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A Model Predictive Control Approach to the Load Shifting Problem in a Household Equipped with an Energy Storage Unit

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Abstract— **This paper deals with the load shifting problem in a household equipped with smart appliances and an energy storage unit with conversion losses. The problem is faced by establishing an event driven Model Predictive Control framework aiming to meet the real life dynamics of a household and to keep low the impact of the control system on the total electric energy consumption. The proposed approach allows the consumer to minimize the daily energy cost in scenarios characterized by Time of Use tariffs and Demand Side Management, by dynamically evaluating the best time to run of the appliances and the optimal evolution of the battery level of charge. A proper set of realistic simulations validates the proposed approach, showing the relevance of the energy storage unit in the domestic load shifting architecture.**

I. INTRODUCTION

As a consequence of the increasing share of renewable and volatile energy sources into distribution electricity grids, a growing need for positive and negative balancing power is expected for the coming years [1]. Demand Side Management (DSM) programs can provide the required flexibility and stability to the power system, by influencing the temporal shift of energy demand. Based on the exchange of price and volume signals among proper market actors and the consumers under their contractual control, DSM policies can address short-term grid requirements, such as: (*i*) the provision of positive tertiary reserve capacity by decreasing the demand when the electricity system falls short of providing sufficient capacity and (*ii*) the provision of negative tertiary reserve capacity by increasing demand if an oversupply occurs.

In Europe the residential sector covers about 28% of the total electricity consumption [2]; this explains the significant expectation about the possibility for this domain to contribute to an efficient operation of the network, resulting in the application of Time of Use (ToU) tariffs and stimulating the research about feasible implementations of the DSM concept [3]. Due to the variety of pricing programs and the need of guaranteing a proper level of comfort and cost-effective operation to the consumers, a domestic energy management system appears as the natural solution to minimize the cost related to the energy consumption and to automatically react to DSM messages coming from outside, by shifting the activation of the electric devices.

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Load shifting problems have recently attracted a significant attention from different research and application fields; among the criteria that can be used to shift loads, there are for example the overload management, the minimization of energy cost when a ToU tariff and DSM are used, the maximization of self consumption or thermal comfort. Two of the most relevant works for the industrial sector, [4] and [5], are based on the optimization framework. The main drawback of these works is the inabilty to manage external disturbances or inaccurate system models; re-optimization is not included to compensate for these deficiencies and then there is no active control of the load shifting process. In [6], the load shifting problem is modeled as a discrete time optimal control problem, and solved by using standard optimization techniques. Basically, the objective is to control a set of conveyor belts used to load coal on trains that transport the product to a final terminal. A new optimization is performed each time a train arrives; so doing the controller takes into account the dynamic evolution of the environment. The approach adopted in [6] provides useful suggestions for the residential case in order to meet the real life dynamics of a household.

Focusing on the residential sector, preliminary works on energy management systems date back to the beginning of the 90’s (e.g. [7][8]). Among the most recent and relevant works, in [9] a simulation model that generates daily load profiles under flat tariffs is presented, and an optimization framework is established to evaluate the changes in these profiles when households are equipped with smart appliances and face time-based electricity prices. Real time interaction with the users, overload management and the non-deferrable loads are not considered in the problem formalization. Further, an interesting architecture and different approaches for load shifting are presented in [10], [11], [12]. In particular, in [11] a distributed scheduling system based on artificial neural networks in a house equipped with local solar panels and a storage unit is considered. The system provides a feasible load shifting schedule enhancing self-consumption for the next day. Once again the approach cannot be used in a real time framework, since the user must provide a list with the appliances to be executed within the next 24 hours. Moreover, non-deferrable loads are not explicitly taken into account: these drawbacks are rather recurring in the relevant literature. A solution is proposed in [13] for a dwelling characterized by a set of household appliances (without energy storage units and micro generation) through the event driven optimization approach suggested in [6] and by introducing a Virtual Power Threshold (VPT), which takes

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into account the forecast of not plannable loads built by using proper load estimators.

The aim of this paper is to extend the work made in [13]. We face a more general problem consisting in the dynamical shift of houdehold appliances programs and, at the same time, in the determination of the control for the recharge/discharge process of a domestic energy storage unit. The presence of a battery in the domain makes the problem an event driven decision and control problem. In particular the proposed framework makes use of an event driven Model Predictive Control (MPC) approach. In typical MPC problems the current control action is obtained by solving, at each sampling instant, a finite horizon open-loop optimal control problem, using the current state of the plant as the initial state; the optimization yields an optimal control sequence and the first control in this sequence is applied to the plant [14]. In this work an open loop optimal control problem is solved each time an event such as a request from the consumer for the execution of a load or a DSM signal triggers the controller; there is no need to perform the optimization task at each sampling instant, which allows to maintain the controller in stand-by hot mode for most of the time, then minimizing the impact of the controller on the energy consumption. So doing the control eventually applied to the inverter of the storage unit is the concatenation of the control between each pair of time instants at which two consecutive events occur. Furthermore, the best time to run of each appliance evolves up to the moment in which the load actually starts, since it depends on the sequence of events triggering the controller. This approach allows to meet the real life dynamics of a household, achieving cost effective use of energy while preserving the quality of experience. The proposed formalization of the problem allows the controller to work in two different scenarios:

- Scenario 1: *Optimization of economic saving in case of ToU Tariffs*. In this scenario, the control system has to shift loads and control the storage unit in order to minimize the energy cost based on a given time varying energy price, while assuring overload management.
- Scenario 2: *Demand Side Management*. This scenario takes into account intra-day interaction between the consumer and an upper level actor. The control system shifts the loads and updates the control law to be applied to the storage unit in reaction to a DSM message which modifies the energy price or the power threshold during a specific temporal slot.

