Preprints of the 2012 20th Mediterranean Conference on Control & Automation (MED), Barcelona, Spain, July 3-6, 2012

Autonomous Deployment of Heterogeneous Mobile Agents with Arbitrarily Anisotropic Sensing Patterns

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***Abstract*— This article examines the problem of autonomous optimal deployment of the nodes in a sensor network with heterogeneous anisotropic patterns. The sensing footprints of the latter are allowed to be any arbitrary convex set instead of circular, while the members of the network are considered heterogeneous, as far as the scaling factor of the aforementioned pattern is concerned. The proposed coordination algorithm relies on suitable partitioning of the sensed space, based on certain Helly–type theorems for planar convex curves, guaranteeing distributed information flow. Results are further confirmed via simulation studies in comparison to circular–approximation–based ones.**

***Index Terms*— Heterogeneous Sensor Networks, Distributed Optimization, Coverage Control, Voronoi Diagrams**

I. INTRODUCTION

Wireless sensor networks have sparked the research interest of the scientific community in the last decade due to their numerous applications. Mobile robotic platforms are spread in areas in order to distributively survey different physical quantities via their on–board sensors [1, 2]. Mobility of the nodes is a crucial advantage in comparison to static networks; in most cases the latter are deployed randomly in areas of interest, while in the sequel the mobile agents autonomously reform their configuration in a way that certain prespecified criteria are satisfied/optimized [3].

In order for the members of the network to attain sufficient information from their neighbouring nodes for coordination purposes, radio transceivers are attached on their platforms. Subsequently, the nodes can broadcast/receive spatial information amongst them or route crucial sensing data back to a central base station in case of emergency [4, 5].

Assuming networks with homogeneous capabilities among their mobile nodes, distributed coordination algorithms have been developed in order to lead the nodes towards a desired topological configuration [6]. Cooperative action of agents via nearest neighbour rules for flocking purposes have been proposed in [7, 8], while several approaches base their action in geometric characteristics of the Voronoi–tessellation–based assigned spaces among the nodes [9–11].

The distributed nature of the previous approaches is either guaranteed via proper adjustment of the RF power of the nodes' antennas [12], or via incorporation of extra constraints in the control design procedure [13–15]. Coordination of heterogeneous groups of agents considering their sensing abilities has been performed via utilization of alternate schemes for partitioning of the configuration space [16–20].

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In these works the nodes' sensing patterns were assumed circular of fixed radii, though the latter were allowed to be unequal among the members of the swarm.

Most of the works in the literature consider networks where the nodes' sensing footprints are node–centred circular ones [9, 10, 17, 19]; however, in most applications this is rather unusual. This article examines the case where a node's sensing region is any arbitrary convex set, while the node is not demanded to be centred at the pattern's centroid (equivalently to the circle–scenarios). Coordination schemes have been developed in the case where the nodes have wedge–shaped sensing patterns [21, 22], while allowing rotation of the pattern during the nodes' motion. In this work though, since the core for developing the proposed scheme relies in certain Helly–type theorems concerning homothets of planar convex curves [23, 24], the rotation of the sensing domain of each node is not assumed. However, the nodes' sensing domains are not restricted to wedge–shaped ones, but are considered as *any* convex compact planar set, instead.

The rest of the article is organized as follows. In Section II the main assumptions are defined, while the area–coverage problem by a heterogeneous swarm is introduced. Considering non–circular sensor footprints, the tessellation of the space into subsets assigned at the nodes is defined in Section III, while the proposed distributed coordination algorithm is developed in Section IV. Illustrative examples are provided in Section V that emphasize in the advantages of the proposed law over approximating the sensing domain of the nodes as node–centred circular ones. Concluding remarks are discussed in the last section.

