION

Due to the high present impact of the electrical wind generation and future expected increases, the power control of wind turbines that use doubly fed induction generator (DFIG) deserves a great attention, especially because it is one of the most widely used until now [1], [2]. There are several objectives that must be accomplished when using DFIG, taking into account that the power is extracted from a non predictable energy source with a high variability along time (the wind over the turbine blades): the maximization of the extracted power; a power generation as soft as possible to assimilate the production from conventional electrical plants; the limitation of the rotation speed variation, due to the decrease of useful life because of fatigue failures that increase the cost of maintenance; or the ability of the wind farms to contribute to the stability of the electrical grid frequency [3]–[5] using for that purpose the kinetic energy stored in the rotating blades.

The most widely used power controllers in the bibliography are PI controllers [4], however, in more recent works, more complex controllers [6] that try to improve their performance can be found. Although some advances have been produced, there is still the need to find a control scheme that achieves an optimization of the generated power at the same time that contributes to system frequency control, as it is explained in [2]. In the present work, a new way to address the problem is developed, translating the goals and restrictions that must be satisfied with the use of wind generators into a computationally tractable optimization problem. For that objective to be achieved, a polynomial dynamical model of the controlled system, the signal characterization via their mathematical norms, and the use of semidefinite programming tools via polynomials [7], [8] are proposed, allowing to obtain numerical solutions to the problems stated in [9]–[11].

The structure of this work is as follows. First, the problem statement is addressed in Section II, where the different mathematical models that explain the behavior of the wind generators are shown, and a novel strategy to analyze the performance of existing power control algorithms is addressed. In the third section, the proposed wind and state observer is developed. In Section IV, the computation of the different performance indices is addressed. In Section V, the proposed analysis and prediction tool is tested with several simulations. The main conclusions and future research work are summarized in Section VI.

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## II. PROBLEM STATEMENT

### A. Wind turbine mathematical model

A mathematical model for a wind generator with doubly fed induction machine connected to the electrical grid will be developed including the the aerodynamic effects, the mechanical drive train, the maximum power point track controller, and the electronic converter. This is a simplified but complex enough model to achieve with sufficient accuracy the proposed goals (performance quantification). The mechanical system is modeled by means of two spring-damper-connected masses, leading to equations

$$H_t\dot{\omega}_t = T_t - D_{tg}\,(\omega_t - \omega_g) - K_{tg}\,\theta \tag{1}$$

$$H_g\dot{\omega}_g = D_{tg}\,(\omega_t - \omega_g) + K_{tg}\,\theta - T_{em} \tag{2}$$

$$\dot{\theta} = \omega_t - \omega_g \tag{3}$$

being $\omega_t$ the slow shaft rotational speed (i.e., the wind turbine), $\omega_g$ the rotational speed of the fast shaft connected to the generator rotor, $H_t$ the wind turbine inertia constant, $H_g$ the generator inertia constant, $T_t$ the torque developed by the wind turbine due to wind action and $T_{em}$ the electromagnetic torque of the generator. Speeds and torques are expressed in per-unit system (pu) with respect to their nominal value. $\theta$ is the angular difference between shafts.

The torque developed by the wind turbine can be expressed approximately by means of static functions of the wind speed, $v$, rotor rotational speed and blade (pitch) angle, $\beta$. Several functions for this torque can be found in the

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literature. In this work, this torque has been approximated by a polynomial function as

$$T_t = \sum_{i=0}^{3}\sum_{j=0}^{3} c_{ikj} v^i \omega_t^j, \quad (4)$$

where the pitch angle is assumed to be zero, as in this work only low and medium wind speeds (the more probable ones) are assumed.

The wind speed can be characterized by means of its mean value and a turbulence component, as stated in IEC standards [12], [13]

$$v(t) = \bar{v}(t) + \delta v(t).$$

The mean value of the wind speed $\bar{v}(t)$ is assumed to change slowly in time (in a scale of hours) and it can be modeled by the Van der Hoven's spectral model plus a Weibull probability distribution. A Kaimal model has been used for the turbulence part. In the short-term, a periodical variation due to tower shadow can be also added. Those models have been implemented in order to generate the wind speed used in both the simulation verification of the proposed analysis and the prediction methods explained later, but the details are omitted for brevity (they can be found in [12], [13]).