The paper is organized as follows. Section II presents the system architecture and particularly the devices involved in the event driven MPC problem. Section III is dedicated to loads and energy storage unit modeling. In section IV the working logic is presented. In section V the formalization of the event driven optimal control problem is given. Finally sections VI and VII are dedicated to the presentation of simulation results, related discussion and conclusions.

II. SYSTEM ARCHITECTURE

System architecture is the same considered in [13] with the introduction of the energy storage unit. The equipment and the related features are here briefly recalled. The device hosting the control strategy is the *Smart Home Controller* (SHC), which is in charge of managing the loads and the storage unit based on the data received from the domain and on DSM messages. The role of the SHC is played by an *Home Residential Gateway*, which establishes a ZigBee connection with the following devices:

- the *Smart Appliances*, able to provide power consumption forecast for each requested program and to be remotely activated;
- the *Energy Storage Unit*, constituted by a controllable inverter and a battery;
- the *Smart Plugs* and the *Smart Meter*, able to provide local and aggregated power metering respectively.

III. MODELING

A. Loads

In this section the models of the loads are briefly recalled. For an extensive explanation of the motivations and the implications related to the modeling choices the reader can refer to [13]. By evaluating the information made available by the devices involved in the system architecture, basically the loads can be classified in two main categories:

- *plannable loads* (PLs): those loads for which consumption forecasts are available and it is possible to choose start times, typically smart appliances;
- *not plannable loads* (NPLs): all the loads that cannot be shifted but can be monitored in a local and/or aggregated way through the smart meter and the smart plugs.

It is straightforward that only PLs can directly partecipate to an open loop optimal control problem consisting in the choice of the optimal loads start times. NPLs can only indirectly partecipate by properly estimating their aggregated consumption during the day.

As in [13] a smart appliance program consists of two active power discrete time sequences:

- an *average power time sequence*, that can be used for the evaluation of the cost related to the execution of the program; it will be expressed as $\bar{P}[n] = \bar{P}[nT_s], n = 0, 1, ..., N-1$, where T_s denotes the sampling time and NT_s is the duration of the program;
- a *peak power time sequence*, that can be used to check for undesidered oveloads and can be expressed as $\hat{P}[n] = \hat{P}[nT_s], n = 0, 1, ..., N-1$.

The model here considered for NPLs is given by the estimation of their aggregated power consumption. It can be expressed by the daily time sequence

$$p^*[k] = \bar{p}[k] + \mu S(p[k]) \qquad k = 0, 1, ..., \frac{60}{T_s} \cdot 24 - 1 \quad (1)$$

where $\bar{p}[k]$ and $S(p[k])$ are the estimators of the expected value $E(p[k])$ and variance $\sigma^2(p[k])$ at time instant kT_s

respectively, calculated over proper sets of historical data, taking into account seasonal variations in the energy consumption and diversifying working days and holidays. Finally, μ is a tuning parameter to be properly chosen. The model (1) plays a key role in the overload management. If P_T denotes the power threshold established by the energy contract, the SHC can work considering as power constraint the VPT given by the sequence

$$P_T^*[k] = P_T - p^*[k] \qquad k = 0, 1, ..., \frac{60}{T_s} \cdot 24 - 1 \quad (2)$$

B. Storage Unit

In the following a discrete time dynamical model of the storage unit is given. The device is supposed to be characterized by a fixed and equal level of power ΔP during release and charging operations; then, when used in a discrete time framework, it always exchanges with the grid the same quantum of energy ΔPT_s during a sampling period. Due to inefficiency in the conversion process a portion ξ of this quantum is typically loss; then the energy effectively stored is $\Delta PT_s(1-\xi)$, while the released one needed to guarantee a power supply ΔP is $\Delta PT_s(1+\xi)$. By introducing the boolean control variables u_k and v_k allowing the activation of the storage unit, the amount of the stored energy x at the generic time instant $(k+1)T_s$ is given by

$$x[k+1] = x[k] + \Delta PT_s(1-\xi)u_k - \Delta PT_s(1+\xi)v_k \quad (3)$$

Since recharge and release operations are not allowed at the same time, the following *storage activation constraint* holds:

$$u_k + v_k \leq 1 \quad (4)$$

In order to put the storage dynamics in a significant form from the energy efficiency point of view, (3) can be handled as follows:

$$x[k+1] = x[k] + \Delta PT_s(u_k - v_k) - \xi\Delta PT_s(u_k + v_k) \quad (5)$$

So doing it is natural to perform the following transformation in the control variables space,

$$U[k] = u_k - v_k \qquad V[k] = u_k + v_k = |U[k]| \quad (6)$$

by which the discrete time dynamical model of the quantized storage unit is finally given by