II. AREA COVERAGE IN HETEROGENEOUS SWARMS

Assume n in number mobile agents responsible for the sensing coverage of a convex compact region $\Omega \subset \mathbb{R}^2$. Let I_n stand for the set of unique identifiers of the nodes, i.e. $I_n = \{1, 2, \dots n\}$. The nodes positions are denoted by $x_i,\ i \in I_n$. The motion of the latter is assumed to be controlled via the corresponding velocities, i.e.

$$\dot{x}_i = u_i, \quad x_i \in \Omega,\ u_i \in \mathbb{R}^2,\ i \in I_n. \qquad (1)$$

Let the density function $\phi: \Omega \subset \mathbb{R}^2 \to \mathbb{R}_+$ describe the importance (in sensing terms) of any point $x \in \Omega$. Considering area coverage applications, an on–board sensor is mounted on each of the mobile platforms, the sensing domain of which is denoted by $C_i,\ i \in I_n$. In the majority of the works in the existing literature, that domain is assumed circular, node–centred and identical for all nodes (homogeneous isotropic case). In this article, however, the sensing pattern is allowed

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be any compact convex set, not necessarily node–centred, while the scaling factor of the pattern differs among the nodes (i.e. where the heterogeneity lies). Apparently, the sensed domain of the network can be expressed as $C := \Omega \cap \bigcup_{i \in I_n} C_i$.

Considering sensing coverage applications, the nodes should move in such spatial coordinates, so that an area–based criterion is optimized. Let the nodes' coverage performance be defined via the indicator function, i.e. $\mathbf{1}_{C_i}(x;x_i) = 1\,(0)$ if $x \in (\notin) C_i$, $i \in I_n$, for any sensing domain C_i.

The selected performance criterion under optimization is the total area sensed by the robotic network, corresponding to

$$\mathscr{H} = \int_{\Omega} \max_{i \in I_n} \mathbf{1}_{C_i}(x;x_i)\,\phi(x) = \int_{C} \phi(x)\,. \qquad (2)$$

The integration variable is omitted, since it is apparent by the dimension of the integration domain C. The main objective from here on is the design of a spatially distributed coordination algorithm that leads the robotic swarm in obtaining an area–optimal topology.

In this article, the sensing region of the nodes is assumed any arbitrary convex compact set. In some cases, though, the sensors' footprints are by nature concave sets, usually met in internal antennas' radiation patterns [25]. The standard approach for utilization of previously developed distributed coordination schemes [10, 12, 26] is to rely on the largest node–centred disk inscribed in the pattern, and use this set as the node's sensing domain C_i in the sequel (see Fig. 1). In this

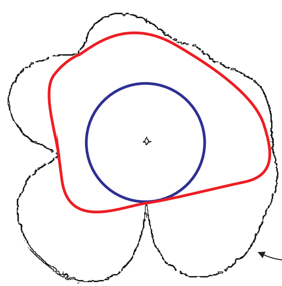

Fig. 1. Graphical example to indicate the benefits arisen by allowing non–node–centred sensing domains in coverage applications. (The node's pattern is the antenna's radiation pattern in T-mote Sky platforms as provided in [25]).

article, though, the demand for node–symmetric regions is relaxed, and larger (area–wise) subsets as the nodes' sensing domains can be used, as shown in Fig. 1, resulting in a more efficient network's optimal state.

III. Partitioning of the Space

Considering distributed coordination of mobile networks for area coverage applications, efficient geometric tools as Voronoi diagrams [27], weighted–Voronoi diagrams [28] and power diagrams [16] (a.k.a. Voronoi–Laguerre diagrams) are utilized in order to partition the space, so that geodesic–based distributed approaches can be applied. The main philosophy is that the space under consideration is partitioned into n subsets, assigned at the mobile agents responsible for coverage of Ω.

As discussed in the previous section, the usual approach for overcoming the anisotropy in the (original) sensing patterns of the nodes is to approximate them with circular node–centred ones. Thus, already existing tessellation techniques can be used to divide the space among the nodes. More specifically, when all nodes have identical patterns (homogeneous case), standard Voronoi diagram can be used, while in the case where the approximated circles have different radii (due to the uneven scaling among the nodes' patterns), optimal partitioning for the "approximated network" is achieved via power diagram.

