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Robust Wiener Optimal Nonlinear Estimation for Uncertain Systems

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***Abstract*—A nonlinear operator based approach to robust estimation is introduced for discrete-time systems. It involves a signal entering a communications channel with nonlinearities, transport delay and uncertainties. The measurements are assumed to be corrupted by coloured noise which is correlated with the signal to be estimated. The signal and noise model parameters are assumed to be subject to perturbations represented by random variables with known means and covariances. The theoretical solution does not involve linearization or approximations. In the limiting case of a linear system the estimator has the form of a Wiener filter in discrete-time polynomial matrix system form.**

I. Introduction

THE optimal filter for linear systems are well known, like the Wiener ([1]) and the Kalman filter ([2, 3]) that have proved their value in applications. These are based on minimizing a statistical criterion and are optimal in an average sense. Over the last few years a new nonlinear estimation paradigm has emerged which leads to simple filters, smoothers and predictors for classes of nonlinear stochastic systems ([4-7]).

There are many techniques for nonlinear estimation and the best known is the Extended Kalman Filter (EKF). The EKF has a similar structure to the Kalman filter but has a nonlinear model within the loop. To accommodate the nonlinearity the model is linearized at each time step to estimate the transition matrix and this is used to update the estimated covariance matrix. The EKF does not include a model for channel dynamics that will be included in the following and the solution involves approximations.

In the following a related frequency domain or polynomial system approach to robust nonlinear estimation problems is presented. The system, signal and noise models are assumed to include uncertain elements that can be represented by linear models with probabilistic parameter deviations. The optimal robust filter, smoother, or predictor can be obtained from the results of a frequency weighted estimation problem. The estimation problem involves inferential estimation of a signal which enters a communication channel that contains nonlinearities and transport delays. The measurements are assumed to be corrupted by a coloured noise signal correlated with the signal to be estimated ([8]). The solution of the nonlinear estimation problem is obtained using nonlinear operators. The cost-function to be minimized involves the averaged variance of the estimation error and requires a very simple optimization procedure ([9]). The averaged mean square error has been previously used in literature by Grimble ([10]), Speyer and Gustfson ([11]), and Sternad and Ahlén ([12]). In the latter was demonstrated that if the uncertainty in the system elements is described by soft bounds, the optimal robust estimator can be found for the solution of the minimum variance problem. The estimator presented in this paper is relatively simple to understand and to implement. It has potential applications in control systems, fault monitoring, communications and signal processing systems.

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II. Signal Processing System Description

The signal and noise models are assumed to be time-invariant, asymptotically stable and discrete-time and represented in transfer-function or polynomial matrix form.

The signal to be estimated passes through a transmitting channel which possesses a delay z^{-k}, linear dynamics W_l and nonlinear dynamics W_{nl}. The signal generated by white noise goes into a colouring filter and then enters the linear subsystem W_l representing part of the channel dynamics which can be non-minimum phase (inverse unstable). It then enters the nonlinear subsystem W_{nl} which is assumed to be stable. The measurements are assumed to be corrupted by a noise signal $n(t)$. The message signal to be estimated is at the output of a linear block $s = W_c y$.

For greater generality a dynamic cost weighting function is introduced that penalizes the signal in a particular frequency range $s_q = W_q s$ and this becomes the signal to be estimated. The signal processing problem is illustrated in Fig. 1. An additional nonlinear parallel channel with dynamics F_{ip} and delay z^{-k} is also introduced in our nonlinear filtering problem (shown by dotted lines in Fig.1). This channel will not exist physically but can be used to represent uncertainties in the nonlinear subsystem. This provides additional design freedom.

In Fig. 1 the white noise signals ε and ω are assumed to be mutually statistically independent and trend free. The covariance matrices for the white noise signals are defined as $\mathrm{cov}[\varepsilon(t), \varepsilon^T(t)] = Q_s \delta_{t\tau}$ and $\mathrm{cov}[\omega(t), \omega^T(t)] = Q_n \delta_{t\tau}$,

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respectively, where $\delta_{t\tau}$ denotes the *Kronecker delta* function.