$$x[k+1] = x[k] + \Delta PT_s U[k] - \xi\Delta PT_s V[k] \quad (7)$$

The interpretation of the variables introduced in (6) is the following. The signal $U[k]$ represents the control variable related to an ideal exchange of energy with the grid ($\xi = 0$), while $V[k]$ can be seen as a disturbance resulting in the loss of a quantum of energy $\xi\Delta PT_s$ each time the storage unit is kept active. Taking into account a specific time period from a start time instant ST_s to an end time instant ET_s, the aggregated loss of energy is given by

$$\sum_{k=S}^{E-1} \xi\Delta PT_s V[k] \quad (8)$$

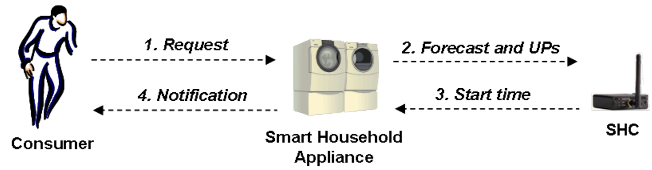

Fig. 1. Consumer/appliance interaction.

IV. SHC WORKING LOGIC

In this section the SHC working logic is presented. The general idea is to perform an event driven control acting on the smart household appliances and the storage unit Two different kind of events can trigger the controller: (*i*) user requests (R events), (*ii*) price/volume signals, sent by an upper level actor (DSM events). Depending on the event triggering the SHC, a proper control law is actuated.

The most frequent event is the R one. In order to clearly define it and the related SHC reaction it is fundamental to explain the interaction here considered between the user and the smart appliance, which can be summarized as follows:

1) The consumer prepares the appliance, asks for a program and indicates the preferred first possible start time and the last acceptable end time; this couple of time instants are referred in the following as the user preferences (UPs). Notice that the preferred first possible start time could be the current time at which the user makes the request, but in general it could be a later time instant.
2) The appliance performs a forecast of the power consumption related to the execution of the program, as described in section III-A. It sends the forecast together with UPs to the SHC. This is what in the following is called an R event.
3) The SHC decides for the best time to run and calculates the control law acting on the storage unit. A notification is sent to the appliance.
4) The appliance shows the notification to the user on a display and remains in stand-by up to the start time.

This interaction is illustrated in Fig. 1. In this simple case the SHC is in charge of performing a single-load and storage control aiming to optimize the economic saving and/or to avoid virtual power threshold violation, while assuring UPs.

Now let's consider a sequence of R events that follow the first one, one by one. In this case, once the SHC is reached by a new request, the control law is updated, taking into account the current event, the appliance that have not started yet and the storage state feedback. Loads already started are considered as further contributions to the VPT constraint.

The same working logic allows a proper reaction to a DSM event. In this case the energy tariff or the available power threshold are modified by an actor outside the home domain; then the control law is updated and a check for user convenience is performed.

In the light of above, the SHC is in charge of performing an *event driven optimal control*. The working logic and closed loop control scheme is summarized in Fig. 2.

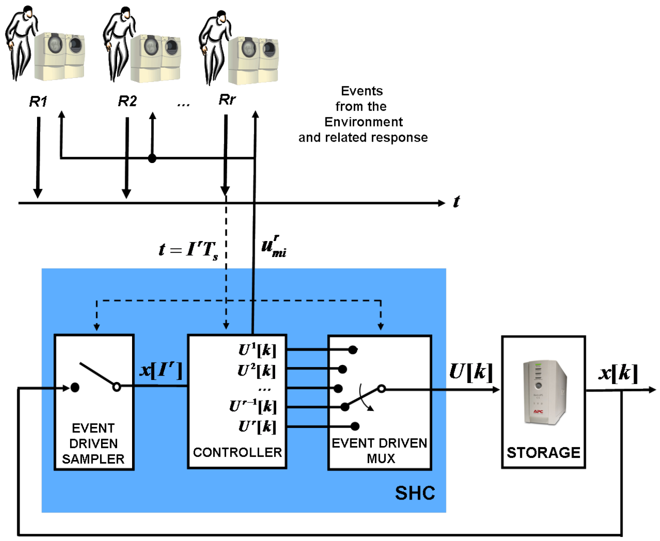

Fig. 2. Control system scheme.

V. SHC Problem Formalization

In this section the mathematical formulation of the event driven optimal control problem to be solved after the generic *r-th* event is given. In what follows the superscript *r* is used on parameters, sets and variables that are evaluated after the event. As a clarification about time, we premise that an event can trigger the SHC in whatever continuous time instant. The next discrete time instant I^rT_s is in the following referred to as the *trigger time* and represents the first available time instant for the actuation of the updated control law.

