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# A method for torque-ripple compensation in non-ideal permanent magnet machines

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*Abstract* — **Real permanent magnet synchronous machines often suffer from significant deviations from ideal ones. They usually occur as the result of simplified and cost-effective manufacturing, but they are also the source of performance degradation. The main problem is the torque-ripple, which is inadmissible in most of the applications. This paper analyses the main sources of this torque-ripple and presents a method to compensate it. The technique used is based on measurements and off-line calculations, followed by real-time control algorithms. The effectiveness of the method is demonstrated on an experimental setup.**

## I. Introduction

Permanent magnet synchronous machines are used in a wide range of applications. The greatest benefit of them the permanent magnet used, which can create high flux densities in the air gap, thus high torque can be generated with less copper losses compared to other types of machines. The capability to produce high torque effectively also makes them ideal for direct-drive applications. In this case, gears are omitted, but more complex control algorithms are necessary.

However, an existing problem is that the torque generated by these machines is not constant, but changes periodically as the rotor rotates. This torque-ripple is undesirable in most of the applications.

A special field of direct-drive systems is the automobile industry. Significant research and development is taken place in direct wheel-drive systems. As the necessity of these systems is increasing, more cost-effective solutions are needed. Low-cost machines are not precisely designed and constructed, thus torque-ripple is even more persisting. Accordingly, compensation of these uncertainties is moving from hardware and mechanics to software. It may be a trend in the future to integrate less precise, but more inexpensive machines with complex algorithms in powerful embedded systems to reduce costs and enhance reliability. This paper presents a method for torque-ripple minimization to realize precise motor drives using low-cost, non-ideal machines. Results are demonstrated on an experimental setup.

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## II. Analysis of Permanent Magnet Machines

Permanent magnet synchronous machines can be grouped into two categories, based on the shape of the magnetic field in the air gap. The field follows either a sinusoidal or a square wave along the perimeter of the air gap. Accordingly, the induced voltage (back-emf) of these machines is either sinusoidal or trapezoidal, respectively [1], [2].

Trapezoidal emf machines are preferable for wheel-drive in automobile industry because of their simple physical design and lower manufacturing costs. Therefore this paper focuses on this type.

In-wheel motors are often constructed as “inside-out” motors. This means that the rotor is outside of the stator, and the permanent magnets are fixed on the inner surface of the rotor’s cylinder, as axially aligned magnet strips. Pole pairs are created by two magnets fixed on the opposite side of the rotor, and adjacent magnets are fixed in the opposite direction. The stator is in the inner side of the machine. It has three-phase, distributed winding.

### A. Ideal machines

Ideally, the magnetic flux is constant under the magnet pole, the winding is symmetrical, the axis of the coils are 120° out of phase and the spatial distribution of the magnetic flux generated by each phase is sinusoidal. According to these assumptions, the emf of each phase follows a trapezoidal waveform [1], [3].

If the magnetic field created by a magnetic pole pair rotates with $\omega$ angular rate, the voltage induced in phase **a** is $u_{ea}$. If the winding contains $N$ turns, the emf of the phase is

$$u_{ea} = k_a^{'} \omega \sum_{j=1}^{N} B_{aj} , \quad (1)$$

where $k_a^{'}$ is a constant which depends only on the physical properties of the machine, and $B_{aj}$ is the induction at the $j$-th wire. Equation (1) can be written in a simpler form:

$$u_{ea} = K_a(\alpha)\omega , \quad (2)$$

where $K_a(\alpha)$ is called the voltage or emf constant and is function of the $\alpha$ rotor angle. Ultimately, the emf is the function of the angle-dependent $K_a(\alpha)$ machine constant and the angular rate. Similar expressions can be written for

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the other two phases. The ideal emf waveforms can be seen on Fig. 1.