The electromagnetic torque is achieved by means of a current control loop in the power electronics converter that presents a much faster dynamics than the one being analyzed. It can be approximately modeled by a first order model that depends on the generator speed as

$$\dot{T}_{em} = \frac{\omega_g}{\tau_{em}}(T^\star_{em} - T_{em}). \quad (5)$$

The generated active power is given by

$$P_g = T_{em}\omega_g, \quad (6)$$

whose value is measured indirectly through currents and voltages.

Nowadays, most of the controllers proposed in the literature have the following structure. First, the use of the wind measurement is avoided due to its technical difficulties and, instead of that, the generated electrical power is measured at each instant of time and is used to determine the generator rotational speed needed for maximizing the captured power. The relationship between rotor speed and captured power at each wind speed is shown in Fig. 1, where the maximum power extraction curve in steady state has been drawn. The resulting curve can be approximated by a polynomial of order $N$ (generally $N \geq 3$) leading to

$$\omega_g^\star(P_g) = \sum_{i=0}^{N} a_i P_g^i \quad (7)$$

Due to wind variability, the generated active power and, therefore, the speed reference obtained with the above curve have also an important variability. For that reason, the reference signal is filtered and then sent to a PI controller, whose control action is the desired electromagnetic torque $T^\star_{em}$ (Fig. 1). When the generated power reaches its nominal

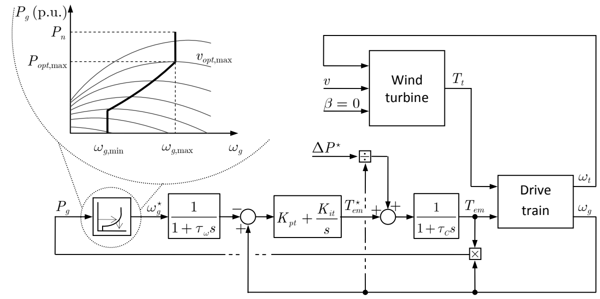


Fig. 1. Speed control. Detail of maximum power tracking.

value, the pitch control is activated and used to prevent a production beyond that value, avoiding the machine acceleration.

The design of this controllers is usually based in linearized models, using robust control techniques or linear time parameter varying models. One of the main drawbacks of those control methods is that the generated power has a frequency spectrum similar to the one wind speed has and, therefore, the quality of service is lost and stability problems on the power system frequency appear.

Another possibility that is being exploited on the wind generation systems is their contribution to frequency control. The idea is to help power systems to quickly restore their nominal frequency after an incident. The power generated by the wind turbine is transiently increased by means of decelerating the machine and injecting its stored kinetic energy. Different approaches are proposed that try to make the power track a given power reference during a short period of time. In fact, the amount of extra generated power and the duration is sometimes fixed by the grid codes of the countries. The transient power demand leads to an increment of the demanded $T^\star_{em}$ by the PI controller in the following way

$$T^{\star\prime}_{em} = T^\star_{em} + \frac{\Delta P^\star}{\omega_g}, \quad (8)$$

where $\Delta P^\star$ is the power demand increment (usually a short-time duration signal), and $T^{\star\prime}_{em}$ the new electromagnetic torque demand sent to the power converter. The quotient $\frac{\Delta P^\star}{\omega_g}$ can be approximated by a polynomial function for the normal speed range of a DFIG ($\omega_g \in [0.8,\ 1.2]$)

$$\frac{\Delta P^\star}{\omega_g} \approx \Delta P^\star \sum_{i=0}^{N} c_i \omega_g^i \quad (9)$$

When this demand is applied, the higher transient production is always followed by a certain time in order to restore the machine to its normal operation. This behavior will be analyzed in several examples.

At the same time that several properties or performances are desired on the wind turbines, there are also some restrictions that must be satisfied to keep the turbine in working order. These restrictions refer to: the steady state mean value of the rotational speed must be restricted between $\bar{\omega}_g, \bar{\omega}_t \in [\bar{\omega}_{\min}, \bar{\omega}_{\max}]$ (usually $\bar{\omega}_{\min} = 0.8$ and $\bar{\omega}_{\max} = 1.2$ pu); the instantaneous speeds and accelerations of the rotational parts must also be restricted to avoid mechanical failures, including fatigue ($\omega_g, \omega_t \in [\omega_{\min}, \omega_{\max}]$, and $|\dot{\omega}_g|, |\dot{\omega}_t| \in [0, \dot{\omega}_{\max}]$).