A. System Equation

In this section a mathematical description of the signals in the system is introduced. The estimation problem shown in Fig. 1 is first modified to the problem in Fig. 2. The mathematical justification and derivation of the innovation signals in Fig. 2 is described in [6].

Input Signal: $y(t) = W_s Q_s C_s^* D_{f0}^{*-1} \varepsilon(t)$ (1)

Noise Signal: $n(t) = W_n Q_n C_n^* D_{f0}^{*-1} \varepsilon(t)$ (2)

where $\varepsilon(t)$ is white driving noise

$$W_s(z^{-1}) = A_0^{-1}(z^{-1}) C_s(z^{-1}), \quad (3)$$

$$W_n(z^{-1}) = A_0^{-1}(z^{-1}) C_n(z^{-1}) \quad (4)$$

and $W_l(z^{-1}) W_s(z^{-1}) = A_0^{-1}(z^{-1}) C_{ls}(z^{-1})$ (5)

Linear channel subsystem output:

$$s_0(t) = W_l y(t) = W_l W_s Q_s C_s^* D_{f0}^{*-1} \varepsilon(t) \quad (6)$$

Channel input: $f(t) = s_0(t) + n(t)$ (7)

Nonlinear parallel channel:

$$\mathcal{F}_c(z^{-1}) = \mathcal{F}_{ip}(z^{-1}) z^{-k} \quad (8)$$

Channel Interference: $n_c(t) = (\mathcal{F}_c f)(t)$ (9)

Nonlinear channel input output:

$$s_d(t) = z^{-k} f(t) = f(t-k) \quad (10)$$

Nonlinear channel subsystem output:

$$s_c(t) = \left(\mathcal{W}_{nl} s_d\right)(t) \quad (11)$$

Observations signal $z_c(t) = n_c(t) + s_c(t)$ (12)

Message signal to be estimated:

$$s(t) = W_c y(t) = W_c W_s Q_s C_s^* D_{f0}^{*-1} \varepsilon(t) \quad (13)$$

Weighted message signal:

$$s_q(t) = W_q s(t) = W_q W_c y(t) = W_q W_c W_s Q_s C_s^* D_{f0}^{*-1} \varepsilon(t) \quad (14)$$

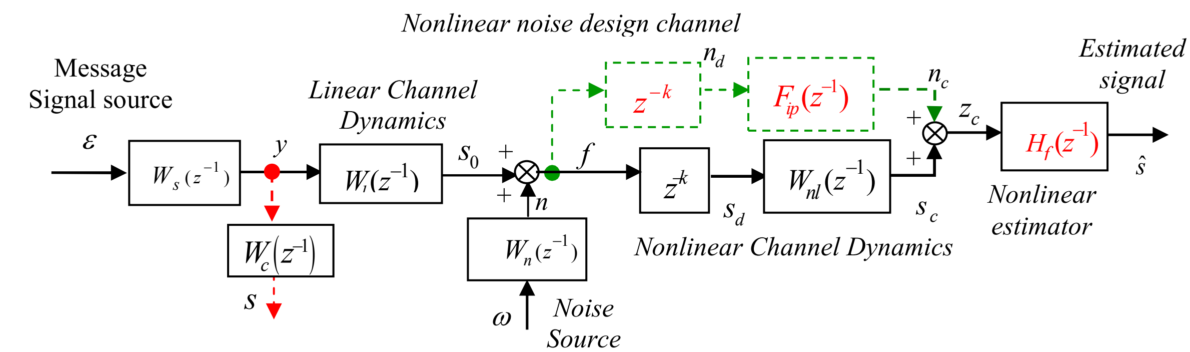

Fig. 1: Signal Source, Noise Sources and Nonlinear Communication Channel Dynamics