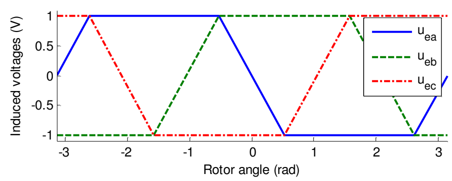


Fig. 1. Induced voltages of an ideal machine

On the other hand, if $i_a$ current flows in phase **a**, the phase creates $m_a$ torque with the rotor magnetic field. This can be formulated similar to (1) as

$$m_a = k_a^{'} i_a \sum_{j=1}^{N} B_{aj} \qquad (3)$$

or in a simpler form

$$m_a = K_a(\alpha) i_a , \qquad (4)$$

where $K_a$ is called the torque constant. Similar expression can be written to the other two phases. The resulting torque is the sum of the three phase torques:

$$m = m_a + m_b + m_c . \qquad (5)$$

The $K$ constants in (2) and (4) are equivalent, therefore the mechanical power of the machine can be expressed as

$$p_m = m\omega = u_{ea} i_a + u_{eb} i_b + u_{ec} i_c . \qquad (6)$$

To generate constant, ripple-free torque, such phase currents are required that (6) remains constant at a given $\omega$ angular rate. These currents can be derived intuitively from Fig. 1, and result in the well-known two-phase rectangular current waveforms or commutated DC currents.

### B. Non-ideal machines

In real machines, there are several sources of torque-ripple. The spatial distribution of the magnetic field in the air gap does not exactly follow a square wave. Also, at the edges of the magnets the field is distorted. Another source is the non-ideal winding. The windings are in slots, thus not perfectly distributed and the shape of the teeth is also not ideal. These deviations affects the induced emf voltages, thus the ideal trapezoidal waveforms are no longer valid. Applying the two-phase rectangular currents to the machine, according to (6) results significant torque-ripple.

From the stator winding point of view, the shape of emf only depends on the flux linked to the asymmetrical, non-ideal winding, the real air gap flux is irrelevant. Thus all the deviations from the ideal construction can be taken into consideration by the $\phi_a(\alpha)$ flux linkage, which is the function of the rotor position. It can be shown [1] that the emf or torque constant can be expressed by this flux linkage:

$$K_a(\alpha) = k_a^{''} \phi_a(\alpha) , \qquad (7)$$

and the (2) expression of the induced voltage is still valid. According to (6), the torque-ripple can be eliminated, if – instead of the two-phase rectangular currents – three-phase continuous currents that are matched to the induced voltages are applied to the machine.

To formulate this, first define the so called Park- or Rácz-transformation in the following form [3]:

$$\mathbf{i} = \frac{2}{3}\left(i_a + \mathbf{a} i_b + \mathbf{a}^2 i_c\right) , \qquad (8)$$

where $\mathbf{i}$ is called Park-vector and $\mathbf{a} = e^{j\frac{2\pi}{3}}$. Apply this to the phase induced voltages and phase currents in stator coordinate-frame results in the following:

$$\mathbf{u}_e = \frac{2}{3}\left(u_{ea} + \mathbf{a} u_{eb} + \mathbf{a}^2 u_{ec}\right) \qquad \mathbf{i} = \frac{2}{3}\left(i_a + \mathbf{a} i_b + \mathbf{a}^2 i_c\right) . \qquad (9)$$

Equation (6) can be rewritten in the following form:

$$p_m = m\omega = \frac{3}{2}\mathbf{u}_p \cdot \mathbf{i} + 3u_{p0} i_0 , \qquad (10)$$

where the product means scalar product, and the torque can be expressed as

$$m = \frac{p_m}{\omega} = \frac{3}{2}\mathbf{K} \cdot \mathbf{i} + 3K_0 i_0 . \qquad (11)$$

If the neutral point of the machine is not connected, the $K_0 i_0$ zero sequence torque can be neglected, because $i_0 = 0$ , thus

$$m = \frac{3}{2}\mathbf{K}(\alpha) \cdot \mathbf{i}(\alpha) . \qquad (12)$$

To produce constant, ripple-free torque, (12) scalar product has to be constant for all $\alpha$ rotor angles [1].