### B. Analysis and prediction procedure

In this work several tools that allow to analyze, predict and redesign the power controllers for wind turbines are presented. These procedures can be used to estimate the goodness of existing controllers (i.e., to analyze if the previous restrictions are satisfied), or to predict their behavior under different situations (as short-time overproduction demands or limiting power production via pitch control).

The prediction procedure starts estimating the wind by means of an observer based on an Extended Kalman Filter that uses the available measurements and control actions (generator rotor speed, generated power and electromagnetic torque). The observed wind is then used to estimate its mean and covariance for a given temporal window. This mean and covariance is then used to predict the behavior of the machine (rotational speeds and accelerations, generated power and available overproduction) by means of several polynomial functions that allow to bound the future behavior for a given mean wind speed. Those polynomial functions must be previously obtained via optimization techniques, based on a model of the system and the applied controllers and making use of induced norms.

The following bounds will be used to analyze the behavior due to wind turbulence,

$$\|y\|_\infty < \gamma_1(\bar{v})\|\delta v\|_\infty \tag{10}$$

$$\|y\|_{RMS} < \gamma_2(\bar{v})\|\delta v\|_{RMS} \tag{11}$$

where $y$ can be any of $\omega_g$, $\dot{\omega}_g$, $\omega_t$, $\dot{\omega}_t$, $P_g$ or $\dot{P}_g$. $\bar{v}$ represents the mean observed wind, while $\delta v$ is the instantaneous variation around that mean wind speed. The first bound allows to detect possible deviations from allowed regions, while the second one allows to analyze the ability of the controller to attenuate the wind fluctuations.

When analyzing the behavior under overproduction demands, a bound on the maximum demanding energy that assures the stability or fine working of the wind turbine (depending on the wind conditions) will be first obtained, leading to

$$\|\Delta P^\star\|_2 < \mu(\bar{v}) \tag{12}$$

where $\|\Delta P^\star\|_2 = \int_0^\infty (\Delta P^\star)^2\, \mathrm{d}t$ is related with the energy demand. Then, a bound on the norms of each of the $y$ signals mentioned before will be obtained as

$$\|y\|_\infty < \gamma_3(\bar{v})\|\Delta P^\star\|_2, \quad \forall \|\Delta P^\star\|_2 < \mu(\bar{v}) \tag{13}$$

$$\|y\|_2 < \gamma_4(\bar{v})\|\Delta P^\star\|_2, \quad \forall \|\Delta P^\star\|_2 < \mu(\bar{v}) \tag{14}$$

The first bound will allow to detect, for instance, the maximum power overproduction or the minimum rotational speed achieved. The second one will help to predict the amount of injected kinetic energy for a given demand. The procedure developed in the next section will also allow to obtain bounds on the evolution of the transient of some signals, as generated power when restoring the wind turbine after an overproduction transient.

## III. State and wind observer

For the state and wind observer, a simple random walk is used for the wind generation model

$$\dot{v} = \omega_v,$$

being $\omega_v$ a white gaussian noise. With this wind model, a forward difference approximation of the DFIG model is defined with a sufficiently small period $T$, leading to

$$\begin{aligned} x_k &= \begin{bmatrix} v_{k-1} + T\,\omega_v \\ \omega_{t,k-1} + \frac{T}{H_t}\left(T_{t,k-1} - D_{tg}\Delta\omega - K_{tg}\theta_{k-1}\right) \\ \omega_{g,k-1} + \frac{T}{H_g}\left(-T_{em,k-1} + D_{tg}\Delta\omega + K_{tg}\theta_{k-1}\right) \\ T_{em,k-1} + \frac{T\,\omega_{g,k-1}}{\tau_{em}}(T^\star_{em,k-1} - T_{em,k-1}) \\ \theta_{k-1} + T\,\Delta\omega \end{bmatrix} \\ \omega_{g,k} &= \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \end{bmatrix} x_k \end{aligned} \tag{15}$$

where $\Delta\omega = \omega_{t,k-1} - \omega_{g,k-1}$, and $T_{t,k-1} = \sum_{i=0}^{3}\sum_{j=0}^{3} c_{ij} v_{k-1}^i \omega_{t,k-1}^j$, and being the state vector $x_k = [v_k,\ \omega_{t,k},\ \omega_{g,k},\ T_{em,k},\ \theta_k]^T$ .Let us now express the previous model as