Figure 2 represents graphically the way that the space is partitioned and assigned among the nodes for two networks. In the left part, the network is consisted of identical

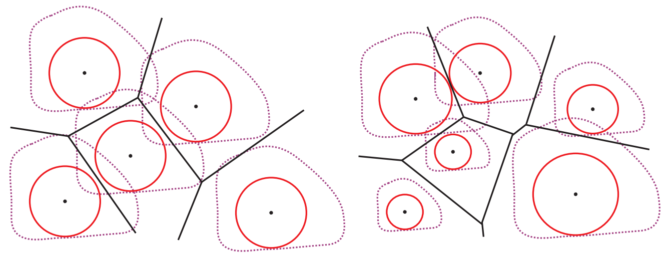

Fig. 2. Graphical representation of partitioning the space based on the inscribed node–centred disks for a homogeneous (via Voronoi diagram) [left] and a heterogeneous network (via power diagram) [right].

nodes (homogeneous case) and standard Voronoi partitioning is utilized, while in the right part of the figure, due to the unevenness in the scaling of the sensing patterns (and consequently in the approximated radii, too), power diagrams are used to tessellate the space. One can verify, that although these tessellations are optimal assuming circular node–centred patterns, the same does not hold if we observe the original (anisotropic non–circular) footprints.

Thus, since the sensing regions of the nodes are arbitrary, an alternative way for space partitioning is sought. This need arises from the fact that distance–based tessellations are formed strictly on the nodes coordinates (and/or the radii of the approximated inscribed disks), while the coverage performance indices $\mathbf{1}_{C_i}$ in (2) are not node–symmetric in our case. Before proceeding to the design of the partitioning scheme, the following preliminaries on Helly–type properties for planar convex curves are needed.

Definition 1: A planar convex curve $\mathscr{C}$ is defined as the boundary of a proper convex subset $D \subset \mathbb{R}^2$, i.e. $\mathscr{C} = \partial D$. A strictly convex curve is a convex curve containing no line segment.

Definition 2: Given a set $D \subset \mathbb{R}^2$, any set of the form $\lambda D + \upsilon$, $\lambda > 0$, $\upsilon \in \mathbb{R}^2$ is a homothet of D. Equivalently are defined the homothets of a curve $\mathscr{C}$.

Lemma 1: [23, 24] Any two distinct homothets of a planar strictly convex curve $\mathscr{C}$ have at most two intersection points.

Let $\mathscr{W} = \mathscr{O} \cup \{W_i,\ i \in I_n\}$ be a tessellation of the sensed space C, where the sets W_i are parts of the latter assigned at node i, while $\mathscr{O}$ are possibly existing regions in C which are not assigned at any node. Each cell W_i is defined as

$$W_i = \bigcap_{j \in I_n} W_{ij}, \tag{3}$$

where the sets W_{ij} are convex compact subsets of C defined in the sequel.

Definition 3: Given two arbitrary nodes i, j, let $\partial C_i, \partial C_j$ be the minimum–length strictly convex curves enclosing their sensing domains. By denoting as $|\cdot|$ the cardinality of the set–argument, the sets W_{ij} are defined as follows:

- If $C_i \subset C_j$, then $W_{ij} = \emptyset$.
- Else if $\left|\partial C_i \cap \partial C_j\right| \leq 1$, then $W_{ij} := C_i \cap \Omega$.
- Otherwise, i.e. $\left|\partial C_i \cap \partial C_j\right| = 2$ (Lemma 1), let $x_{ij}^{(1)}, x_{ij}^{(2)}$ be the intersection points of the curves. The latter define a separating line, that tessellates $C_i \cup C_j$ into two disjoint compact subsets, $C_{ij}^{(1)}, C_{ij}^{(2)}$, respectively. Then

$$W_{ij} := C_{ij}^{(\ell)} \cap \Omega \;\Leftrightarrow\; C_{ij}^{(\ell)} \subseteq C_i, \quad \ell \in \{1,2\}. \tag{4}$$

Apparently, the domain W_{ji} is then defined univocally as $W_{ji} := \Omega \cap \left\{C_i \cup C_j\right\} \setminus W_{ij}$.