The analytical expression of $\mathbf{K}(\alpha)$ would be difficult to derive, because all the asymmetries and deviations from the ideal should be modeled.

According to the above, current matching can be evaluated in two steps. First, the induced voltages have to be measured in no-load generator mode at a given $\omega$ angular rate, then $\mathbf{K}(\alpha)$ has to be constructed. Second, the current matching has to be done based on (12), thus the matched $\mathbf{i}(\alpha)$ current vector has to be computed for every $\alpha$ angle by

$$\mathbf{i}(\alpha) = \frac{2}{3} m \frac{1}{\mathbf{K}^*(\alpha)} , \qquad (13)$$

where the $m$ torque is constant.

Fig. 2. depicts the emf voltages measured on the motor used in the experimental setup. The associated matched currents can be seen on Fig. 3, the current Park-vector is depicted on Fig. 4.

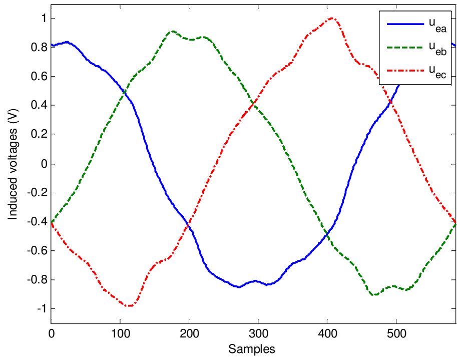


Fig. 2. Measured induced voltages

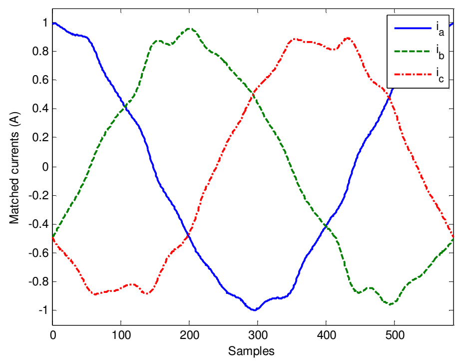


Fig. 3. Matched phase currents

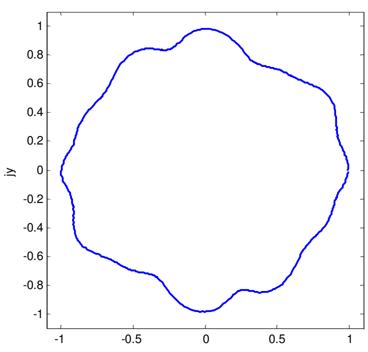


Fig. 4. Matched current Park-vector

For ripple-free torque generation this current vector has to be applied to the machine, depending on the actual rotor position. This type of control is similar to the method used with sinusoidal synchronous machines, where it is called filed-oriented vector control. These matched currents also result in minimal copper losses.

Note, that in the ideal case with the two-phase rectangular currents, the current Park-vectors would be identical to the vertices of a hexagon.

It should also be noted that in the narrower sense, the Park-transformation is the basis of the field vector approach of electrical machines, and in this sense it can only be applied when the stator windings are symmetrical and the flux they generate is sinusoidal. However, in the wider sense, the Park-transformation is only a mathematical transformation, independently from any spatial layout, and can be interpreted as a 3/2 phase conversion. According to this, using Park-transformation in the non-ideal and not symmetrical case is still correct [3].

## III. TORQUE CONTROL SYSTEM

The voltage vector equation of the machine in stator coordinate frame is:

$$\mathbf{u} = \mathbf{R}\mathbf{i} + \mathbf{L}\frac{d\mathbf{i}}{dt} + \mathbf{u}_e , \qquad (14)$$

where $\mathbf{R}$ and $\mathbf{L}$ is the stator winding resistance and inductance matrix, respectively.
According to expression (7) of the flux, the elements of the $\mathbf{L}$ inductance matrix are all functions of the $\alpha$ rotor angle.