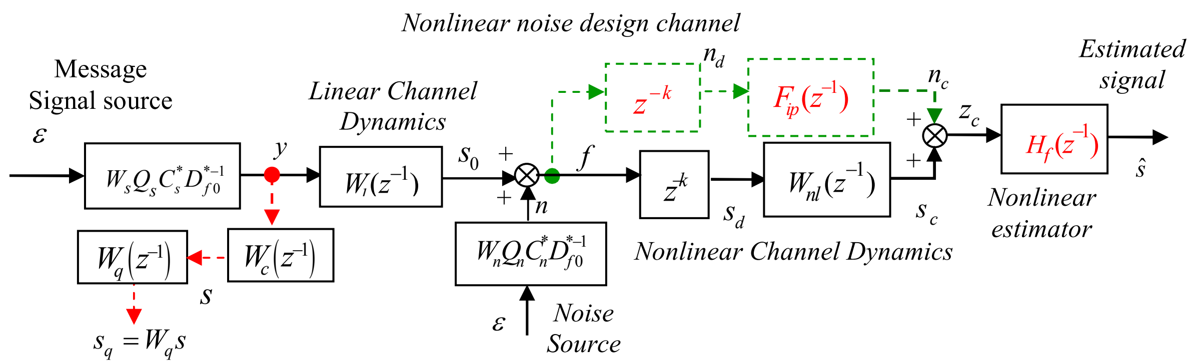

Fig. 2: Nonlinear Filtering Problem with Innovation Signals

B. Uncertain System Model Representation

The system models W_s, W_n and W_l are assumed to be uncertain, hence the notation for their models will now be modified to allow for the uncertainty:

$$W_s = \overline{W}_s \delta W_s$$

$$W_n = \overline{W}_n \delta W_n$$

$$W_l = \overline{W}_l \delta W_l$$

where $\overline{W}_i$ for i=s,n,l represents the nominal model and δW_s, δW_n and δW_l are linear in the random parameters.

For simplicity a scalar uncertain problem is considered. Let $E_p\{\cdot\}$ denote the expectation taken with respect to the random parameters, then for the scalar system the transfer function of the uncertainty δW_i for $i = s, n, l$ is assumed to have the following polynomial form: $\delta W_i = \delta W_{inum} / \delta W_{iden}$, where $E_p\{\delta W_{inum}\} = E_p\{\delta W_{iden}\} = 1$

The numerator and denominator terms, denoted as δW_{inum} and δW_{iden} respectively, can be written in the form $\delta W_{inum} = 1 + \delta\tilde{W}_{inum}$ and $\delta W_{iden} = 1 + \delta\tilde{W}_{iden}$ where $E_p\{\delta\tilde{W}_{inum}\} = E_p\{\delta\tilde{W}_{iden}\} = 0$.

For example, the second-order uncertain polynomials δW_{inum} and δW_{iden} may be represented in the linear form:

$$\delta W_{inum} = (1 + \alpha_1 z^{-1} + \alpha_2 z^{-2})$$

$$\delta W_{iden} = (1 + \beta_1 z^{-1} + \beta_2 z^{-2})$$

where the means and the variances of the random parameters are as follow:

$$E_p\{\alpha_1\} = E_p\{\alpha_2\} = E_p\{\beta_1\} = E_p\{\beta_2\} = 0$$

$$E_p\{\alpha_1^2\} = \sigma_{n1}^2, E_p\{\alpha_2^2\} = \sigma_{n2}^2, E_p\{\beta_1^2\} = \sigma_{d1}^2, E_p\{\beta_2^2\} = \sigma_{d2}^2$$

These random parameters are, for simplicity, taken to be independent.

The work by Grimble ([10]), reveals that the solution of the Wiener filter problem for an uncertain system in the form described is very similar to the traditional Wiener filtering problem. However, the spectral factor involving products of random variables include the corresponding covariance terms. The remaining polynomial models describing the plant, signal and noise transfers functions are simply the polynomials with mean levels of parameters included.