A. Target function

The target function has to take into account all the cost items related to PLs and the storage unit. As far as concerns PLs let's consider the generic *m-th* smart appliance program, characterized by a duration N_mT_s, a forecast $(\bar{P}_m[\cdot], \hat{P}_m[\cdot])$ and UPs $(I_{Sm}T_s, I_{Em}T_s)$. Given a daily energy price time sequence $C[n]$, the cost related to the execution of the program starting at time instant iT_s is given by

$$\sum_{n=i}^{i+(N_m-1)} \bar{P}_m[n-i]T_sC[n] \tag{9}$$

Now in order to take into account all the costs related to the appliances in a compact form, we introduce the generic boolean decision variable u^r_{mi} and sum over all the feasible start times. So doing we have

$$\sum_{i=I^*_{Sm}}^{I_{Em}-N_m} \left[\sum_{n=i}^{i+(N_m-1)} \bar{P}_m[n-i]T_sC[n] \right] u^r_{mi} \tag{10}$$

where

$$I^*_{Sm} = max\left\{I_{Sm}, I^r\right\} \tag{11}$$

is introduced in order to guarantee causality, then in case excluding as possible start times the time instants which are compatible with the UPs but are prior to the trigger time I^rT_s. Moreover notice that the last feasible start time is $(I_{Em} - N_m)T_s$, since $I_{Em}T_s$ represents the last time instant within the program has to be terminated, according to user preferences. Then, the cost related to the whole set of smart household appliances to be controlled is given by

$$\sum_{m\in M^r} \sum_{i=I^*_{Sm}}^{I_{Em}-N_m} \left\{ \sum_{n=i}^{i+(N_m-1)} \bar{P}_m[n-i]T_sC[n] \right\} u^r_{mi} \tag{12}$$

where M^r represents the set of appliances for which a request exists and not started yet at trigger time I^r. As far as concerns the storage unit we have to consider the cost related to the charging process and the saving given by the supply operation over all the control horizon. The net cost is given by

$$\sum_{k=S^{r*}}^{E^r-1} \Delta PT_sC[k]U^r[k] \tag{13}$$

where S^{r*} and E^r, defined by

$$\begin{aligned} S^{r*} &= \max\left\{S^r, I^r\right\} \\ S^r &= \min\left\{I_{Sm} : m \in M^r\right\} \\ E^r &= \max\left\{I_{Em} : m \in M^r\right\} \end{aligned} \tag{14}$$

represents the boundaries of the control horizon.

Finally, the target function is given by

$$\begin{aligned} &\sum_{m\in M^r} \sum_{i=I^*_{Sm}}^{I_{Em}-N_m} \left[\sum_{n=i}^{i+(N_m-1)} \bar{P}_m[n-i]T_sC[n] \right] u^r_{mi} + \\ &\qquad + \sum_{k=S^{r*}}^{E^r-1} \Delta PT_sC\left[k\right]U^r\left[k\right] \end{aligned} \tag{15}$$

B. Prediction model

The prediction model is obtained by introducing in (7) the control variable U^r, which results in

$$x[k+1] = x[k] + \Delta PT_sU^r[k] - \xi\Delta PT_s\left|U^r[k]\right| \tag{16}$$

C. Control constraints

Control constraints are related either to the nature of the variables or to the values that can be assumed in relation to overload management and the activation of the storage unit. Decision and control variables must satisfy

$$u^r_{mi} \in \{0,1\} \quad m \in M^r \quad i = I^*_{Sm}, ..., I_{Em} - N_m \tag{17}$$

$$U^r[k] \in \{-1,0,1\} \qquad k = S^{r*}, ..., E^r - 1 \tag{18}$$

the latter one coming from the definition of the control variable given in (6), subject to the *storage activation constraint* (4).

Since there exists only one real start time for each smart household appliance program, and according to the boolean nature of the decision variables, the following constraints hold:

$$\sum_{i=I^*_{Sm}}^{I_{Em}-N_m} u^r_{mi} = 1 \qquad m \in M^r \tag{19}$$

As far as concerns the constraint for overload avoidance, we have to check for the peak power contribution each program gives at each time instant belonging to the control horizon. At generic time instant kT_s the peak power consumption of the *m-th* load can be written in the compact form as

$$\sum_{i=k-(N_m-1)}^{k} \hat{P}_m[k-i]u_{mi}^r \tag{20}$$

Nevertheless, in order to take into account the boundaries of the user preference interval, summation index limits must be modified to provide the feasible contribution

$$\sum_{i=(k-(N_m-1))^{I_{Sm}^*}}^{k-(k-I_{Em}+N_m)^+} \hat{P}_m[k-i]u_{mi}^r \tag{21}$$

where $(\cdot)^+ = max[\cdot, 0]$ and $(\cdot)^\alpha = max[\cdot, \alpha], \alpha \in N$. Then, by considering all the programs involved in the planning, the aggregated peak power contribution can be expressed as

$$\sum_{m \in M_k^r} \left[\sum_{i=(k-(N_m-1))^{I_{Sm}^*}}^{k-(k-I_{Em}+N_m)^+} \hat{P}_m[k-i]u_{mi}^r \right] \tag{22}$$

where

$$M_k^r = \{m \in M^r : I_{Sm}^* \leq k \leq I_{Em} - 1\} \tag{23}$$

represents the subset of loads giving a possible contribution at time instant kT_s. This contribution has to be increased with the aggregated peak power $\hat{P}_S[k]$ from programs already started and thus no more subject subject to control, and finally with the power absorbed by the storage unit ΔP. All this power cannot exceed the VPT augmented with the power released by the storage unit. In formula

$$\begin{aligned} \sum_{m \in M_k^r} \left[\sum_{i=(k-(N_m-1))^{I_{Sm}^*}}^{k-(k-I_{Em}+N_m)^+} \hat{P}_m[k-i]u_{mi}^r \right] + \\ + \hat{P}_S[k] + \Delta P U^r[k] \leq P_T^*[k] \\ k = S^{r*}, S^{r*}+1, ..., E^r - 1 \end{aligned} \tag{24}$$