The equation of motion of the machine is:

$$m = \Theta\frac{d\omega}{dt} + D\omega + m_l , \qquad (15)$$

where $\Theta$ is the rotor inertia, $D$ represents the friction and $m_l$ is the loading torque.

Based on (14) and (15) the dynamic model of the machine is:

$$\begin{aligned} \mathbf{L}\frac{d\mathbf{i}}{dt} &= -\mathbf{R}\mathbf{i} - \mathbf{u}_e + \mathbf{u} \\ \Theta\frac{d\omega}{dt} &= m - D\omega - m_l \, . \\ \frac{d\alpha}{dt} &= \omega \end{aligned} \qquad (16)$$

To generate ripple-free torque, the computed current vector has to be applied to the machine. The control scheme belonging to the case when two independent controllers are applied can be seen in Fig. 5.

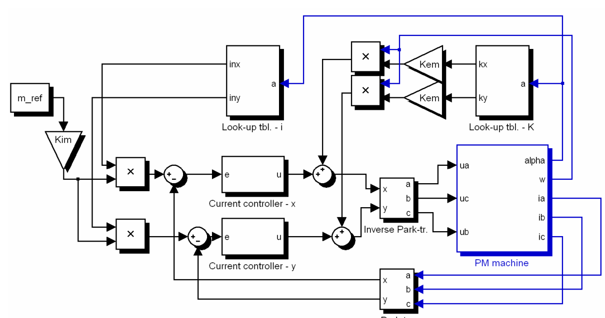


Fig. 5. Block diagram of the control scheme

There are two current controllers for the two orthogonal components of the Park-vector. Thus Park-transform and inverse Park-transform has to be applied to the three measured phase-currents and the computed phase-voltages, respectively.

The previously measured emf and off-line calculated current vectors are stored in look-up tables. The waveforms are re-sampled to get as many samples as it fits to the resolution of the rotor angle measurement. Furthermore, these values are normalized by their peak value, thus all of them fall between ±1, as it can be seen on Fig. 2-4. The constant $K_{em}$ is defined as the maximum of $K_a(\alpha)$, $K_b(\alpha)$, $K_c(\alpha)$ for all $\alpha$.

The reference signal of the torque controller is scaled by an appropriate constant $K_{im}$ and modulated by the normalized current vector components that are addressed from a look-up table by the rotor angle. These provide the reference signals for the current controllers. The measured and transformed currents are subtracted from the reference signals and then the control algorithms are applied. The computed control signals are modified by the addition of the actual emfs, to realize real torque control. The actual emfs are computed by the product of the measured angular rate and the normalized emf constants addressed form a look-up table by the rotor angle, scaled by $K_{em}$. The control signals are then inverse Park-transformed and the three-phase voltages are applied to the machine. The normalized, orthogonal components of the current Park-vector can be seen on Fig. 6.

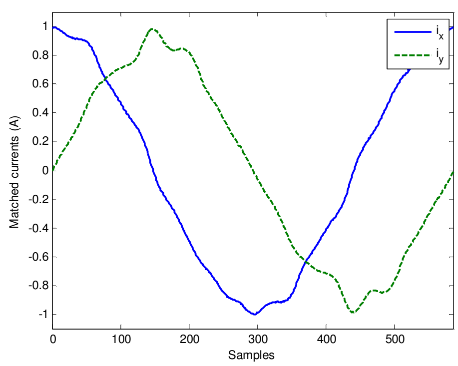


Fig. 6. Orthogonal components of the current Park-vector

Although, numerical values are calculated off-line and stored in look-up tables, and transformations require high computation power, since phase currents are not independent if the neutral point of the motor is not connected, and cannot be controlled separately, thus the Park-transformation is required as it decouples the three currents into two independent currents. In this scheme the controllers are operating in the stator coordinate frame, thus the reference signals are periodic AC signals. As a summary, it can be said that the presented method generates only the reference signals for the current controllers.