In the limiting case when the channel dynamics are absent and the problem reverts to a Wiener filtering problem we require the nonlinear proposed estimator to be identical to the optimal Wiener filter for the uncertain system.

The approximation taken is therefore to make these substitutions before solving the nonlinear estimation problem. For simplicity of notation it will assumed the polynomials therefore include mean values but where products of uncertain terms are present the expectation E_p will be included which signifies the variances of uncertain parameters will be involved.

III. Robust Optimal Estimator

The theory of the *NonLinear Minimum Variance (NMV) filter*, was introduced by *Grimble* ([4]) using polynomial system models ([9, 13, 14]). These models are summarized in the previous section. The *NMV filter* involves the minimization of variance of the estimation error:

$$\tilde{s}(t \,|\, t-l) = s(t) - \hat{\bar{s}}(t \,|\, t-l) \tag{15}$$

where $\hat{\bar{s}}(t \,|\, t-l)$ denotes the average with respect to the random parameters of the weighted estimation signal at time *t*, given observations $z_c(t)$ up to time $t-l$. The value of l may be positive or negative according to the following conditions ([15]):

$l = 0$, for estimation

$l > 0$, for prediction

$l < 0$, for fixed-lag smoothing

The criterion to judge optimality for the uncertain system models minimum variance estimation problem can be expressed as below ([16]):

$$J = trace\{E_s\{(W_q\tilde{s}(t \,|\, t-\ell))(W_q\tilde{s}(t \,|\, t-\ell)^T\}\} \tag{16}$$

where $E_s\{\cdot\}$ denotes the expectations with respect to the stochastic signals. The W_q ([9]) denotes a linear dynamic weighting function matrix which is assumed to be strictly minimum phase, square and invertible and may be represented in polynomial matrix form as:. $W_q(z^{-1}) = A_q^{-1}(z^{-1})C_q(z^{-1})$

A. Spectral Factorization

The solution of the nonlinear estimation problem requires the introduction of an average spectral factorization of the signal *f*. The *power spectrum* for the combined linear models can be calculated by using the Parseval's theorem ([9])

$$\Phi_{ff} = E_p\left\{(W_l W_s \varepsilon + W_n \omega)(W_l W_s \varepsilon + W_n \omega)^*\right\} \tag{17}$$

where the notation for the adjoint of W_s implies $W_s^*(z^{-1}) = W_s^T(z)$, and in this case the z denotes the z-domain complex number. The *averaged generalized spectral-factor* Y_f that is required may be computed using

$$Y_f Y_f^* = \Phi_{ff} \text{, where} \tag{18}$$

$$Y_f = A_0^{-1} D_{f0} \tag{19}$$

The system models are assumed such that D_{f0} is a *strictly Schur* polynomial matrix ([11, 17]) satisfying:

$$D_{f0} D_{f0}^* = E_p\left\{C_{ls} Q_s C_{ls}^* + C_n Q_n C_n^*\right\} \tag{20}$$

A realization of the averaged signal $\bar{f}$ with respect to the random parameters can be obtained from the average spectral factor: $\bar{f}(t) = E_p\{f(t)\} = Y_f \varepsilon(t)$ (21)

B. The Robust Wiener Optimal Estimator Solution

The estimator can be designed from the *spectral factor and Diophantine equation* to minimize the variance of the estimation error ([4] and [6]) given in equation (15).

The estimate $\hat{\bar{s}}(t \,|\, t-\ell)$ can be generated from a nonlinear estimator of the form:

$$\hat{\bar{s}}(t \,|\, t-\ell) = H_f(t, z^{-1}) z_c(t-\ell) \tag{22}$$

and it is shown in the next section that for an uncertain system this will be of the form:

$$H_f(t, z^{-1}) = G_f A^{-1}(\mathrm{F}_{ip} + W_{nl})^{-1} Y_f^{-1} \tag{23}$$

where $\mathcal{H}_f(t, z^{-1})$ denotes a minimal realization of the optimal nonlinear estimator. The block diagram representation of $\mathcal{H}_f(t, z^{-1})$ will be as shown in Fig.3.