$$x_k = f(x_{k-1}, T^\star_{em,k-1}) + \omega_{k-1} \tag{16}$$

$$\omega_{g,k} = C\,x_k + \nu_k, \tag{17}$$

being $\omega_{k-1}$ a white noise disturbance vector taking into account the wind speed variations and possible model errors, and $\nu_k$ the measurement noise, assumed to be a white noise signal with known variance $\mathcal{E}\{\nu_k^2\} = R$. The algorithm, based on the Extended Kalman Filter, that must be computed at each sampling period to estimate the wind speed, its mean and root mean square value is

$$\hat{x}_k^- = f(\hat{x}_{k-1}, T^\star_{em,k-1}) \tag{18a}$$

$$P_k^- = F_{k-1} P_{k-1} F_{k-1}^T + Q \tag{18b}$$

$$L_k = P_k^- C^T (C\,P_k^-\,C^T + R)^{-1} \tag{18c}$$

$$\hat{x}_k = \hat{x}_k^- + L_k(w_{g,k} - C\,\hat{x}_k^-) \tag{18d}$$

$$P_k = (I - L_k\,C)P_k^- \tag{18e}$$

$$\hat{\bar{v}}_k = p \cdot \hat{\bar{v}}_{k-1} + (1-p)\cdot \hat{v}_k \tag{18f}$$

$$\hat{\sigma}_{v2,k} = p \cdot \hat{\sigma}_{v2,k-1} + (1-p)\cdot (\hat{\bar{v}}_k - \hat{v}_k)^2 \tag{18g}$$

$$\hat{\sigma}_{v,k} = \sqrt{\hat{\sigma}_{v2,k}} \tag{18h}$$

where $Q$ is used as a tuning parameter (see the examples), $p$ is a slow discrete time pole ($0 < p \lesssim 1$) chosen to be the discrete-time equivalent of a continuous-time pole similar to the model that generates the mean wind speed variations (about $600s^{-1}$). The matrix $F_{k-1}$ is given by

$$F_{k-1} = \left.\frac{\partial f}{\partial x}\right|_{\hat{x}_{k-1}, T^\star_{em,k-1}}.$$

Note that this algorithm is useful for both wind estimation and state observation, and can be used to implement more complex control algorithms based on polynomial state feedback control. This idea will be explored in future works.

## IV. Behavior characterization

In this section the procedure to obtain the polynomials that bound the different norms of the involved signals is explained. The main tool that has been used is the analysis of induced norms via Lyapunov functions and sum of squares techniques [9]. In these techniques a dynamical polynomial model of the system is assumed to be available, fulfilling

$$\dot{x} = f(x) + g(x)\omega, \qquad f(0,0) = 0 \tag{19}$$

where $x$ is the state vector, $\omega$ are the state disturbances inputs, and $f(x)$ and $g(x)$ are given polynomial vectorial functions. For analysis purposes, the wind will be modeled as a slowly time varying mean value $\bar{v}$ plus a signal generated by a bounded white noise $w$ filtered by a first order system with a low time constant, leading to

$$\delta\dot{v} = \frac{1}{\tau_v}(w - \delta v), \tag{20}$$

$$v = \bar{v} + \delta v. \tag{21}$$

The model used for analysis purposes, including the incremental power demand (8) is then defined as

$$\begin{bmatrix} \delta\dot{v} \\ \dot{\omega}_t \\ \dot{\omega}_g \\ \dot{\theta} \\ \dot{\omega}_{gf} \\ \dot{I} \\ \dot{T}_{em} \end{bmatrix} = \begin{bmatrix} \frac{1}{\tau_v}(w - \delta v) \\ \frac{1}{H_t}\left(T_t(v,\omega_t) + D_{tg}(\omega_t - \omega_g) - K_t\theta\right) \\ \frac{1}{H_g}\left(-T_{em} - D_{tg}(\omega_t - \omega_g) + K_t\theta\right) \\ \omega_t - \omega_g \\ \frac{1}{\tau_w}(\omega_g^\star(P_g) - \omega_{gf}) \\ \omega_{gf} - \omega_g \\ \frac{\omega_g}{\tau_{em}}(K_p(\omega_{gf} - \omega_g) + K_i I + \Delta P^\star\, p(\omega_g) - T_{em}) \end{bmatrix} \tag{22}$$