Figure 3 depicts graphically the way that the sensed space of a network consisted of seven nodes is partitioned. At

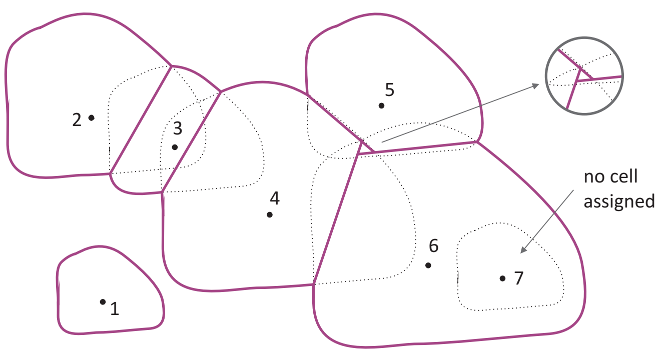

Fig. 3. Illustrative example of the partitioning of the sensed domain of a network via Definition 3.

first, one can verify that if an arbitrary node i (take for example node 1) is compared with any other node j for which $\left|\partial C_i \cap \partial C_j\right| \leq 1$ holds, then this comparison does not affect W_i, considering (3) and Definition 3. As far as concerns the case of two intersection points, the nodes' sensing domain is split correspondingly, via the line that connects the two points where their boundaries intersect. Furthermore, by observing the region among the nodes 4–5–6, one can verify that $\{W_i, i \in I_n\}$ is not a tessellation of the sensed space C, since there exists a part of the plane that is sensed by the nodes, although it is not assigned to any of them. Let us call the set of these regions as *sole cells*, denoted as $\mathscr{O}$, previously.

Remark 1: Since $\mathscr{O} = C \setminus \{W_i, i \in I_n\}$, then $\mathscr{W} = \mathscr{O} \cup \{W_i,\ i \in I_n\}$ consists a tessellation of C.

IV. Spatially Distributed Coordination Algorithm

The core objective of this section is the design of a distributed coordination law that leads the network in a covered–area–optimum spatial topology. More specifically, considering (1), the nodes' control actions u_i are determined so that the nodes' trajectories represent a gradient ascending flow of the criterion under optimization, $\mathscr{H}$, while only local information is required, i.e. spatial information from a subset of neighbouring nodes. Each node identifies the subset of Ω that is simultaneously sensed and assigned by/to itself, while the control action is based on the geometric characteristics of this set, i.e. W_i. At this stage, let us state the main result of the article in the following Theorem, the structure of which is based on that of [10].

Theorem 1: Considering a heterogeneous mobile sensor network consisted of nodes with unevenly scaled arbitrarily convex sensing patterns, governed by the kinodynamics described in (1), the coordination scheme

$$u_i = \int_{\partial W_i \cap \partial C_i} \phi\, n_i, \quad i \in I_n, \tag{5}$$

maximizes the performance criterion (2) along the nodes' trajectories in a monotonic manner, leading in an area–optimal configuration of the network.