The generalized spectral factor $Y_f = A_0^{-1} D_{f0}$ used in this filter can be obtained from the equation (20) where D_{f0} is required to be asymptotically stable. The *minimal degree* solution of G_0 and F_0 can be obtained with the help of the following Diophantine equation:

$$A_{cs} A_q F_0 + G_0 D_{f0}^* z^{-g} = E_p\left\{C_{cs} Q_s C_s^* z^{(k+l-g)}\right\} \tag{24}$$

The minimum value of theoretical variance in this case will be as follows:

$$J = trace\left\{F_0 D_{f0}^{*-1} D_{f0}^{-1} F_0^*\right\} \tag{25}$$

C. The Robust Wiener Optimal Estimator Solution Proof

To obtain a proof of the estimator we start from the expression of the weighted estimation error:

$$\tilde{s}_q(t \mid t-l) = s_q(t) - \hat{\hat{s}}_q(t \mid t-l) \tag{26}$$

Using equation (14) and (22):

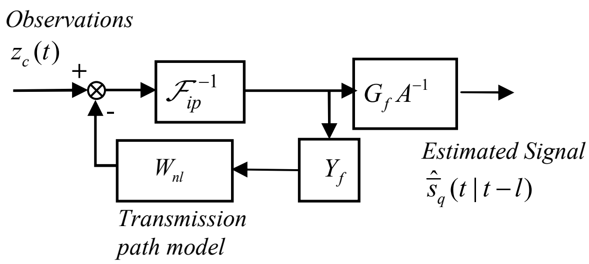

Fig. 3: Block Diagram of the Robust Optimal Estimator for Uncertain System.

$$\begin{aligned}&\tilde{s}_q(t \mid t-l) = s_q(t) - \hat{\hat{s}}_q(t \mid t-l) = \\ &E_p\left\{W_q W_c W_s Q_s C_s^* D_{f0}^{*-1}\varepsilon(t)\right\} - E_p\left\{W_q H_f z_m(t-l)\right\}\end{aligned} \tag{27}$$

Recall (12) and substitute in (27):

$$\begin{aligned}&\tilde{s}_q(t \mid t-l) = E_p\left\{W_q W_c W_s Q_s C_s^* D_{f0}^{*-1}\varepsilon(t)\right\} \\ &-E_p\left\{W_q H_f(n_c(t-l) + s_c(t-l))\right\}\end{aligned} \tag{28}$$

Considering equation from (8) to (11) and after simple manipulations we obtain:

$$\begin{aligned}&\tilde{s}_q(t \mid t-l) = E_p\left\{W_q W_c W_s Q_s C_s^* D_{f0}^{*-1}\varepsilon(t)\right\} \\ &-W_q H_f(F_{ip} + W_{nl}) E_p\left\{f(t-k-l)\right\}\end{aligned} \tag{29}$$

Recall (21) and substitute in (29):

$$\begin{aligned}&\tilde{s}_q(t \mid t-l) = E_p\left\{W_q W_c W_s Q_s C_s^* D_{f0}^{*-1}\varepsilon(t)\right\} \\ &-W_q H_f(F_{ip} + W_{nl}) Y_f \varepsilon(t-k-l)\end{aligned} \tag{30}$$

Advancing by $t+l+k$ in (30) we obtain:

$$\begin{aligned}&\tilde{s}_q(t+k+l \mid t+k) = \\ &E_p\left\{W_q W_c W_s Q_S C_s^* D_{f0}^{*-1} z^{(k+l)}\varepsilon(t)\right\} \\ &-W_q H_f(F_{ip} + W_{nl}) Y_f \varepsilon(t)\end{aligned} \tag{31}$$