where $\omega_{gf}$ is the filtered speed reference $\omega_g^\star$, $T_t(v,\omega_t)$ is the polynomial defined in (4), $\omega_g^\star(P_g)$ the one defined in (7), and $p(\omega_g)$ the one defined by (9). Noting that $P_g = \omega_g T_{em}$, this is a polynomial dynamic model with inputs $\omega$ and $\Delta P^\star$ that can be written as

$$\dot{x} = f(x,\bar{v}) + g_w(x)w + g_P(x)\Delta P^\star$$

Using model (22), general bounds for the different signals could be obtained for all the working range. However, those bound would be very convervative and hence, would not give a precise enough information. In order to obtain more useful results, a gridding approach along different mean values for the wind speed will be applied. For that purpose, for each value of the mean speed wind in the grid, the model (22) is proposed to be rewritten in the following way. First, the operating point $\bar{x}$ for a static wind $\bar{v}$ and null inputs is obtained from equations

$$f(\bar{x},\bar{v}) = 0$$

and a new state vector $\delta x$ is defined as $x = \bar{x} + \delta x$. Substituting on (22), it leads to

$$\delta\dot{x} = f(\delta x,\bar{v}) + g_w(\delta x)\omega + g_P(\delta x)\Delta P^\star \tag{23}$$

where $\delta x$ is the new state vector. For brevity of notation, $\delta x$ will be replaced by $x$ in the results to come.

The following result, needed to obtain the performances indices, can be derived from the Positivstellensatz [10].

*Lemma 4.1:* If there exist sum of squares polynomials $s_i(x)$ ($i = 1,\ldots,n$, $x \in \mathbb{R}^n$) and polynomial $q(x)$ such that

$$f(x) - \sum_{i=1}^{n} s_i(x)\, g_i(x) + q(x)\, l(x) \in \Sigma(x),$$

being $\Sigma(x)$ the set of sum of squares polynomials in $\mathbb{R}^n$, then, the following condition holds

$$f(x) \geq 0, \quad \forall\, g_i(x) \geq 0,\ l(x) = 0.$$

In the following, $y$ represents the output whose bound is being obtained and, in general, can be expressed as $y = h(x)$. Some of the most interesting signals to be analyzed are

$$y = P_g = T_{em}\,\omega_g = x_3\,x_7$$
$$y = \dot{\omega}_g = f_3(x,\bar{v})$$

being $x_i$ and $f_i$ the $i$-th component of the state vector and the derivative function in (22).

### A. Behavior under wind turbulence

Under wind turbulence, it is useful to know the variances of the generated power (it should be as low as possible to not disturb the electrical system), and of the rotational speeds (it should also be as low as possible to avoid mechanical failures for high torque demands or fatigue). It is also useful to know the possible minimum and maximum speeds, accelerations and generated power for a given peak on the wind that affects the turbine. For a given mean wind speed, the Kaimal model allows us to obtain a probable bound for the wind speed. Let us assume that this bound is known and defined as a function of the mean wind speed by

$$\|w\|_\infty < \alpha(\bar{v}) \tag{24}$$

The following theorem allows to bound a set in which the state will be caught for all the disturbances bounded by (24).

*Theorem 4.1:* If there exist a positive $\epsilon$, a sum of squares polynomial $s(x)$, and polynomials $V(x)$ and $q(x)$ such that

$$V(x) - \epsilon x^T x \in \Sigma(x) \tag{25}$$
$$-\dot{V}(x) - s(x)(\alpha(\bar{v})^2 - w^2) + q(x)(V(x) - 1) \in \Sigma(x)$$

with $\dot{V}(x) = \frac{\partial V(x)}{\partial x} f(x,\bar{v}) + g_w(x)w$, then, under null power demand $\Delta P^\star$, $V(x) \leq 1$ is an invariant set for system (23) with wind turbulence bounded by $|w| \leq \alpha(\bar{v})$.

*Proof:* Applying lemma 4.1, if conditions (25) are satisfied, then $V(x) > 0\ \forall x \neq 0$, and $\dot{V}(x) \leq 0$, $\forall |w| \leq \alpha(\bar{v})$, $V(x) = 1$, i. e., $V(x)$ is a Lyapunov function that is assured to decrease on the boundary defined by $V(x) = 1$ for all inputs fulfilling $|w| \leq \alpha(\bar{v})$, and, therefore, the state will always be contained in the set $V(x) \leq 1$. ∎

If conditions (25) are assumed, two interesting results can be derived, leading to bounds on the maximum value of the outputs, and also to a relationship between the RMS values of inputs and outputs.