Proof: Considering (2), along with Remark 1, and taking into account that the sets $W_i,\ i \in I_n$ are mutually disjoint, the criterion under optimization is written as

$$\mathscr{H} = \int_C \phi = \int_{\mathscr{O}} \phi + \sum_{i \in I_n} \int_{W_i} \phi.$$

Taking the partial derivative of $\mathscr{H}$ with respect to x_i we have

$$\frac{\partial \mathscr{H}}{\partial x_i} = \frac{\partial}{\partial x_i}\int_{\mathscr{O}} \phi + \frac{\partial}{\partial x_i}\int_{W_i} \phi + \sum_{j \neq i} \frac{\partial}{\partial x_i}\int_{W_j} \phi.$$

At this point, the generalized Leibniz integral rule [29] is utilized. Since infinitesimal motion of x_i may affect W_j *iff* $\partial W_i \cap \partial W_j \neq \emptyset$, while $\partial \mathscr{O}$ is affected by x_i at $\partial \mathscr{O} \cap \partial W_i$, the above expression can be rewritten as

$$\frac{\partial \mathscr{H}}{\partial x_i} = \int_{\partial W_i \cap \partial \mathscr{O}} \upsilon_0^i n_0\, \phi + \int_{\partial W_i} \upsilon_i^i n_i\, \phi + \sum_{j \neq i} \int_{\partial W_i \cap \partial W_j} \upsilon_j^i n_j\, \phi,$$

where n_i (n_j) is the outward unit normal at ∂W_i (∂W_j) and $\upsilon_i^i, \upsilon_j^i$ stand for the Jacobian matrices with respect to x_i of the points $x \in \partial W_i,\ x \in \partial W_j$, respectively, while n_0 stands for the outward unit normal at $\partial \mathscr{O}$ and υ_0^i is defined as $\upsilon_0^i(x) := \frac{\partial x}{\partial x_i},\quad x \in \partial \mathscr{O},\ i \in I_n$.

Considering the second integral, parts of ∂W_i may possibly lie either on $\partial \Omega$, ∂C_i, $\partial \mathscr{O}$ (possible neighbouring sole cells), or on the boundary of neighbouring sensed assigned cells, i.e. $\bigcup_{j \neq i} \partial W_i \cap \partial W_j$. But, since the above sets are disjoint, $\frac{\partial \mathscr{H}}{\partial x_i}$ can be written as

$$\begin{aligned}\frac{\partial \mathscr{H}}{\partial x_i} = &\int_{\partial W_i \cap \partial \mathscr{O}} \upsilon_0^i n_0\, \phi + \int_{\partial W_i \cap \partial \Omega} \upsilon_i^i n_i\, \phi + \\ &\int_{\partial W_i \cap \partial C_i} \upsilon_i^i n_i\, \phi + \int_{\partial W_i \cap \partial \mathscr{O}} \upsilon_i^i n_i\, \phi + \\ &\sum_{j \neq i} \int_{\partial W_i \cap \partial W_j} \upsilon_i^i n_i\, \phi + \sum_{j \neq i} \int_{\partial W_i \cap \partial W_j} \upsilon_j^i n_j\, \phi.\end{aligned}$$

As far as the second integral is concerned, it is apparent that $\upsilon_i^i = 0$ at $x \in \partial W_i \cap \partial \Omega$ since all $x \in \partial\Omega$ remain unaltered by infinitesimal motions of x_i, considering that the region of interest Ω is fixed and independent of the nodes' positions. Grouping integrals over the same sets results in

$$\frac{\partial \mathscr{H}}{\partial x_i} = \int_{\partial W_i \cap \partial \mathscr{O}} \left(\upsilon_0^i n_i + \upsilon_i^i n_i\right) \phi + \int_{\partial W_i \cap \partial C_i} \upsilon_i^i n_i \phi + \sum_{j \neq i} \int_{\partial W_i \cap \partial W_j} \left(\upsilon_i^i n_i + \upsilon_j^i n_j\right) \phi.$$

Considering the first (third) integral, one can remarkably verify that $\upsilon_0^i(x) n_0(x) = -\upsilon_i^i(x) n_i(x)$ at $x \in \partial W_i \cap \partial \mathscr{O}$ ($\upsilon_i^i(x) n_i(x) = -\upsilon_j^i(x) n_j(x)$ at $x \in \partial W_i \cap \partial W_j$), respectively. More specifically, $\upsilon_0^i(x) = \upsilon_i^i(x)$, $\partial W_i \cap \partial \mathscr{O}$ and $\upsilon_i^i(x) = \upsilon_j^i(x)$, $\partial W_i \cap \partial W_j$ hold by definition of the Jacobian matrices υ, as provided previously, while $n_0 = -n_i$ at $x \in \partial W_i \cap \partial \mathscr{O}$ and $n_i = -n_j$ at $x \in \partial W_i \cap \partial W_j$, since the aforementioned sets are the common part of the lines separating $W_i, \mathscr{O}$ and W_i, W_j, respectively.