Now introduce the left-coprime polynomial matrices for the weighted signal model ($W_q = A_q^{-1} C_q$):

$$A_{cs}^{-1} C_{cs} = C_q W_c W_s \tag{32}$$

Using (32) in (31) we can write:

$$\begin{aligned}&\tilde{s}_q(t+k+l \mid t+k) = \\ &E_p\left\{A_q^{-1} A_{cs}^{-1} C_{cs} Q_S C_s^*\right\} D_{f0}^{*-1} z^{(k+l)}\varepsilon(t) \\ &-W_q H_f(F_{ip} + W_{nl}) Y_f \varepsilon(t)\end{aligned} \tag{33}$$

The equation in (33) can be simplified using the Diophantine equation defined in (24):

$$\begin{aligned}&\tilde{s}_q(t+k+l \mid t+k) = (F_0 D_{f0}^{*-1} z^g + A_q^{-1} A_{cs}^{-1} G_0)\varepsilon(t) \\ &-A_q^{-1} C_q H_f(F_{ip} + W_{nl}) Y_f \varepsilon(t) = \\ &F_0 D_{f0}^{*-1}\varepsilon(t+g) \\ &+A_q^{-1}\left[A_{cs}^{-1} G_0 - C_q H_f(F_{ip} + W_{nl}) Y_f\right]\varepsilon(t)\end{aligned} \tag{34}$$

The second group of terms in the square brackets in (34) is all dependent upon past values of the white noise signals, whereas the first term depends only upon future values. It follows that these two groups of terms are statistically independent and the expected value of the cross terms is null.

Also note that the first term of (34) is independent of the choice of estimator. It follows that the smallest variance is achieved when the remaining terms are set to zero. Assuming the existence of a finite gain stable causal inverse to the non-linear operator the *optimal estimator* is obtained by setting this second group of terms to zero:

$$A_{cs}^{-1} G_0 - C_q H_f(F_{ip} + W_{nl}) Y_f = 0$$

From the above equation we obtain the following result:

$$H_f = C_q^{-1} A_{cs}^{-1} G_0 (\mathrm{F}_{ip} + W_{nl})^{-1} Y_f^{-1}$$

This relationship may be simplified by defining the following right coprime polynomial matrix as:

$$C_q^{-1} A_{cs}^{-1} G_0 = G_f A^{-1}$$

So that we obtain the result: shown in the previous section:

$$H_f(t, z^{-1}) = G_f A^{-1} (\mathrm{F}_{ip} + W_{nl})^{-1} Y_f^{-1}$$

IV. Experimental Results

The robust filter is computed below for a typical application and a simulation is used to verify the results. Consider a system having linear non-minimum phase channel characteristics as:

$$H(z^{-1}) = 0.3482 + 0.8704 z^{-1} + 0.3482 z^{-2}$$

The model is generally used in channel equalization case studies. The nonlinearity in the signal channel is modelled as $z(t) = \tanh\left(f(t - \Lambda_0)\right)$, where Λ_0 is the channel delay. The nonlinearity is a function of the signal output of the linear channel dynamics $H(z^{-1})$, and takes saturation effects, due to the transmitting amplifier, into account.

The overall system when implemented with the robust Wiener filter will be as shown in Fig.4.

In Fig. 5 is shown a comparison between actual and estimated signals using the Wiener NMV estimator ([6, 7]) and the robust Wiener estimator for the system described above without any uncertainty. The results from the estimators are the same as we could have expected from the theory, in fact with no uncertain models the robust wiener filter correspond to the Wiener NMV filter.

When the system plant differs from the nominal model, the estimation using the robust estimator is lower in term of variance of estimation error than the Wiener NMV filter (see Fig. 6).

In Fig. 7 an uncertainty is introduced in the signal and noise model. The robust filter variance error for this simulation also happens to be less than the variance error for the Wiener NMV estimation. Simulations were also carried out for either system plant and signal/noise models differing from their nominal models (see Fig. 8).