*Theorem 4.2:* For a given $V(x)$ fulfilling (25), and an output $y = h(x)$, if there exist a positive $\gamma$ and a sum of squares polynomial $s(x)$ such that

$$(\gamma^2 - h(x)^2) - s(x)(1 - V(x)) \in \Sigma(x), \tag{26}$$

then, $\|y\|_\infty \leq \gamma$ if the system initially lies inside the invariant set $V(x) \leq 1$.

*Proof:* Applying again lemma 4.1, the previous constraint implies that $h(x)^2 \leq \gamma^2$, $\forall V(x) \leq 1$. Then, if initial inclusion on the invariant set is assumed, and (25) is fulfilled, then the state will always fulfill $V(x) \leq 1$, and therefore, $|w| \leq \alpha(\bar{v})$ implies $\|y\|_\infty \leq \gamma$. ■

*Theorem 4.3:* For a given $V(x)$ fulfilling (25), and an output $y = h(x)$, if there exist positive $\gamma$, $\epsilon$, a polynomial $W(x)$ and sum of squares polynomials $s_1(x)$, $s_2(x)$ such that

$$\begin{aligned} & W(x) - \epsilon x^T x \in \Sigma(x) \\ & -\dot{W}(x) - h(x)^2 + \gamma^2 w^2 - s_1(x)(\alpha(\bar{v})^2 - w^2) \\ & \quad + s_2(x)(1 - V(x)) \in \Sigma(x) \end{aligned} \tag{27}$$

with $\dot{W}(x) = \frac{\partial W(x)}{\partial x} f(x, \bar{v}) + g_w(x) w$, then, under null initial conditions, $\|y\|_{RMS} \leq \gamma \|w\|_{RMS}$.

*Proof:* These conditions imply

$$\begin{aligned} & W(x) > 0, \ \forall x \neq 0 \\ & \dot{W}(x) + h(x)^2 < \gamma^2 w^2, \quad \forall\, w^2 < \alpha(\bar{v})^2, \forall x : V(x) \leq 1 \end{aligned}$$

Integrating the second inequality from 0 to $T$, it follows $W(T) - W(0) + \int_0^T h(x)^2\, dt \leq \gamma^2 \int_0^T w^2\, \mathrm{d}t$. If null initial conditions are assumed, $W(0) = 0$, and taking into account that $W(T) > 0$, the previous expression can be divided by $T$, and taking the limit when $T$ tends to $\infty$, it finally leads $\|y\|_{RMS} \leq \gamma \|w\|_{RMS}$. ■

### B. Behavior under overproduction demand

The transient that is produced during over-production demands is usually much shorter than the wind fluctuations. For that reason, in this case the wind disturbance $w$ will be neglected, and only the over-production demand signal ($\Delta P^\star$) will be taken into account. This will be assumed to be a pulse signal (of finite energy).

*Theorem 4.4:* If there exist a positive $\epsilon$, a sum of squares polynomial $s_3(x)$, and polynomial $V(x)$ such that

$$\begin{aligned} & V(x) - \epsilon x^T x \in \Sigma(x) \\ & -\dot{V}(x) + \frac{1}{\mu(\bar{v})^2} \Delta P^{\star 2} - s_3(x)(1 - V(x)) \in \Sigma(x) \end{aligned} \tag{28}$$

with $\dot{V}(x) = \frac{\partial V(x)}{\partial x} f(x, \bar{v}) + g_P(x) \Delta P^\star$, then, under null initial conditions, if $\Delta P^\star$ fulfills $\int_0^T \Delta P^{\star 2}\, dt \leq \mu(\bar{v})^2$, $T \geq 0$, then $V(x) \leq 1$ $\forall t$. Furthermore, under null disturbances, the Lyapunov function decreases for all $V(x) \leq 1$.