With respect to the parts of the sensing boundary curve that lie in W_i, i.e. $\partial W_i \cap \partial C_i$, any point x on that set moves along the direction of x_i with the same rate, assuming pure translational motion of x_i. Hence, it is apparent that $\upsilon_i^i(x) = \mathbb{I}_N$ at $x \in \partial W_i \cap \partial C_i$, where $\mathbb{I}_N$ stands for the $N \times N$ identity matrix.

Substituting the above into the former expression of $\frac{\partial \mathscr{H}}{\partial x_i}$ results in vanishing the first and third integral, i.e.

$$\frac{\partial \mathscr{H}}{\partial x_i} = \int_{\partial W_i \cap \partial C_i} n_i \phi.$$

Thus, recalling (1), the proposed control law (5), along with the above expression for $\frac{\partial \mathscr{H}}{\partial x_i}$ it is concluded that (5) leads to a gradient flow of $\mathscr{H}$ along the nodes trajectories, while $\mathscr{H}$ increases monotonically, since

$$\frac{d\mathscr{H}}{dt} = \sum_{i \in I_n} \frac{\partial \mathscr{H}}{\partial x_i} \cdot \dot{x}_i = \sum_{i \in I_n} \left\| \int_{\partial W_i \cap \partial C_i} n_i \phi \right\|^2 \geq 0.$$

■

The main concept behind coordination scheme (5) is that the network should organize its action in a distributed manner; thus, an arbitrary node i should be able to identify the nodes in its neighbourhood and evaluate the part of the sensed region that is assigned to it, i.e. W_i, without requiring to acquire knowledge of the state/configuration of the whole network.

Lemma 2: The subset of nodes required for distributed evaluation of W_i are the ones in range of

$$\sup\{\|x_i - x_j\| \mid |\partial C_i \cap \partial \bar{C}| = 1\},$$

where $\bar{C}$ is the largest sensing footprint in the network (i.e. the one with the largest scaling factor)

Proof: Considering Definition 3, the worst case for a node is to share exactly one intersection point with the node of the network that has the largest sensing set. Consequently, the corresponding bound to ensure distributedness of the partitioning scheme (and consequently the proposed control law) is the largest of these distances among the two nodes (worst case scenario topology). ■

It should be noted that, although the bound of Lemma 2 differs among the nodes, each node can determine it a–priori and not during the coordination stage, since they are only dependent on the nodes' sensing pattern.

V. Simulation Studies

In this section, the control scheme proposed in this article is verified via simulation studies. The nodes in the networks under consideration are initially deployed randomly in a convex compact planar region. The latter is selected identical to that used in [10] for consistency. As far as the sensing pattern of the nodes is concerned, a non–trivial arbitrary convex one is selected. More specifically the latter is chosen as the (approximated) maximal convex set inscribed in the interior of T–mote Sky footprints [25], as depicted graphically initially in the left part of Fig. 1.

Remark 2: In the case where the boundaries of the nodes' sensing domains are polygons instead of strictly convex curves, then the result of Lemma 1 does not hold. In such cases, the aforementioned method under–approximates slightly the sensing domain, in order to provide the necessary conditions for Theorem 1.