The elements of the inductance matrix in (16) are all functions of the rotor angle. Consequently, it is recommended to use current controllers, which can cope with the varying parameter in the system and robust enough to satisfy the appropriate performance criteria. However, only the dynamic part of the model (the winding inductance) depends on the rotor angle, the static part (the winding resistance) remains constant, thus if the velocity is low and the sampling frequency is high enough, conventional controllers can be used.

As the compensating method based upon the induced voltage and computed current value pairs, accurate rotor angle measurement is required. If the position sensor resolution is low, or the measurement is noisy, false readout from the look-up table can produce increased torque-ripple. As a result, an accurate position sensor is needed or some type of position estimation algorithm also has to be implemented, which can increase costs. This can be a disadvantage of the method.

If angular rate control is also needed, it should be implemented as a higher hierarchical level controller above the closed-loop torque control system in a cascaded scheme.

It should be noted, that the presented method only eliminates torque-ripple that comes from the asymmetries and deviations from the ideal construction. The other component of torque-ripple which comes from the reluctance torque between the magnets and the teeth (cogging torque) is not compensated in this scheme. But it simply can be measured and eliminated by feed-forward compensation.

## IV. Experimental Setup

The presented algorithm was implemented and tested in an embedded control system, based on a Texas Instruments C2000 digital signal controller. The power electronics are built from MOSFETs, thus the control signals are phase voltages. The rotor angle and angular rate is measured by a 12-bit quadrature encoder. Hence, the above mentioned waveforms with the seven-pole-pairs machine that is used, are 585 increments long. All the three phase currents are measured by shunt resistors and Hall-element current transducers.

The frame of the software is written is C language, the controllers are evaluated in MATLAB/Simulink and compiled by Real-Time Workshop.

The current controllers are conventional PI controllers with a sampling time of 54 us, which is low enough for accurate current control. The floating-point DPS's clock frequency is 150 MHz, thus all the signal processing algorithms, transformations and control loops can be computed in one sampling period.

Tests have been carried out using the machine mentioned above and applying rectangular and matched current waveforms at different angular rates. The results can be seen on Fig. 7. and Fig.8.

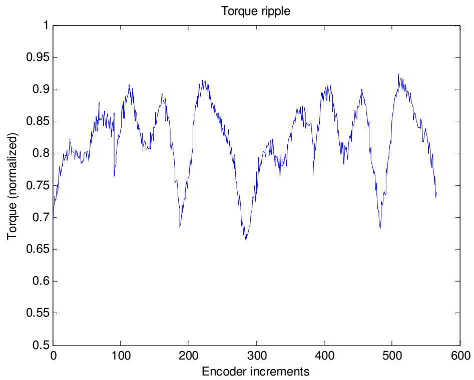


Fig. 7. Torque-ripple using rectangular currents

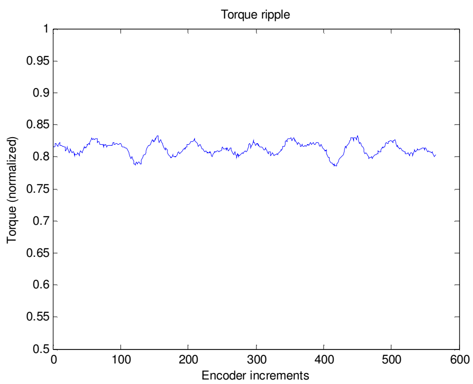


Fig. 8. Torque-ripple using matched currents

It can be observed that using the matched currents, torque-ripple can be reasonably reduced.

## V. CONCLUSION

The method presented in this paper shows, that precise, torque-ripple-free motor drives can be realized by integrating low-cost, less precisely constructed machines with complex control algorithms. One of its most important benefits, it that the asymmetries and deviations from an ideal machine are not needed to be modeled analytically. Instead, look-up tables can be used based upon measurements and off-line calculations. However, as a result, the matched currents are only valid for one individual series of machine. But these measurement and calculations can be done in a simple way, thereafter significant improvements can be achieved in torque-ripple minimization.

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