*Proof:* Applying Lemma 4.1, restrictions (28) imply $\dot{V}(x) \leq \frac{1}{\mu(\bar{v})^2} \Delta P^{\star 2}$, $\forall V(x) \leq 1$. If null initial conditions are assumed, integrating both sides of the inequality leads to $V(T) \leq \int_0^T \frac{1}{\mu(\bar{v})^2} \Delta P^{\star 2}\, dt \leq 1$. On the other hand, if null disturbances are assumed, it leads to $\dot{V}(x) \leq 0$, $\forall V(x) \leq 1$. ■

*Remark 4.1:* The maximum non destabilizing energy demand $\mu(\bar{v})$ can be obtained by finfing the maximum $\mu(\bar{v})$ such that conditions stated in theorem 4.4 hold.

If, after an overproduction demand, $\Delta P^\star$ becomes 0, the system will return to its equilibrium point. The convergence ratio when returning to that point can be obtained by means of the following theorem.

*Theorem 4.5:* Given a Lyapunov function $V(x)$ fulfilling (28), if there exist a positive constant $\gamma > 0$, and a sum of squares polynomial $s_4(x)$ such that

$$-\dot{V}(x) - \gamma V(x) - s_4(x)(1 - V(x)) \in \Sigma(x), \tag{29}$$

being $\dot{V}(x) = \frac{\partial V(x)}{\partial x} f(x, \bar{v})$, then, the decay ratio from a point in the boundary of the reachable set with $\int_0^T \Delta P^{\star 2}\, dt \leq \mu(\bar{v})^2$ to the origin is bounded by $\gamma$.

*Proof:* Using lemma 4.1, constraint (29) implies $\dot{V}(x) < -\gamma V(x)$, $\forall V(x) \leq 1$, $\Delta P^\star = 0$, that, integrating, leads to $V(x(t)) < e^{-\gamma t} V(x(0))$. If the degree of the Lyapunov polynomial function is $d$, this is equivalent to $\|x(t)\|_d < c\, e^{-\frac{\gamma}{d} t} \|x(0)\|_d$, $c > 0$. ■

*Remark 4.2:* $\gamma$ is used to approximate the required return time to the equilibrium point after an overproduction demand. If a quadratic Lyapunov function is used, the time to reach the 98% of the state values at the equilibrium point will be $8/\gamma$ s.

*Theorem 4.6:* Given a Lyapunov function $V(x)$ fulfilling (28), if there exist a positive constant $\alpha > 0$, and a sum of squares polynomial $s_5(x)$, such that

$$\alpha^2 - h(x)^2 - s_5(x)(1 - V(x)) \in \Sigma(x), \tag{30}$$

then, the system output defined by $y = h(x)$ will be bounded by $\|y\|_\infty \leq \alpha$ for all inputs $\Delta P^\star$ bounded in energy by $\mu(\bar{v})$.

*Proof:* Using lemma 4.1, constraint (30) implies $\|y\| \leq \alpha$, $\forall V(x) \leq 1$. If (28) is fulfilled, $V(x) \leq 1$ is the reachable set for all inputs $\Delta P^\star$ bounded in energy by $\mu(\bar{v})$. ■

This last result can also be used to find the maximum allowable overproduction demand that assures that the operation of the wind turbine remains inside a given operation region (for instance, to assure that the speeds are between 0.8 and 1.2). The idea is to find the maximum $\mu(\bar{v})$ fulfilling both the statements of theorem 4.4 and 4.6.

## V. Simulation results

The techniques developed before have been tested for a given wind turbine model and controllers. First, a simulation model has been implemented, including all the effects considered in this work and a wind generation model as stated in section 2. Then, using the parser Yalmip for defining optimization problems subject to sum of squares constraints, and the SDP solver PENBMI, the bound of the norms for different outputs and operating ranges have been obtained (and approximated by polynomials $\gamma(\bar{v})$ depending on the mean speed wind). They have been compared with those obtained via simulation.

For instance, the relationship between RMS norms of the generated power and wind variations under wind turbulence has been obtained for different mean speeds in simulations, and compared to those obtained with a gridding approach along different mean wind speeds, applying the following optimization problem: minimize $\gamma$ subject to (25) and (27).

Also, the relationship between the maximum acceleration and the maximum wind turbulence $\|w\|_\infty$ has been explored through simulation and via the following optimization problem: minimize $\gamma$ subject to (25) and (26). The polynomials that approximate the performances as a function of the mean speed wind are shown in Fig. 2. Furthermore, a prediction strategy has been implemented including the wind observer explained in section III, and the estimated mean and standard deviation of the wind speed have been used together with those polynomials to predict the bounds ($\pm 3\sigma$ confidence interval) on power and acceleration fluctuations (see Fig. 3).