In table 1 there is a comparison between Wiener NMV estimation and robust optimal estimation carried out for two different values of the parallel channel dynamic (Case1 and Case2).

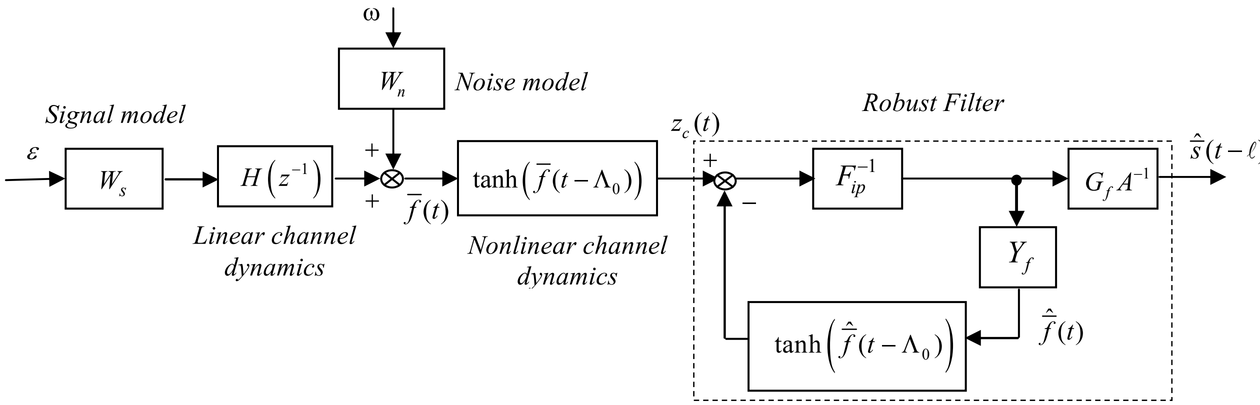

Fig. 4: System Model Along with Channel Dynamics and Robust Estimator

The expectation is that the robust estimator has benefits compared to the Wiener NMV estimator since it can have less sensitivity to dynamic model uncertainties.

It is useful to consider the limiting form of the estimator so that it may be related to existing filter solutions. In the limiting case as the nonlinear channel dynamics tend to the identity and the uncertainty weighting F_{ip} tends to zero, the estimator becomes equivalent to a *Wiener deconvolution estimator with uncertain models* in the linear polynomial matrix equation form $H_f = G_f A^{-1} Y_f^{-1}$

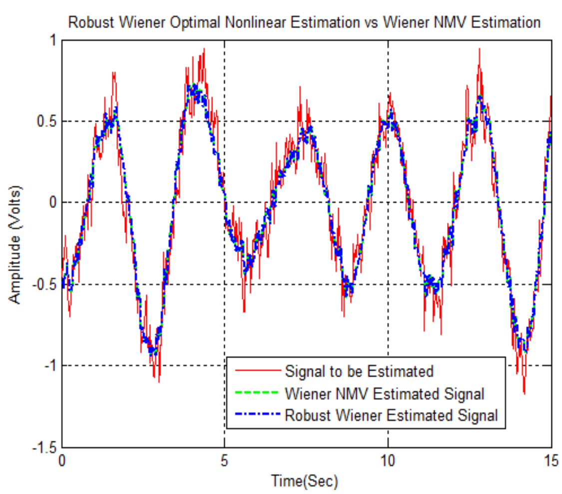

Fig. 5: Comparison between actual and estimated signals using Wiener NMV estimator and robust Wiener estimator without any uncertainty