As discussed in section II, most control techniques in the existing literature consider networks with circular node–centred sensing footprints. However, in cases where the sensors are anisotropic, a standard approach is to approximate their patterns with the maximal node–centred disk inscribed in the sensor footprint. In this article, the control scheme proposed in section IV is compared via simulations to the one proposed by the authors in [10], i.e.

$$\tilde{u}_i = \int_{\partial B_i^r \cap \partial V_i^r} n_i, \quad i \in I_n, \tag{6}$$

where the $\sim$–sign is used in order to distinguish between the control laws. In the former expression, $B_i^r := \{x \mid \|x - x_i\| \leq r\}$ represents a disc of radius r around x_i, while $V_i^r := V_i \cap B_i^r$ (r–limited Voronoi cell) is the intersection of the aforementioned disc with the assigned Voronoi/power cell. The reader is encouraged to refer to [10] for more details.

It should be stated clearly that the coordination scheme (6) is proven to lead to area–optimal network configuration when the nodes sensing patterns are circular. The scope of the comparison that follows between the two control schemes is to show the benefits of the proposed law when dealing with anisotropic sensors. In fact, when $C_i \equiv B_i^r$, the coordination scheme defined in (5) degenerates in (6), while the partitioning $\mathscr{W}$ is the standard Voronoi tessellation or power diagram, equivalently.

In the scenario examined, the network is consisted of $n = 9$ heterogeneous nodes, initially deployed randomly in a region of Ω. Taking into account the area of Ω, $\int_\Omega \phi = 5.0809$ units^2, along with that of the sensor footprints, the nodes can achieve coverage ratio, in a best case scenario, equal to 59.5%. This case is when the nodes can self–position themselves (if possible) in a way where there is no overlapping among their patterns or with the boundary Ω.

In the case where control law (6) is to be applied, each sensing domain of the nodes is firstly approximated as a node–centred disc (maximal inscribed node–centred one). In this case, the nodes' initial configuration, their evolution through time, along with the final network's state, are shown in the top row of Fig. 4, in this order.

It is apparent that the nodes have positioned themselves in a way that the discs do not overlap among them or with the boundary of Ω. However, although the coordination scheme assumes uniform omnidirectional coverage performance, one can verify that the (real) coverage performance of the network via the patterns C_i can be increased even more. Figure 5 depicts the evolution the network's coverage ratio, i.e. $\mathscr{H}/\int_\Omega \phi$

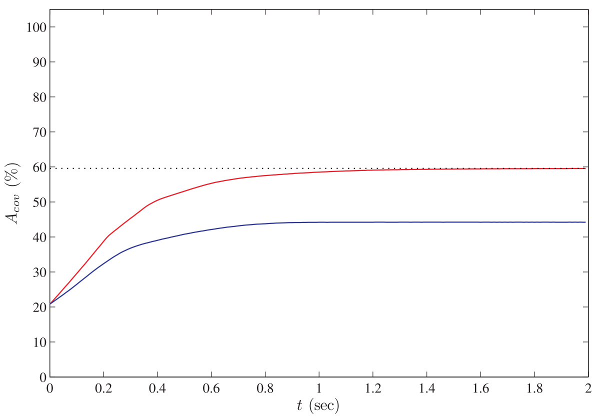

Fig. 5. Percentage of sensed area w.r.t. time, when control schemes (6) [blue line] and (5) [red line] are applied, respectively. The black dotted line represents the maximum possible coverage ratio.

w.r.t. time when coordination scheme (6) is applied [blue solid line]. It is apparent that, although the network's performance increases from 21% to 44% of Ω, this percentage is not satisfactory considering the maximum possible coverage ratio 59.5% (best case scenario). It should be noted, however, that this deficiency is due to the approximation of the sensing patterns as discs and not due to the control law.

On the other hand, when the proposed control action (5) is applied—without the need for "altering"/approximating the nodes' patterns—one can observe the behaviour of the network in the bottom row of Fig. 4. Comparing the figures in the middle column (i.e. network evolution w.r.t. time via the two control laws), one can verify that the nodes tend to "spread" more uniformly in this case, since they (indirectly) try to avoid overlapping amongst their patterns. The red solid line in Fig. 5 depicts the way that the coverage ratio of the network increases to its optimal value, which in this case is equal to the maximum possible one, i.e. around 59%, while monotonic increase of the latter is guaranteed.