Last constraint directly related to control is about the operation of power supply from the storage unit. We assume that released power has to be totally consumpted in the home domain, without any energy injection into the distribution grid. It means that the working in release mode ($U^r[k] = -1$) is allowed only if there is enough planned load. To build a proper constraint we can reason the same way as for the overload management, by considering the average power time sequences instead of the peak power ones. As done before, the aggregated contribution from loads subject to the control task has to be augmented with the aggregated average power $\bar{P}_S[k]$ from programs already started; the resulting power must be greater than the power ΔP in order to allow the storage unit to work in control mode. Finally it is mathematically expressed as

$$\begin{aligned} \sum_{m \in M_k^r} \left[\sum_{i=(k-(N_m-1))^{I_{Sm}^*}}^{k-(k-I_{Em}+N_m)^+} \bar{P}_m[k-i]u_{mi}^r \right] + \bar{P}_S[k] + \\ + \Delta P U^r[k] \geq 0 \\ k = S^{r*}, S^{r*}+1, ..., E^r - 1 \end{aligned} \tag{25}$$

D. State and termination constraints

The evolution of the storage state, based on the prediction model given by (7) with initial condition $x[S^{r*}]$, is bounded due to the limits in the capacity to store energy. We can express the related *state constraint* simply as

$$0 \leq x[k+1] \leq x_{max} \quad k = S^{r*}, ..., E^r - 2 \tag{26}$$

Finally, as usually done in control problem based on prediction model, we put a *termination constraint* aiming to guarantee a minimum level of charge at the end of the control horizon. It can be expressed as

$$x_{ref} \leq x[E^r] \leq x_{max} \tag{27}$$

E. Overall problem definition

In the light of above, the SHC event driven optimal control problem resulting from the analysis of the scenarios and system architecture described in section I and II respectively can be summarized as follows.

SHC event driven optimal control problem. *For a given trigger time $I^r T_s$, a daily energy price time sequence $C[n]$, a virtual power threshold time sequence P_k^*, a storage unit with power ΔP, maximum capacity x_{max} and losses ξ, a set M^r of user requests with related average and peak power time sequences pairs $(\bar{P}_m[n], \hat{P}_m[n])$ and UPs intervals $[I_{Sm}, I_{Em}[$, solve*

$$\min_{u,U} \left\{ \begin{aligned} \sum_{m \in M^r} \sum_{i=I_{Sm}^*}^{I_{Em}-N_m} \left[\sum_{n=i}^{i+(N_m-1)} \bar{P}_m[n-i]T_s C[n] \right] u_{mi}^r + \\ + \sum_{k=S^{r*}}^{E^r-1} \Delta P T_s C[k] U^r[k] \end{aligned} \right\} \tag{28}$$

subject to the dynamics (16) with initial condition $x[S^{r*}]$, the control constraints (17), (18), (19), (24), (25), the state constraint (26), and the termination constraint (27), where the boundaries S^{r*} and E^r of the control horizon are given by (14), while the set M_k^r is defined by (23). The SHC optimal control problem is equivalent to a binary linear programming problem. It is straightforward to verify it symply by using recursively the prediction model in (26), (27), performing the change of variables (6) and adding the constraint (4).

F. Composition of control signals

Reasoning on a sequence of events, we can finally reconstruct the control signal effectively applied to the energy storage unit, which is given by

$$U^1([I^1, I^2)) \cup U^2([I^2, I^3)) \cup ... \cup U^r([I^r, I^c]) \qquad (29)$$

where I^c denotes the current time.

VI. Simulation Results

In this section simulation results are shown and discussed. Simulations have been performed by using Matlab and CPLEX tools, on a PC with 2.4 GHz Intel Core 2 Duo CPU and 4 GB memory. Each optimization problem has been solved using the CPLEX function *cplexbilp* called from the Matlab environment, which is based on well known branch and cut methods [15].Depending on the features of the problem to be solved, and making use of comparison tests among different methods presented in the relevant operations research literature, CPLEX is able to identify the best optimization strategy, then choosing the best algorithms in the steps towards optimality: continuous relaxation solution, cutting and branching processes [16].

Two sets of simulation are presented: at first a simulation is shown as a proof of concept aiming to clarify and validate the approach; then a pair of more complex simulation is discussed, covering both the scenario described in section I.

The household under investigation is characterized by 4.5 kW subscribed power threshold and subject to the following ToU tariff implemented in Italy by the retailer Edison:

Off-Peak	[00:00-08:00) ∪ [19:00-24:00)	16.75 €cent/kWh
Peak	[08:00-19:00)	21,22 €cent/kWh

The VPT is built according to (2); the consumption forecast (1) from NPLs is supposed to be given by the superposition of three gaussian functions, centered respectively in 8:00, 13:00 and 20:00, the maximum amplitudes of which are respectively 0.5 kW, 0.5 kW and 1.4 kW. The storage unit is characterized by $x_{max} = 3$ kWh and $\xi = 0.02$. The initial condition is set to 1 kWh and the termination value is $x_{Ref} = 1$ kWh in order to make a comparison with the situation in which the storage unit is absent. All the simulations are characterized by a sampling time of $T_s = 5$ minutes and have been performed with no timeout, then achieving a benchmark for the SHC performance.