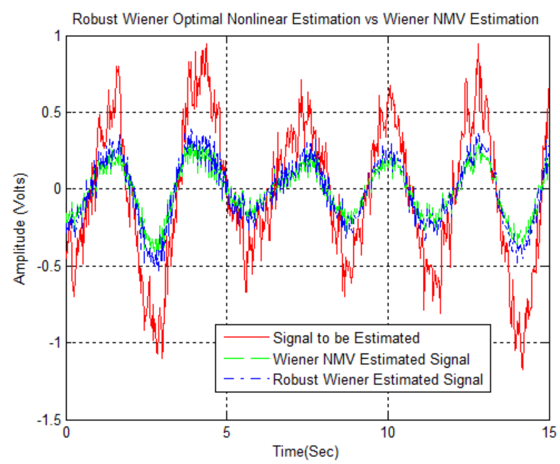

Fig. 6: Comparison between actual and estimated signals using Wiener NMV estimator and robust Wiener estimator with plant differing from its model

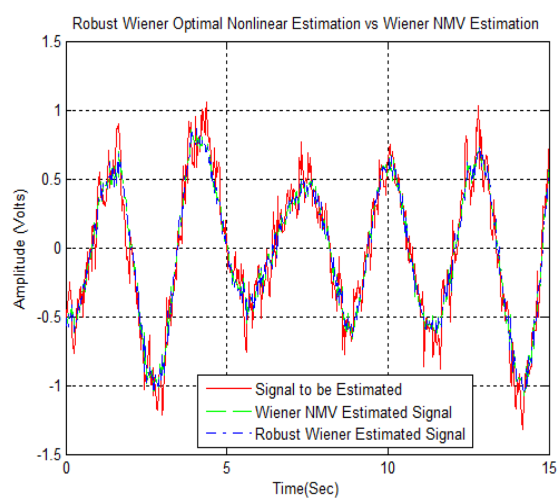

Fig. 7: Comparison between actual and estimated signals using Wiener NMV estimator and robust Wiener estimator for signal and noise uncertainty

Table 1: **Variance of the Estimation error for Wiener NMV estimator and robust Wiener estimator**

Variance of Estimation Error		**Wiener NMV Filter**	**Robust Wiener Filter**
	Uncertainties		
CASE 1	*None*	0.0187	0.0187
	Plant Model	0.1046	0.0826
	Signal & Noise Models	0.0220	0.0219
	Signal, Noise & Plant	0.1134	0.1034
CASE 2	*None*	0.0199	0.0199
	Plant Model	0.1044	0.0823
	Signal & Noise Models	0.0231	0.0228
	Signal, Noise & Plant	0.1135	0.1034

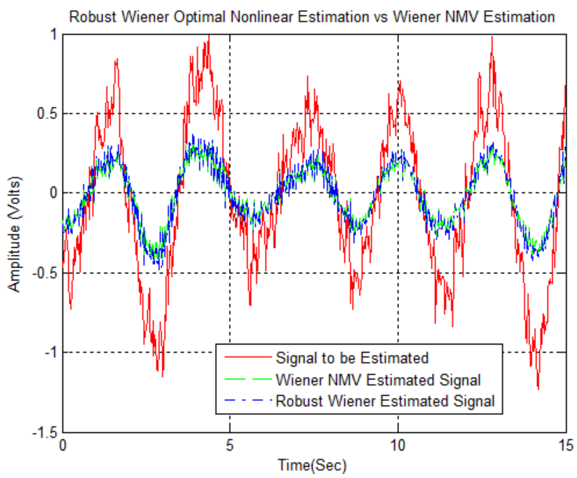

Fig. 8: Comparison between actual and estimated signals using Wiener NMV estimator and robust Wiener estimator for either plant and signal/noise uncertainty

V. Conclusion

The robust filtering problem is particularly appropriate for nonlinear uncertain systems. The uncertainty in signal, noise and plant are represented by a probabilistic system description and the variances are assumed to be given. The channel sub-system W_{nl} is represented by a nonlinear operator, which is a very general approach. Possible nonlinear channel dynamics uncertainties are represented in a parallel channel that also introduces design freedom.

In the limiting case when the dynamics are linear, the estimator has the form of a Wiener filter in polynomial system description form. The advantage of the solution is the relative simplicity of the theoretical approach and ease of implementation.

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