VI. Conclusions

In this paper, a geometric approach for distributed area coverage optimization in heterogeneous anisotropic sensor networks is proposed. Restriction for circular node–centred sensing domain of the nodes is relaxed, allowing it to be any convex compact planar set, while heterogeneity lies in the scaling factor of the aforementioned pattern among the nodes. The sensed space is partitioned based on certain Helly–type theorems for homothetic convex planar curves, while a distributed coordination law for leading the nodes towards their optimal positions autonomously is developed. Efficiency of the proposed scheme is further confirmed by simulation studies, in comparison to circular–approximation–based ones.

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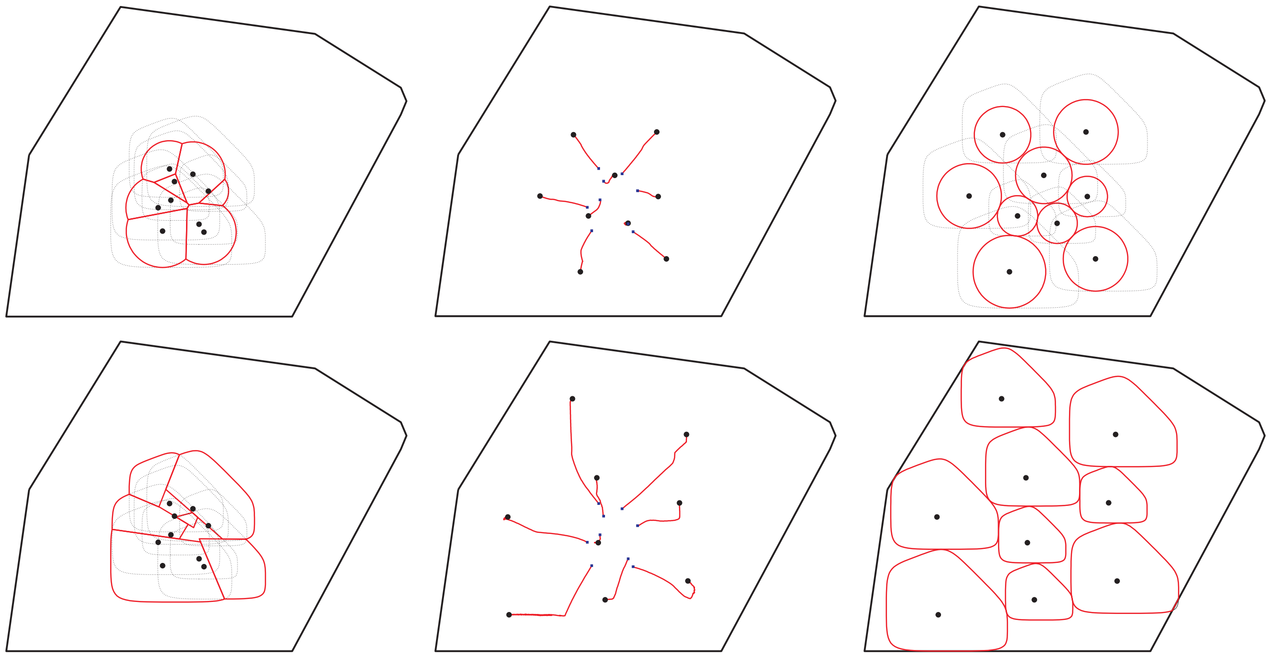

Fig. 4. Coordination results derived via control schemes (6) [top row] and (5) [bottom row], respectively. [Left column] Initial network configuration. [Middle column] Network evolution through time. The black circles (blue squares) represent the nodes' final (initial) positions. [Right column] Final network state.

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