TABLE I
Proof of Concept.

Event ID	*R1*	*R2*	*R3*	*R4*
Trigger time	18:00	18:40	19:20	20:00
UP start time	18:00	18:40	19:20	20:00
UP end time	20:00	20:40	21:20	21:20
Load	*Plan*	*Plan*	*Plan*	*Plan*
1	18:30	18:30	18:30	18:30
2	N.A.	19:10	19:10	19:10
3	N.A.	N.A.	20:45	21:00
4	N.A.	N.A.	N.A.	20:20
c_{op} [€cent]	45,2	86,0	91,6	120,6

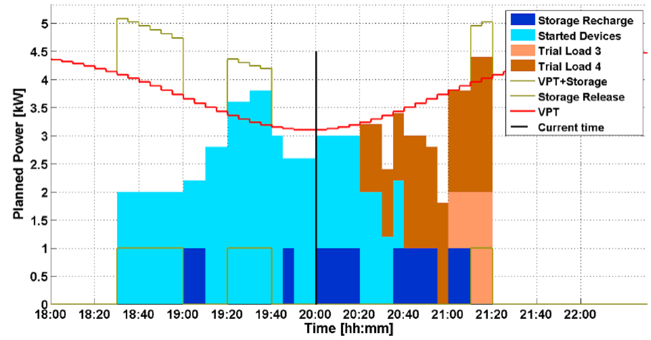

Fig. 3. Proof of concept: SHC planning occurring after R4 event.

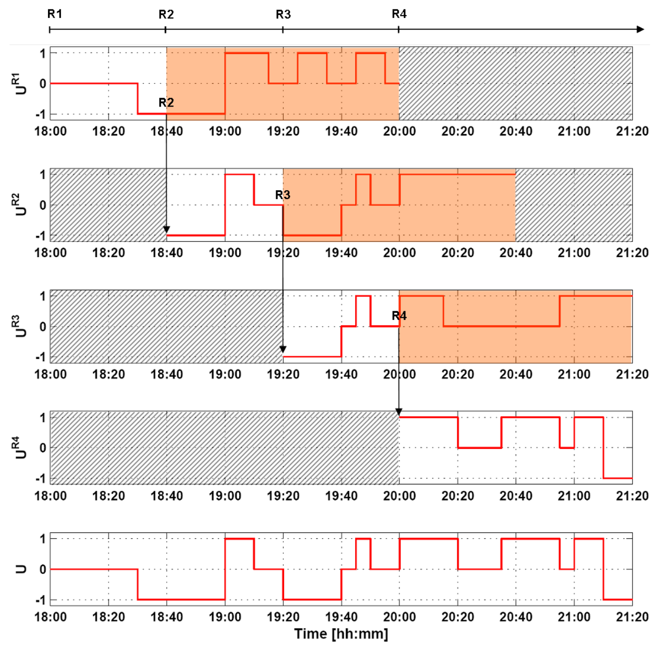

Fig. 4. Proof of concept: control evolution.

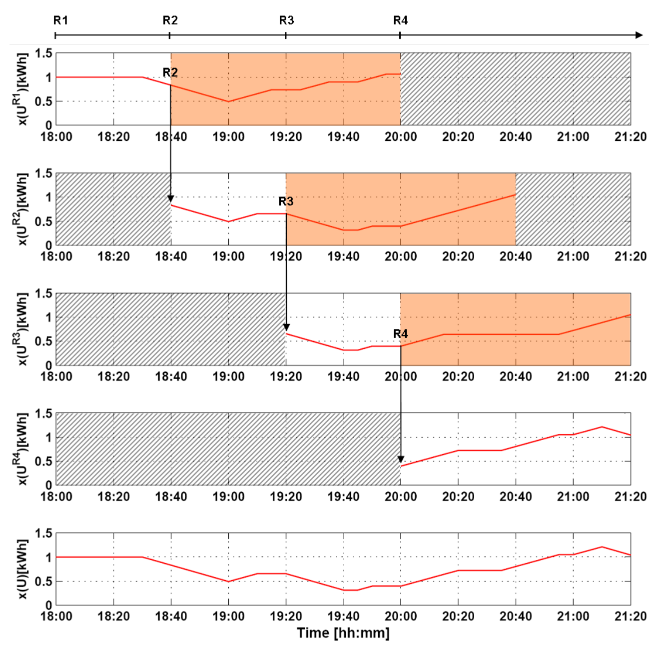

Fig. 5. Proof of concept: state evolution.

A. Proof of Concept

The aim of this simulation is to show how SHC works. A set of sample load profiles has been used here. The SHC is triggered by a sequence of 4 R-events occurring every 40 minutes starting from 18:00. The details of the events, the start times chosen by the SHC and the aggregated optimal cost c_{op} during the day are reported in Table I. Notice that after R2 and R3, the SHC can plan only the start times of the loads 2 and 3 respectively, since the other appliances have already started; on the contrary, after R4 the plan concerns loads 4 and 3 also, because load 3 is not started yet at R4-trigger time. The situation resulting from the reaction to R4 is depicted in Fig. 3, where the action of the storage unit is represented also; it is possible to distinguish the consumption already happened (on the left of the vertical bold line) to what is established to happen (on the right).