The model has also been tested to analyze the behavior under overproduction demands $\Delta P^\star$ with two different speed PI controllers (with $\frac{K_{p1}}{K_{p2}} = 4$). First of all, the maximum non destabilizing power demand has been obtained iteratively through simulations, and also via the optimization technique detailed in remark 4.1. Also, the recovery time has been estimated through the decay rate as explained in remark 4.2. Fig. 4 shows these values, and also the dynamic response to a power demand of 0.4 pu during 10 seconds, where it can be observed that the recovery time estimation obtained via optimization fits simulation data for both controllers.

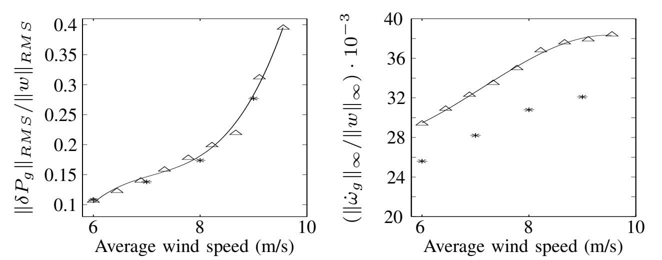


Fig. 2. Performance index as a function of the mean wind speed. $*$: simulation results; $\triangle$: gridding approach; –: polynomial approximation.

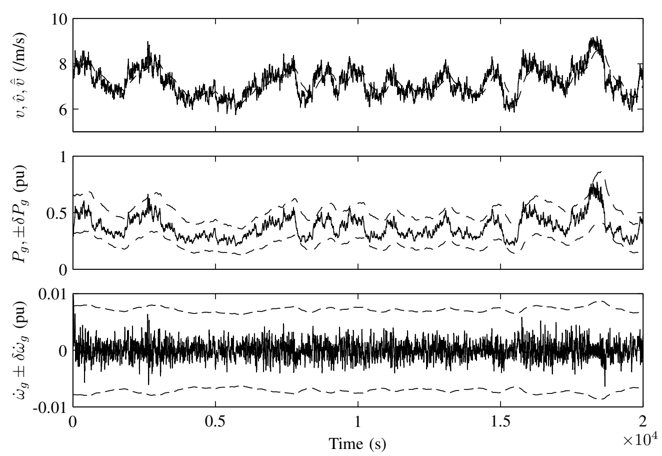


Fig. 3. Wind speed and its mean estimation; measured power; and acceleration (dashed lines: predicted bounds on the transient behavior).

## VI. Conclusions

In this work, a polynomial model for a controlled DFIG has been developed. Based on that model, a wind and state observer has been proposed, leading to an algorithm that estimates the mean and variance of the wind speed on real-time during wind turbine operation. For the proposed model, sum of squares techniques have been used to obtain different performance indices that allow to bound the norm

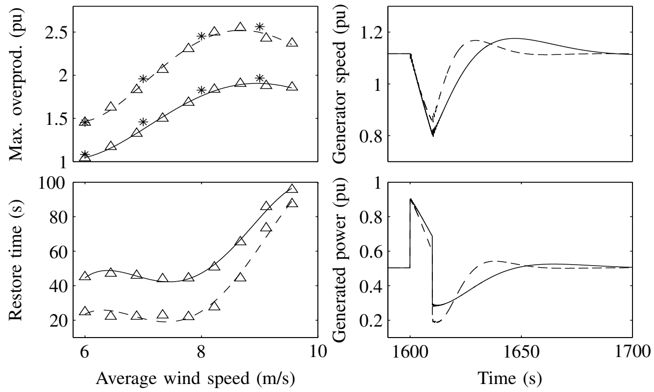


Fig. 4. Left: maximum non destabilizing overproduction demand, and restore time. Right: response to power demand. $K_{p1}$ ('- -') and $K_{p2}$ ('–')

of different outputs that are important to quantify the wind turbine behavior in two common situations: wind turbulence and overproduction demand. The wind observer plus the performance indices allow to know online the capabilities of the wind turbine to attenuate wind fluctuations or to transiently increase the generated power (that can be useful to grid support), giving also information about the needed time to restore reliably the turbine to normal operation. Some simulated examples demonstrate the validity of the approach.

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