The storage unit works three times in release mode: the first one during the peak period to avoid the purchase of energy at peak price; the second and third ones to allow the appliances to run during time periods in which the peak consumption exceeds the VPT. The recharge is split in 5 time periods for which this additional consumption is allowed: for those periods, characterized by congestion, notice how the VPT is followed from below.

We remark that the storage control is the result of the composition of the controls evaluated after each R-event in the sequence, as it is shown in Fig. 4. For each row of the figure, it is represented the control calculated after the corresponding event, which can be divided in the portion actually applied to the plant (white background) and the portion which is updated as a consequence of the upcoming event; the last row represents the whole optimal control actually applied to plant, the one represented in Fig. 3 also. The same considerations hold for the state evolution in Fig. 5: each row represents the evolution predicted after an event, while the last one shows the actual evolution of the charge level resulting from the actuation of the optimal control; this evolution is the optimal one that, together with the shifted appliances start times, allows the consumer to keep minimum the energy cost.

B. Scenarios Simulation

In order to evaluate the performances of the system and in particular the relevance of the storage unit, two simulations of the scenarios described in section I have been performed. The used load profiles have been provided by the appliances manufacturer Electrolux S.p.a. and are summarized in Table II. Scenarios simulations have a common part, which is characterized by a sequence of 7 R-events and the related SHC reactions; after R7 we consider two different simulations: (*i*) the SHC is triggered by the event R8, (*ii*) at first the SHC is triggered by a volume signal notifying the limitation of the power threshold to 3 kW during the time period 20:30-21:30 and then by R8. Simulation results are shown in Table III, where each column reports: scenario ID, the details of the event, loads start times, cost of execution c in absence of the control system, optimal costs c_{op} and c^s_{op} resulting from the

TABLE II
LOAD PROFILES.

Load	Phases	$\Delta t[min]$	$\bar{P}[kW]$	$\hat{P}[kW]$
1	6	[5,10,15, 5,5,10]	[0.02,2.0,0.02, 0.02,0.02,0.05]	[0.15,2.1,0.15, 0.15,0.2,0.55]
2	6	[20,15,35, 10,20,50]	[0.07,2,0.07, 0.07,1.8,0.01]	[0.1,2.1,0.1, 0.25,2.3,0.02]
3	7	[5,25,20,5, 10,10,20]	[0.04,2.0,0.3,0.06, 0.06,0.06,0.08]	[0.2,2.1,2.1,0.2, 0.3,0.3,0.5]
4	6	[15,30,10, 5,20,50]	[0.07,1.4,0.1, 0.07,2.0,0.01]	[0.1,2.1,1.2, 0.1,2.2,0.02]
5	8	[25,5,60,20, 10,10,10,20]	[0.3,0.05,2.1,0.1, 0.1,0.1,0.1,0.3]	[2.1,0.3,2.2,0.2, 0.6,0.8,0.8,1.1]
6	1	[105]	[2.4]	[2.7]
7	3	[50,20,50]	[0.8,0.5,1]	[1,0.8,1.2]
8	4	[20,20,10,15]	[1.4,0.5,0.6,1]	[1.6,0.8,0.6,1]

TABLE III
SIMULATIONS RESULTS.

Scenario	1-2	1-2	1-2	1-2	1-2	1-2	1-2	1	2	2
Event ID	*R1*	*R2*	*R3*	*R4*	*R5*	*R6*	*R7*	*R8*	*DSM*	*R8*
Trigger Time	11.00	12.00	13.00	14.00	15.00	16.00	17.00	18:00	17.20	18.00
Time	11.00	12.00	13.00	14.00	15.00	16.00	17.00	18:00	20.30	18.00
period	18.00	20.00	24.00	24.00	24.00	24.00	24.00	24:00	21.30	24.00
Load ID	Plan	Plan	Plan	Plan	Plan	Plan	Plan	Plan	Plan	Plan
1	11:00	11:00	11:00	11:00	11:00	11:00	11:00	11:00	11:00	11:00
2	N.A.	17:30	17:30	17:30	17:30	17:30	17:30	17:30	17:30	17:30
3	N.A.	N.A.	22:05	22:25	20:25	22:05	21:25	21:20	21:25	21:20
4	N.A.	N.A.	N.A.	21:50	21:50	20:40	19:00	21:40	19:00	21:35
5	N.A.	N.A.	N.A.	N.A.	21:20	19:25	20:35	19:20	21:20	18:40
6	N.A.	N.A.	N.A.	N.A.	N.A.	22:05	22:15	22:15	22:15	22:15
7	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	19:00	19:05	19:10	19:50
8	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	20:50	N.A.	19:30
c [€cent]	8,7	33,9	54,5	84,7	134,7	222,5	257,8	276,9	257,8	276,9
c_{op} [€cent]	8,7	32,5	48,8	72,6	112,1	181,4	210,7	226,3	216,9	232,0
Saving [%]	0,00	4,07	10,51	14,27	16,79	18,48	18,27	18,29	15,86	16,22
c^s_{op} [€cent]	8,7	32,0	48,3	72,1	111,6	180,9	208,8	223,9	208,8	224,3
Saving [%]	0,00	5,45	11,37	14,82	17,14	18,69	19,01	19,16	19,01	19,01

SHC reaction in absence and presence of the storage unit respectively, related savings. The aggregated power profiles resulting from SHC reaction to R8 in scenarios 1 and 2 are shown in Figs. 6 and 8 respectively, while the related optimal control and state evolutions are depicted in Figs. 7 and 9.

The observation of results raises several remarks. In general, at the end of the day (after R8) the control systems guarantees an economic saving greater than 19% with respect to the cost the consumer would experience if all the load would start at the time of the requests; it holds for both the scenarios. Further, making a comparison between c_{op} and c^s_{op}, it can be noticed that in scenario 1 the portion of saving due to the storage unit is c_{op}(R8) - c^s_{op}(R8) = 2.4 €cents; this value increases significantly in scenario 2, where it is 7.7 €cents: we remark that these values are affected by the choice of UPs, and are expected to be greater when an higher number of UPs time periods have no intersection with the off-peak period.

Finally the relevance of the storage unit appears significant in relation to the additional cost which results from the SHC reaction to DSM; in the case there is no storage in the architecture, this cost is given by c_{op}(DSM) - c_{op}(R7) = 6.2 €cents, while the same value evaluated in presence of the storage unit is c^s_{op}(DSM) - c^s_{op}(R7) = 0 €cents. This result raises several non trivial consequences from the business model point of view in relation to the identification of the storage unit owner or provider: for example if the investment

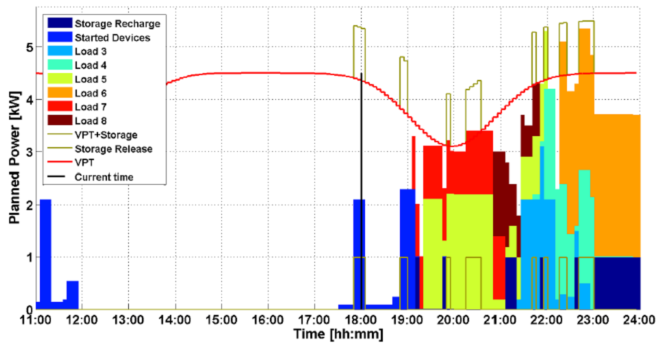

Fig. 6. Scenario 1: SHC planning occurring after R8 event.

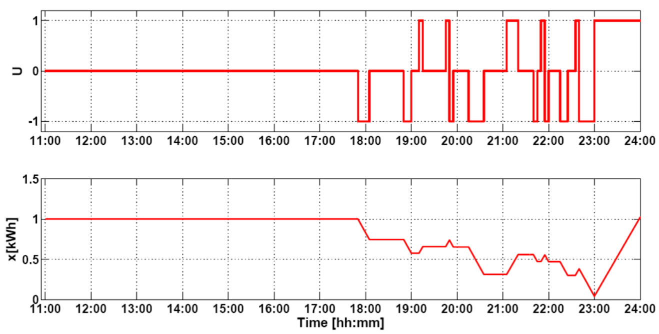

Fig. 7. Scenario 1: control and state evolutions.

for the purchase is made by the consumer, he should be allowed to claim 6.6 €cents as minimum rebate for positive reaction to the DSM event.

VII. Conclusions

In this work the load shifting problem in a household characterized by smart appliances and an energy storage unit with conversion losses has been faced. An event driven MPC framework has been established, which implies that a discrete time open loop optimal control problem is solved by using the current battery level of charge as initial condition, each time the controller is triggered by a proper event. The optimal control allows the consumer to keep the daily energy cost minimum in scenarios characterized by a ToU pricing model and DSM. Simulation results show the relevance of the energy storage unit in the load shifting architecture for the home domain, which gives rise to interesting issues from the business model point of view. Promising directions for future works consider the extension of the given formalization to local photovoltaic generation and real time pricing models.

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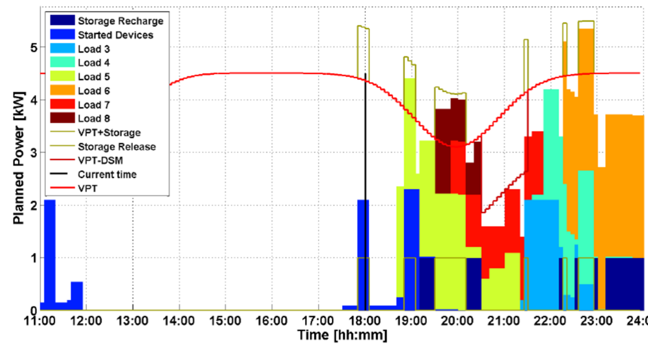

Fig. 8. Scenario 2: SHC planning occurring after R8 event.

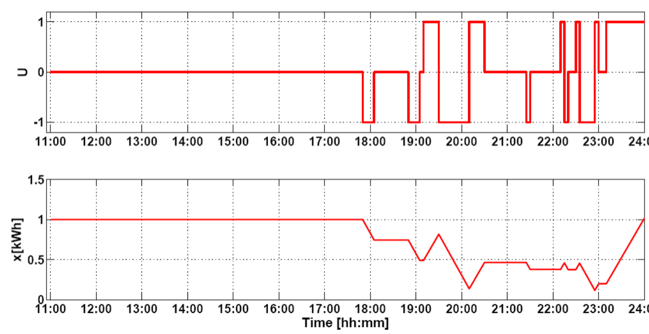

Fig. 9. Scenario 2: control and state evolution.

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